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ELECTRICITY, ELECTROMETER MAGNETISM, AND ELECTROLYSIS.

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University of California

ELECTRICITY

Intro-
duction.

THE word *Electricity* is derived from the Greek word ἤλεκτρον, meaning *amber*. The term was invented by Gilbert,¹ who used it with reference to the attractions and repulsions excited by friction in certain bodies of which amber may be taken as the type. To the cause of these forces was given the name *Electricity*; and out of the study of these and kindred phenomena arose the science of electricity, of which it is the purpose of the present article to give a brief outline.

The science has been divided into three branches—*Electrostatics*, which deals with electricity at rest; *Electrokinetics*, which considers the passage of electricity from place to place; and *Electromagnetism*, which treats of the relation of electricity to magnetism. We shall, however, make no attempt to adhere to this division, but shall exhibit the different parts of the subject in such order and connection as seems most clear and natural in the present state of the science. For the sake of the non-scientific reader we prefix a brief history² of the science of electricity, wherein mention is made of some of the more striking electrical discoveries and of the steps by which our knowledge of the subject has advanced to its present condition.

HISTORICAL SKETCH.

Thales, 600 B.C. The name of the philosopher who first observed that amber when rubbed possesses the property of attracting and repelling light bodies has not been handed down to our times. Thales of Miletus is said to have described this remarkable property, and both Theophrastus (321 B.C.) and Pliny (70 A.D.) mention the power of amber to attract straws and dry leaves. The same authors speak of the *lapis lycurinus*, which is supposed to be a mineral called *tourmaline*, as possessing the same property. The electricity of the torpedo was also known to the ancients. **Animal Electricity.** Pliny informs us, that when touched by a spear it paralyzes the muscles and arrests the feet, however swift; and Aristotle adds that it possesses the power of benumbing men, as well as the fishes which serve for its prey. The influence of electricity on the human body, and the electricity of the human body itself, were also known in ancient times. Anthero, a freedman of Tiberius, was cured of the gout by the shocks of the torpedo; and Wolimer, the king of the Goths, was able to emit sparks from his own body. Eustathius, who records this fact, also states that a certain philosopher, while dressing and

undressing, emitted occasionally sudden crackling sparks, while at other times flames blazed from him without burning his clothes. Such are the scanty gleanings of electrical knowledge which we derive from the ancient philosophy; and though several writers of the Middle Ages have made occasional references to these facts, and even attempted to speculate upon them, yet they added nothing to the science, and left an open field for the researches of modern philosophers.

Dr Gilbert of Colchester may be considered as the founder of the science, as he appears to have been the first philosopher who carefully repeated the observations of the ancients, and applied to them the principles of philosophical investigation. In order to determine if other bodies possessed the same property as amber, he balanced a light metallic needle on a pivot, and observed whether or not it was affected by causing the excited or rubbed body to approach to it. In this way he discovered that the following bodies possess the property of attracting light substances:—amber, gages or jet, diamond, sapphire, carbuncle, rock-crystal, opal, amethyst, vincentina or Bristol stone, beryl, glass, paste for false gems, glass of antimony, alags, belemnites, sulphur, gum-mastic, sealing-wax of lac, hard resin, arsenic, rock salt, mica, and alum. These various bodies attracted, with different degrees of force, not only straws and light films, but likewise metals, stones, earths, wood, leaves, thick smoke, and all solid and fluid bodies. Among the substances which are not excited by friction Gilbert enumerated emerald, agate, carnelian, pearls, jasper, calcedony, alabaster, porphyry, coral, marble, Lydian stone, flints, hematites, smyris (emery or corundum), bones, ivory, hard woods, such as cedar, ebony, juniper, and cypress, metals, and natural magnets. Gilbert also discovered that the state of the atmosphere affects the production of electricity; dryness with north or east wind being a favourable condition, while moisture with south wind is unfavourable. An account of Gilbert's experiments will be found in his book *De Magnete*, lib. ii. cap. 2.

Robert Boyle added many new facts to the science of electricity, and he has given a full account of them in (1672-1675) his *Experiments on the Origin of Electricity*. By means of a suspended needle, he discovered that amber retained its attractive virtue after the friction which excited it had ceased; and though smoothness of surface had been regarded as advantageous for excitation, yet he found a diamond which in its rough state exceeded all the polished ones and all the electrics which he had tried, having been able to move a needle three minutes after he had ceased to rub it. He found also that heat and *terison* (or the cleaning or wiping of any body) increased its susceptibility of excitation; and that if the attracted body were fixed, and the attracting body movable, their mutual approach would still take place. To Gilbert's list of "electrics" Boyle added the resinous cake which remained after evaporating one-fourth part of good oil of turpentine, the dry mass which remains after distilling a mixture of petroleum and

¹ *De Magnete Magneticisque Corporibus*.

² A portion of this historical sketch was written by Sir David Brewster, and formed the introduction to his article "Electricity" in last edition of the *Encyclopædia*. It has been modified by suppressions and alterations here and there, and by large additions at the end which were thought necessary to make it suit the present state of science. For the sake of the student in search of original sources of information, pretty copious reference to such has been added throughout. Valuable for information of this kind the student will find Bress's *Reibungselectricität*, Young's *Natural Philosophy*, Wiedemann's *Galvanismus*, and the recent work on electricity by Prof. Mascart, of the Collège de France.

strong spirit of nitre, glass of lead, caput mortuum of amber, white sapphire, white amethyst, diaphanous ore of lead, carnelian, and a green stone supposed to be a sapphire.

Otto von Guericke (1602-68). To these discoveries of Boyle his contemporary Otto von Guericke added the highly important one of *electric light* (*Experimenta Nova Magdeburgica*, lib. iv. cap. 15). Having cast a globe of sulphur in a glass sphere, and broken off the glass, he mounted the sulphur ball upon a revolving axis, and excited it by the friction of the hand. By this means he discovered that light and sound accompanied strong electrical excitation, and he compares the light to that which is exhibited by breaking lump sugar in the dark. With this powerful apparatus Guericke verified on a greater scale the results obtained by his predecessors, and obtained several new ones of very considerable importance. He found that a light body, when once attracted by an excited electric, was repelled by it, and was incapable of a second attraction until it had been touched by some other body; and that light bodies suspended within the sphere of influence of an excited electric possessed the same properties as if they had been excited.

Newton (1643-1727). To our illustrious countryman Sir Isaac Newton the science of electricity owes some important observations. He used in his electrical experiments a globe of glass rubbed by the hand instead of the sulphur globe of Von Guericke. It would appear that Newton was the first to use *glass* in this way (*Optics*, query 8th). We owe also to Sir Isaac a beautiful experiment on the excitation of electricity which has since become very popular. Having fixed a round disc of glass in a short brass cylinder, he placed small pieces of thin paper within the cylinder and upon a table, so that the lower surface of the glass was one-eighth of an inch distant from the table. He then rubbed the upper surface of the glass, and he observed the pieces of paper "leap from one part of the glass to the other, and twirl about in the air." This experiment, after a previous unsuccessful trial, was repeated by the Royal Society in 1676 (*Brewster's Life of Newton*, p. 307).

Hawksbee (1705). Francis Hawksbee, one of the most active experimental philosophers of his age, added many new facts to the science. In 1705 he communicated to the Royal Society several curious experiments on what he calls "the mercurial phosphorus." He showed that light could be produced by passing common air through mercury placed in a well-exhausted receiver. The air rushing through the mercury, blew it up against the sides of the glass that held it, "appearing all around like a body of fire, consisting of abundance of glowing globules." The phenomenon continued till the receiver was half full of air. These phenomena had been observed in the Torricellian vacuum before Hawksbee's time, and various explanations suggested. He suspected that they were due to electricity, and remarked their resemblance to lightning. Like Newton he used a revolving glass globe rubbed by the hand to generate electricity. Besides the experiment above alluded to he made many others on the electric light and on the attractions of electrified bodies. Descriptions of these will be found in his *Physico-Mechanical Experiments*, 1709, and in several memoirs in the *Philosophical Transactions* about 1707.

About the same time Dr Wall (*Phil. Trans.*, 1708) observed the spark and crackling sound accompanying the electrical excitation of amber, and compared them to thunder and lightning.

Stephen Gray (1696-1736). One of the most ardent experimentalists of his time was Stephen Gray, a Fellow of the Royal Society. In his first paper, published in 1720, he showed that electricity could be excited by the friction of feathers,

hair, silk, linen, woollen, paper, leather, wood, parchment, and gold-beaters' skin. Several of these bodies exhibited light in the dark, especially after they had been warmed; but all of them attracted light bodies, and sometimes at the distance of eight or ten inches. An epoch was made in the history of electricity by the discovery of Gray in 1729, that certain bodies had, while others had not, the power of conveying electricity from one body to another, *i.e.*, in modern phrase, *conducting* it. Gray experimented with a glass tube, into the ends of which were fastened two corks; into one of these he fastened a fir rod, and to the end of the rod an ivory ball. On rubbing the glass he found that the ball attracted the light bodies as vigorously as the glass itself. He made a variety of experiments with rods of different length, and with a packthread, by which he suspended his ball from the balcony of an upper story of his house, all with the same result. He then attempted to carry the electricity horizontally on a packthread which he suspended with hempen strings; but the experiment failed. On the occasion of a repetition of the experiments at the house of his friend Wheeler, silk strings were suggested as a support, and found to answer, while metal wires failed. Gray and Wheeler were thus led to the conclusion that it was the material of the supports that was in question, and that whereas packthread had, silk had not the power of transmitting electricity to a distance. Gray and Wheeler managed, by supporting a packthread by silk loops, to convey electricity from a piece of rubbed glass to a distance of 886 feet. The conducting power of fluids, and of the human body, was established by Gray. He also made many curious experiments on the electrical properties of resinous cakes, which he allowed to cool and harden in the ladles in which they had been melted. For an account of these and others the student is referred to memoirs in the *Philosophical Transactions* for 1731, 1735, &c.

Desaguliers made many experiments confirming Gray's conclusions, and found that bodies that have the property of being electrically excitable by friction, or *electrics per se*, have not the power of *conduction*; whereas *conductors* are not *electrics per se*. These terms, introduced by him, were useful in bringing into concise and scientific language the discoveries of Gray.

While Gray was pursuing his career of discovery in England, M. Dufay, of the Academy of Sciences, and superintendent of the Royal Botanic Gardens, was actively employed in the same researches. He found that all bodies, whether solid or fluid, could be electrified by an excited tube, by setting them on a glass stand slightly warmed, or only dried; and that those bodies which are in themselves least electrical received the greatest degree of electricity from the approach of the glass tube. He repeated the experiments of Gray, confirming his results, and found that electricity was transmitted more easily along packthread when it was wetted, and that it might be supported upon glass tubes in place of silk lines. In this way he conveyed it along a string 1256 feet long. He suspended by silken strings and electrified a child as Gray had done; and having suspended himself in a similar manner, he discovered that an electrical spark, accompanied with a crackling noise, took place when any other person touched him, and he has described the prickling sensation like the burning from a spark of fire, which is at the same time felt either through the clothes or on the skin. The great discovery of Dufay, however, was that of two different kinds of electricity. He fully recognized the importance of this fundamental fact, and gave the name of *vitreous* electricity to that which is produced by exciting glass, rock-crystal, precious stones, hair of animals, wool, and many other bodies; and the name of *resinous* to that which is produced by exciting resinous

Vitreous and resinous electricity

bodies, such as amber, copal, gum-lac, silk, paper, thread, and a number of other substances. The characteristic of those two electricities was, that a body with vitreous electricity attracted all bodies with resinous electricity, and repelled all bodies with vitreous electricity; while a body with resinous electricity attracted all bodies with vitreous electricity, and repelled all bodies with resinous electricity. Two electrified silk threads, for example, repel each other, and also two electrified woollen threads, but an electrified silk thread will attract an electrified woollen thread. Hence it is easy to determine whether any body possesses vitreous or resinous electricity. If it *attracts* an electrified silk thread, its electricity will be vitreous; if it *repels* it, it will be resinous.

Gray repeated and varied the experiments of Dufay, and made many new ones. Like Hawksbee and Dr Wall, he recognized the similarity between the phenomena of electricity and those of thunder and lightning; and he expresses a hope "that there may be found out a way to collect a greater quantity of electric fire, and consequently to increase the force of that power, which, by several of these experiments, *si licet magnis componere parva*, seems to be of the same nature with thunder and lightning."

The discoveries which we have now recounted began to rouse the activity of the German and Dutch philosophers. To the electrical machine used by Newton and Hawksbee, Professor Boze of Wittenberg added the *prime* conductor, which at first consisted of an iron or tin tube supported by a man standing upon cakes of rosin; but it was afterwards suspended by silken strings. Professor Winkler of Leipsic substituted a *cushion* in place of the hand for exciting the revolving globe; and Professor Gordon of Erfurt, a Scotch Benedictine monk, first used a glass cylinder, eight inches long and four broad, which he caused to revolve by means of a bow and string. By these means electrical sparks of great size and intensity were produced, and by their aid various combustible substances, both fluid and solid, were inflamed. In 1744 M. Ludolph of Berlin succeeded in firing, by the electrical spark, the ethereal spirit of Frobenius. Winkler did the same by a spark from his finger; and he succeeded in inflaming French brandy and other weaker spirits after they had been heated. Gordon kindled spirits by a jet of electrified water. Dr Miles inflamed phosphorus by the electric spark; and oil, pitch, and sealing-wax, when strongly heated, were set on fire by similar means. We refer the student for lists of the works of the philosophers just mentioned to the admirable bibliography given by Young, *Natural Philosophy*, p. 515.

These striking effects were all produced by the electricity obtained immediately from an excited electric; but a great step was now made in the science by the discovery of a method of accumulating and preserving electricity in large quantities. The author of this great invention is not distinctly known; but there is reason to believe that a monk of the name of Kleist, a person of the name of Cuneus, and Professor Muschenbroeck of Leyden had each the merit of an independent inventor. The invention by which this accumulation was effected was called the *Leyden Jar* or *Phial*, because it was principally in Leyden that it was either invented or tried. Having observed that excited electrics soon lost their electricity in the open air, and that their loss was accelerated when the atmosphere was charged with moisture or other conducting materials, Muschenbroeck conceived that the electricity of bodies might be retained by surrounding them with bodies which did not conduct it. In putting this idea to the test of experiment, he electrified some water in a glass bottle, and a communication having been made between the water and the *prime* conductor, the assistant, who was holding the bottle, on trying to disengage the communicating wire, received a

sudden shock in his arms and breast, and thus established the efficacy of the Leyden jar.

Sir William Watson made some important experiments at this period of our history (*Memoirs in Phil. Trans.* about 1747). He succeeded in firing gunpowder by the electric spark; and by mixing the gunpowder with a little camphor he discharged a musket by the same power. He also fired hydrogen by the electric spark; and he kindled both spirits of wine and hydrogen by means of a drop of cold water, and even with ice. In the German experiments the fluid or solid to be inflamed was set on fire by an electrified body; but Sir William Watson placed the fluid in the hands of an electrified person, and set it on fire by causing a person not electrified to touch it with his finger. Sir William Watson first observed the flash of light which attends the discharge of the Leyden phial, and it is to him that we owe the present improved form of the Leyden phial, in which it is coated both without and within with tinfoil. Dr Bevis indeed had suggested the outside coating, and at Smeaton's recommendation, he coated a pane of glass on both sides, and within an inch of the edge, with tinfoil; but still the idea of coating the jar doubly belongs to Sir William Watson.

A party of the Royal Society, with the president at their head, and Sir William Watson as their chief operator, entered upon a series of magnificent experiments, for the purpose of determining the velocity of the electric fluid, and the distance to which it could be conveyed. The French savans had conveyed the influence of the Leyden jar through a circuit of 12,000 feet; and in one case the basin at the Tuileries, containing about an acre of water, formed part of the circuit; but the English philosophers made a more complete series of experiments, of which the following were the results:—

1. That in all their operations, when the wires have been properly conducted, the electrical commotions from the charged phial have been very considerable only when the observers at the extremities of the wire have touched some substance readily conducting electricity with some part of their bodies.
2. That the electrical commotion is always felt most sensibly in those parts of the bodies of the observers which are between the conducting wires and the nearest and the most non-electric substance, or, in other words, so much of their bodies as comes within the electrical circuit.
3. That on these considerations we infer that the electrical power is conducted between these observers by any non-electric substances which happen to be situated between them, and contribute to form the electrical circuit.
4. That the electrical commotion has been perceptible to two or more observers at considerable distances from each other, even as far as two miles.
5. That when the observers have been shocked at the end of two miles of wire, we infer that the electrical circuit is four miles, viz. two miles of wire, and the space of two miles of the non-electric matter between the observers, whether it be water, earth, or both.
6. That the electrical commotion is equally strong, whether it is conducted by water or dry ground.
7. That if the wires between the electrifying machine and the observers are conducted on dry sticks, or other substances non-electric in a slight degree only, the effects of the electrical power are much greater than when the wires in their progress touch the ground, or moist vegetables, or other substances in a great degree non-electric.
8. That by comparing the respective velocities of electricity and sound, that of electricity, in any of the distances yet experienced, is nearly instantaneous.

In the following year these experiments were resumed with the view of ascertaining the absolute velocity of electricity at a certain distance, and it was found "that through the whole length of a wire 12,276 feet the velocity of electricity was instantaneous."

The theory of positive and negative electricity which was afterwards elaborated by Franklin, was distinctly announced by Sir W. Watson. He lays it down as a law that in electrical operations there is an afflux of "electric fluid" to the globe and the conductor, and also an efflux of the same

case,
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Sir Wm.
Watson
(1715-
1807).

Experi-
ments of
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matter from them. In the case of two insulated persons, the one in contact with the rubber and the other with the conductor, he observed that either of them would communicate a much stronger spark to the other than to any bystander. The electricity of the one, he says, became more rare than it is naturally, and that of the other more dense, so that the density of the electricity in the two insulated persons differed more than that between either of them and a bystander.

A variety of interesting experiments were made about this time by Le Monnier, Nollet, Winckler, Ellicott, Jallabert, Boze, Menon, Smeaton, and Miles. In 1746 Le Monnier confirmed the result previously obtained by Gray, that electricity is communicated to homogeneous bodies in proportion to their surfaces only. Boze discovered that capillary tubes which discharged water by drops afforded a continuous stream when electrified. The Abbé Nollet (*Essai sur l'Electricité*, 1746; *Recherches*, 1749; *Lettres*, 1753), the friend and coadjutor of Dufay, ascertained that electricity increases the natural evaporation of fluids, and that the evaporation is hastened by placing them in non-electric vessels. Jallabert confirmed the result previously obtained by Watson, that electricity passes through the substance of a conducting wire, and not along its surface. Smeaton found that the red hot part of an iron bar could be as strongly electrified as the cold parts on each side of it. Dr Miles kindled common spirits by a stick of black sealing-wax excited by dry flannel. Ellicott conceived that the particles of the electric fluid repel each other, while they attract those of all other bodies. Mowbray concluded that the vegetation of two myrtles was hastened by electrifying them,—a result which Nollet confirmed in the case of vegetating seeds. The Abbé Menon found that cats, pigeons, sparrows, and chaffinches lost weight by being electrified for five or six hours, and that the same result was true of the human body; and hence it was concluded that electricity augments the insensible perspiration of animals.

Franklin
(1706-
90).

A high place in the history of electricity must be allotted to the name of Dr Benjamin Franklin of Philadelphia. His researches did much to extend our theoretical and practical knowledge of electricity, and the clearness and vigour of his style made his writings popular, and spread the study of the subject.

One of the first labours of the American philosopher was to present, in a more distinct form, the theory of positive and negative electricity, which Sir W. Watson had been the first to suggest. He showed that electricity is not created by friction, but merely collected from its state of diffusion through other matter by which it is attracted. He asserted that the glass globe, when rubbed, attracted the electrical fire, and took it from the rubber, the same globe being disposed, when the friction ceases, to give out its electricity to any body which has less. In the case of the charged Leyden jar, the inner coating of tinfoil had received more than its ordinary quantity of electricity, and was therefore electrified *positively* or *plus*, while the outer coating of tinfoil having had its ordinary quantity of electricity diminished, was electrified *negatively* or *minus*. Hence the cause of the shock and spark when the jar is discharged, or when the superabundant plus electricity of the inside is transferred by a conducting body to the defective or minus electricity of the outside. This theory of the Leyden phial Franklin established in the clearest manner, by showing that the outside and the inside coating possessed opposite electricities, and that, in charging it, exactly as much electricity is added on one side as is subtracted from the other. The abundant discharge of electricity by points was observed by Franklin in his earliest experiments, and also the power of points to conduct

it copiously from an electrified body. Hence he was furnished with a simple method of collecting electricity from other bodies; and he was thus enabled to perform those remarkable experiments which we shall now proceed to explain. Hawksbee, Wall, and Nollet had successively suggested the similarity between lightning and the electric spark, and between the artificial snap and the natural thunder. Previous to the year 1750 Franklin drew up a statement, in which he showed that all the general phenomena and effects which were produced by electricity had their counterpart in lightning. After waiting some time for the erection of a spire at Philadelphia, by means of which he thought to bring down the electricity of a thunder-storm, he conceived the idea of sending up a kite among the clouds themselves. With this view he made a small cross of two small light strips of cedar, the arms being sufficiently long to reach to the four corners of a large thin silk handkerchief when extended. The corners of the handkerchief were tied to the extremities of the cross, and when the body of the kite was thus formed, a tail, loop, and string were added to it. The body was made of silk to enable it to bear the violence and wet of a thunder-storm. A very sharp pointed wire was fixed at the top of the upright stick of the cross, so as to rise a foot or more above the wood. A silk ribbon was tied to the end of the twine next the hand, and a key suspended at the junction of the twine and silk. In company with his son, Franklin raised the kite like a common one, in the first thunder-storm, which happened in the month of June 1752. To keep the silk ribbon dry, he stood within a door, taking care that the twine did not touch the frame of the door; and when the thunder-clouds came over the kite he watched the state of the string. A cloud passed without any electrical indications, and he began to despair of success. He saw, however, the loose filaments of the twine standing out every way, and he found them to be attracted by the approach of his finger. The suspended key gave a spark on the application of his knuckle, and when the string had become wet with the rain, the electricity became abundant; a Leyden jar was charged at the key, and by the electric fire thus obtained spirits were inflamed, and all the other electrical experiments performed which had been formerly made by excited electrics. In subsequent trials with another apparatus, he found that the clouds were sometimes positively and sometimes negatively electrified, and so demonstrated the perfect identity of lightning and electricity. Having thus succeeded in drawing the electric fire from the clouds, Franklin conceived the idea of protecting buildings from lightning by erecting on their highest parts pointed iron wire or conductors communicating with the ground. The electricity of a hovering or a passing cloud would thus be carried off slowly and silently; and if the cloud was highly charged, the lightning would strike in preference the elevated conductors.

The most important of Franklin's electrical writings are his *Experiments and Observations on Electricity made at Philadelphia*, 1751-54; his *Letters on Electricity*, and various memoirs and letters, *Phil. Trans.*, 1756, 1760, &c.

About the same time that Franklin was making his kite experiment in America, D'Alibard and others in France had erected a long iron rod at Marli, and obtained results agreeing with those of Franklin. Similar investigations were pursued by many others, among whom Father Becaria deserves especial mention.

These experiments were often dangerous, and in one case ^{Death} a fatal accident occurred. Professor Richman of St Petersburg had erected on his house an iron rod to collect the ^{Rich-} electricity of thunder-clouds. On the 6th August 1753, ^{man,} during a thunder-storm, he was observing, along with his friend Sokolow, the indications of an electrometer which ^{1753.}

formed part of his apparatus, when a tremendous thunder-clap burst over the neighbourhood. Richman bent to observe the electrometer; while in this position, his head being a foot from the iron rod, Sokolow saw a globe of bluish fire about the size of the fist shoot from the iron rod to the professor's head, with a report like that of a pistol. The shock was fatal; Richman fell back upon a chest and instantly expired. Sokolow was stupified and benumbed, and the red hot fragments of a metallic wire struck his clothes, and covered them with burnt marks.

One of the most diligent labourers in the field of electrical science was an Englishman, John Canton (*Phil. Trans.*, 1753-54). Before his time it had been assumed as indisputable that the same kind of electricity was invariably produced by the friction of the same electric,—that glass, for example, yielded always *vitreous*, and amber always *resinous* electricity. Having roughened a glass tube by grinding its surface with emery and sheet lead, he found that it possessed vitreous or positive electricity when excited with oiled silk, but resinous electricity when excited with new flannel. He found, in short, that vitreous or resinous electricity might, in certain cases, be developed at will in the same tube, by altering the surfaces of the tube and the exciting rubber. Removing the polish from one half of the tube, he excited the different electricities with the same rubber at a single stroke, and, curiously enough, the rubber was found to move much more easily over the rough than over the polished half. Canton likewise discovered that glass, amber, sealing-wax, and calcareous spar were all electrified positively when taken out of mercury; and hence he was led to the important practical discovery that an amalgam of mercury and tin was most efficacious in exciting glass when applied to the surface of the rubber. Canton discovered, and to a certain extent explained by the then prevalent theory of "electrical atmospheres," the fundamental fact of *electrification by induction*. He also found that the air in a room could be electrified positively or negatively, and might remain thus electrified for a considerable time.

Beccaria, a celebrated Italian physicist, kept up the spirit of electrical discovery in Italy. He showed that water is a very imperfect conductor of electricity, that its conducting power is proportional to its quantity, and that a small quantity of water opposes a powerful resistance to the passage of electricity. He succeeded in making the electric spark visible in water, by discharging shocks through wires that nearly met in tubes filled with water. In this experiment the tubes, though sometimes eight or ten lines thick, were burst in pieces. Beccaria likewise demonstrated that air adjacent to an electrified body gradually acquired the same electricity, that the electricity of the body is diminished by that of the air, and that the air parts with its electricity very slowly. He considered that there was a mutual repulsion between the particles of the electric fluid and those of air, and that in the passage of the former through the latter a temporary vacuum was formed. Beccaria's experiments on atmospheric electricity are of the greatest interest to the meteorologist. For farther account of his work, see his *Lettere dell' Elettr.*, 1758; *Experimenta*, 1772; and letters, &c., in *Phil. Trans.* about 1770.

The science of electricity owes several practical as well as theoretical observations to Robert Symmer (*Phil. Trans.*, about 1759). In pulling off his stockings in the evening, he had often remarked that they not only gave a crackling noise, but even emitted sparks in the dark. The electricity was most powerful when a silk and a worsted stocking had been worn on the same leg, and it was best exhibited by putting the hand between the leg and the stockings, and pulling them off together. The one stock-

ing being then drawn out of the other, they appeared more or less inflated, and exhibited the attractions and repulsions of electrified bodies. Two white silk stockings, or two black ones, when put on the same leg and taken off, gave no electrical indications. When a black and a white stocking were put on the same leg, and after ten minutes taken off, they were so much inflated when pulled asunder, that each showed the entire shape of the leg, and at the distance of a foot and a half they rushed to meet each other.

"But what appears most extraordinary is, that when they are separated, and removed at a certain distance from each other, their electricity does not appear to have been in the least impaired by the shock they had in meeting. They are again inflated, again attract and repel, and are as ready to rush together as before. When this experiment is performed with two black stockings in one hand, and two white in the other, it exhibits a very curious spectacle; the repulsion of those of the same colour, and the attraction of those of different colours, throws them into an agitation that is not unentertaining, and makes them catch each at that of its opposite colour, at a greater distance than one would expect. When allowed to come together, they all unite in one mass. When separated, they resume their former appearance, and admit of the repetition of the experiment as often as you please, till their electricity, gradually wasting, stands in need of being recruited.

Symmer likewise found that a Leyden jar could be charged by the stockings either positively or negatively, according as the wire from the neck of the jar was presented to the black or the white stocking. When the electricity of the white stocking was thrown into the jar, and then the electricity of the black one, or *vice versa*, the jar was not electrified at all. With the electricity of two stockings he charged the jar to such a degree that the shock from it reached both his elbows; and by means of the electricity of four silk stockings he kindled spirits of wine in a tea-spoon which he held in his hand, and the shock was at the same time felt from the elbows to the breast. Symmer has the merit of having first maintained the theory of two distinct fluids, not independent of each other, as Dufay supposed them to be, but co-existent, and, by counteracting each other, producing all the phenomena of electricity. He conceived that when a body is said to be positively electrified, it is not simply that it is possessed of a larger share of electric matter than in a natural state, nor, when it is said to be negatively electrified, of a less; but that, in the former case, it is possessed of a larger portion of one kind of electricity, and in the latter, of a larger portion of the other; while a body, in its natural state, remains unelectrified, because there is an equal amount of the two everywhere within it.

Contemporary with Symmer were Delaval, Wilson, Cigna, Kinnersley, Wilcke, and Priestley (for the works of these electricians consult Young). Delaval found that the sides of vessels that were perfect conductors were non-conductors, and that animal and vegetable bodies lost their conducting power when reduced to ashes. Wilson concluded that when two electrics are rubbed together, the harder of the two is generally electrified positively and the other negatively, the electricities always being opposite. Cigna made many curious experiments by using silk ribbands in place of the silk stockings of Symmer. Kinnersley, the friend of Franklin, made some important experiments on the elongation and fusion of iron wires, when a strong charge was passed through them in a state of tension (*Phil. Trans.*, 1763); he also experimented on the disruptive discharge in air. Wilcke brought to light many phenomena respecting the electrification produced by the melting of electric substances.

The pyro-electricity of minerals, or the faculty possessed by some minerals of becoming electric by heat, and of exhibiting negative and positive poles, now began to attract the notice of philosophers. There is reason to believe that the *lyncurium* of the ancients, which, according to

Pyro-
electricity of
minerals.

Æpinus
(1724-
1806).

Theophrastus, attracted light bodies, was the *tourmaline*, a Ceylon mineral, in which the Dutch had early recognized the same attractive property, whence it got the name of *Aschenrikker*, or attractor of ashes. In 1717 M. Lemery exhibited to the Academy of Sciences a stone from Ceylon which attracted light bodies; and Linnæus, in mentioning the experiments of Lemery, gives the stone the name of *Lapis Electricus*. The Duke de Noya was led in 1758 to purchase some of the stones called *tourmaline* in Holland, and, assisted by Daubenton and Adanson, he made a series of experiments with them, a description of which was published. The subject, however, had engaged the attention of Æpinus, a celebrated German philosopher, who published an account of them in 1756. Hitherto nothing had been said respecting the necessity of heat to excite the *tourmaline*; but it was shown by Æpinus that a temperature between $99\frac{1}{2}^{\circ}$ and 212° Fahr. was requisite for the development of its attractive powers. Benjamin Wilson (*Phil. Trans.*, 1763, &c.), Priestley, and Canton continued the investigation; but it was reserved for the Abbé Haüy to throw a clear light on this curious branch of the science (*Traité de Mineralogie*). He found that the electricity of the *tourmaline* decreased rapidly from the summits or poles towards the middle of the crystal, where it was imperceptible; and he discovered that if a *tourmaline* is broken into any number of fragments, each fragment, when excited, has two opposite poles. Haüy discovered the same property in the Siberian and Brazilian topaz, borate of magnesia, mesotype, prehnite, sphene, and calamine. He also found that the polarity which minerals receive from heat has a relation to the secondary forms of their crystals,—the *tourmaline*, for example, having its resinous pole at the summit of the crystal which has three faces, and its vitreous pole at the summit which has six faces. In the other pyro-electrical crystals above mentioned, Haüy detected the same deviation from the rules of symmetry in their secondary crystals which occurs in *tourmaline*. Brard discovered that pyro-electricity was a property of the *axinite*; and it was afterwards detected in other minerals. In repeating and extending the experiments of Haüy, Sir David Brewster discovered that various artificial salts were pyro-electrical; and he mentions tartrate of potash and soda, and tartaric acid, as exhibiting this property in a very strong degree. He also made many experiments with the *tourmaline* when cut into thin slices, and reduced to the finest powder, in which state each particle preserved its pyro-electricity; and he showed that *scolezite* and *mesolite*, even when deprived of their water of crystallization and reduced to powder, preserve their property of becoming electrical by heat. When this white powder is heated and stirred about by any substance whatever, it collects in masses like new fallen snow, and adheres to the body with which it is stirred. (For Sir David Brewster's work on pyro-electricity see *Trans. R.S.E.*, 1845; *Phil. Mag.*, Dec. 1847; *Edinburgh Journal of Science*, Oct. 1824 and 1825).

In addition to his experiments on the *tourmaline*, Æpinus made several on the electricity of melted sulphur; and in conjunction with Wilcke, he investigated the subject of electric atmospheres, and discovered a beautiful method of charging a plate of air by suspending large wooden boards coated with tin, and having their surfaces near each other and parallel. Æpinus, however, has been principally distinguished by his ingenious theory of electricity, which he has explained and illustrated in a separate work (*Tentamen Theoriæ Electricitatis et Magnetismi*) which appeared at St Petersburg in 1759. This theory is founded on the following principles. 1. The particles of the electric fluid repel each other with a force decreasing as the distance increases. 2. The particles of the electric fluid

attract the particles of all bodies, and are attracted by them, with a force obeying the same law. 3. The electric fluid exists in the pores of bodies; and while it moves without any obstruction in non-electrics, such as metals, water, &c., it moves with extreme difficulty in electrics, such as glass, rosin, &c. 4. Electrical phenomena are produced either by the transference of the fluid from a body containing more to one containing less of it, or from its attraction and repulsion when no transference takes place.

The electricity of fishes, like that of minerals, now began to excite very general attention. The ancients, as we have seen, were acquainted with the benumbing power of the torpedo, but it was not till 1676 that modern naturalists attended to this remarkable property. The Arabians had long before given this fish the name of *raad* or lightning; but Redi was the first who communicated the fact that the shock was conveyed to the fisherman by means of the line and rod which connected him with the fish. Lorenzini published engravings of its electrical organs; Reaumur described the electrical properties of the fish; Kämpfer compared the effects which it produced to lightning; but Bancroft was the first person who distinctly suspected that the effects of the torpedo were electrical. In 1773 Walsh (*Phil. Trans.*, 1773-5) and Ingenhousz proved, by many curious experiments, that the shock of the torpedo was an electrical one; and Hunter (*Phil. Trans.*, 1773-5) examined and described the anatomical structure of its electrical organs. Humboldt (*Ann. de Chim. et de Phys.*, i. 15), Gay-Lussac, and Geoffroy pursued the subject with success; and Cavendish (*Phil. Trans.*, 1776) constructed an artificial torpedo, by which he imitated the actions of the living animal. The subject was also investigated by Todd, Sir Humphrey Davy (*Phil. Trans.*, 1829), John Davy, and Faraday (*Exp. Res.*, vol. ii.). The power of giving electric shocks has been discovered also in the *Gymnotus electricus*,¹ the *Malapterurus electricus*,² the *Trichiurus electricus*,² and the *Tetraodon electricus*.² The most interesting and the best known of these singular fishes is the *Gymnotus* or Surinam eel. Humboldt gives a very graphic account of the combats which are carried on in South America between the *gymnoti* and the wild horses in the vicinity of Calabozo.

Among the cultivators of electricity Henry Cavendish is entitled to a distinguished place. Before he had any knowledge of the theory of Æpinus, he had communicated to the Royal Society a similar theory of electrical phenomena. As, however, he had carried the theory much further, and considered it under a more accurate point of view, he did not hesitate to give his paper to the world (*Phil. Trans.*, 1771). Cavendish made some accurate experiments on the relative conducting power of different substances. He found that electricity experiences as much resistance in passing through a column of water one inch long as it does in passing through an iron wire of the same diameter 400,000,000 inches long, whence he concluded that iron wire conducts 400,000,000 times as well as rain or distilled water. He found that a solution of one part of salt in one of water conducts a hundred times better than fresh water, and that a saturated solution of sea-salt conducts seven hundred and twenty times better than fresh water. Cavendish likewise determined by nice experiments that the quantity of electricity on coated glass of a certain area increased with the thinness of the glass, and that on different coated plates the quantity was as the area of the coated surface directly, and as the thickness of the glass inversely. Although electricity had been employed as a chemical agent in the oxidation and fusion of metals, yet it is to Cavendish that we owe the first of those brilliant inquiries which have done so much for the

Electricity
of fish

Cavendish
(1731
1810)

¹ Powerful

² Weak.

advancement of modern chemistry. By using different proportions of oxygen and hydrogen, and examining the products which they formed after explosion with the electric spark, he obtained a proportion of which the product was pure water (*Phil. Trans.*, 1784-5). The decomposition of water by the electric spark was first effected by Paets Van Troostwijk and Deiman; improved methods of effecting it were discovered and used by Pearson, Cuthbertson, and Wollaston (*Phil. Trans.*, 1801).

The great discovery made by Galvani in 1790, that the contact of metals produced muscular contraction in the frog, and the invention of the voltaic pile, in 1800, by Volta led to the recognition of a new kind of electricity called *Galvanic* or *Voltaic Electricity*, which is now proved to be identical with frictional electricity. The chemical effects of the voltaic pile far transcend those of ordinary electricity. In 1800 Nicolson and Carlisle discovered the power of the pile to decompose water; and in 1807 (*Bakerian Lecture*) Sir Humphry Davy decomposed the earths and the alkalies, and thus created a new epoch in the history of chemistry.

Contemporaneous with Cavendish was Coulomb, one of the most eminent experimental philosophers of the last century. In order to determine the law of electrical action, he invented an instrument called a *torsion balance*, which has since his time been universally used in all delicate researches, and which is particularly applicable to the measurement of electrical and magnetical actions. *Æpinus* and Cavendish had considered the action of electricity as diminishing with the distance; but Coulomb proved, by a series of elaborate experiments, that it varied, like gravity, in the inverse ratio of the square of the distance. Dr Robison had previously determined, without, however, having published his experiments, that in the mutual repulsion of two similarly electrified spheres, the law was slightly in excess of the inverse duplicate ratio of the distance, while in the attraction of oppositely electrified spheres the deviation from that ratio was in defect; and hence he concluded that the law of electrical action was similar to that of gravity. Adopting the hypothesis of two fluids, Coulomb investigated experimentally and theoretically the distribution of electricity on the surface of bodies. He determined the law of its distribution between two conducting bodies in contact; he measured the density of the electricity at different points of two spheres in contact; he ascertained the distribution of electricity among several spheres (whether equal or unequal) placed in contact in a straight line; he measured the distribution of electricity on the surface of a cylinder, and its distribution between a sphere and cylinder of different lengths but of the same diameter. His experiments on the dissipation of electricity possess also a high value. He found that the momentary dissipation was proportional to the degree of electrification at the time, and that, when the charge was moderate, its dissipation was not altered in bodies of different kinds or shapes. The temperature and pressure of the atmosphere did not produce any sensible change; but he concluded that the dissipation was nearly proportional to the cube of the quantity of moisture in the air. In examining the dissipation which takes place along imperfectly insulating substances, he found that a thread of gum-lac was the most perfect of all insulators; that it insulated ten times as well as a dry silk thread; and that a silk thread covered with fine sealing-wax insulated as powerfully as gum-lac when it had four times its length. He found also that the dissipation of electricity along insulators was chiefly owing to adhering moisture, but in some measure also to a slight conducting power. For the memoirs of Coulomb see *Mém. de Math. et Phys. de l'Acad. de Sc.*, 1785, &c.

Towards the end of the last century a series of experiments was made by Laplace, Lavoisier, and Volta (*Phil. Trans.*, 1782, or *Collezione dell' Op.*), from which it appeared that electricity is developed when solid or fluids bodies pass into the gaseous state. The bodies which were to be evaporated or dissolved were placed upon an insulating stand, and made to communicate by a chain or wire with a Cavallo's electrometer, or with Volta's condenser, when it was suspected that the electricity increased gradually. When sulphuric acid diluted with three parts of water was poured upon iron filings, hydrogen was disengaged with a brisk effervescence; and at the end of a few minutes the condenser was so highly charged as to yield a strong spark of negative electricity. Similar results were obtained when charcoal was burnt on a chafing dish. Volta, who happened to be at Paris when these experiments were made, and who took an active part in them, subsequently observed that the electricity produced by evaporation was always negative. He found that burning charcoal gives out negative electricity; and in other kinds of combustion he obtained distinct electrical indications. In this state of the subject Saussure (*Voyage dans les Alpes*, t. ii. p. 808, *et seqq.*) undertook a series of elaborate experiments on the electricity of evaporation and combustion. In his first trials he found that the electricity was sometimes positive and sometimes negative when water was evaporated from a heated crucible of iron; but he afterwards found it to be always positive both in an iron and a copper crucible. In a silver and a porcelain crucible the electricity was negative. The evaporation of alcohol and of ether in a silver crucible also gave negative electricity. Saussure made many fruitless trials to obtain electricity from combustion, and he likewise failed in his attempt to procure it from evaporation without ebullition. Many valuable additions were about this time made to electrical apparatus, as well as to the science itself, by Van Marum, Cavallo, Nicholson, Cuthbertson, Brooke, Bennet, Read, Morgan, Henley, and Lane; but these cannot here be noticed in detail.

The application of analysis to electrical phenomena may be dated from the commencement of the present century. Coulomb had considered only the distribution of electricity on the surface of spheres; but Laplace undertook to investigate its distribution on the surface of ellipsoids of revolution, and he showed that the thickness of the coating of fluid at the pole was to its thickness at the equator as the polar is to the equatorial diameter. Biot (*Traité de Physique Exp. et Math.*) has extended this investigation to all spheroids differing little from a sphere, whatever may be the irregularity of their figure. He likewise determined analytically that the losses of electricity form a geometrical progression when the two surfaces of a jar or plate of coated glass are discharged by successive contacts; and he found that the same law regulates the discharge when a series of jars or plates are placed in communication with each other. It is to Poisson (*Mém. de l'Inst. Math. et Phys.*, 12, 1811, &c.) however, that we are mainly indebted for having brought the phenomena of electricity under the dominion of analysis, and placed it on the same level as the more exact sciences. Assuming the hypothesis of two fluids, he deduced theorems for determining the distribution of the electric fluid on the surface of two conducting spheres when they are either placed in contact or at any given distance. The truth of these theorems had been established by experiments performed by Coulomb long before the theorems themselves had been investigated.

Voltaic electricity had now absorbed the attention of experimental philosophers. The splendour of its phenomena, as well as its association with chemical discovery, contributed to give it popularity and importance; but the

Laplace,
Lavoisier,
and
Volta.

Saussure.

Application of
analysis to
electricity.

Biot.

Poisson.

Magnetic
action of
electric
current
dis-
covered
by
Oersted.

discoveries of Galvani and Volta were destined, in their turn, to pass into the shade, and the intellectual enterprise of the natural philosophers of Europe was directed to new branches of electrical and magnetical science. Guided by theoretical anticipations, Professor H. C. Oersted of Copenhagen (*Experimenta circa effectum conflictus electrici in acum magneticam*) in 1820 discovered that the electrical current of a galvanic battery, when made to pass through a platinum wire, acted upon a compass needle placed below the wire. He found that a magnetic needle placed in the neighbourhood of an electric current always places itself perpendicular to the plane through the current and the centre of the needle; or, more definitely, that a magnetic north pole, carried at a constant distance round the current in the direction of rotation of an ordinary cork-screw advancing in the positive direction of the current, would always tend to move in the direction in which it is being carried.

Electro-
dynam-
ics.
Am-
père's
theory.

Scarcely had the news of Oersted's discovery reached France when a French philosopher, Ampère, set to work to develop the important consequences which it involved. Physicists had long been looking for the connection between magnetism and electricity, and had, perhaps, inclined to the view that electricity was somehow to be explained as a magnetic phenomenon. It was, in fact, under the influence of such ideas that Oersted was led to his discovery. Ampère showed that the explanation was to be found in an opposite direction. He discovered the ponderomotive action of one electric current on another, and by a series of well-chosen experiments he established the elementary laws of electrodynamical action, starting from which, by a brilliant train of mathematical analysis, he not only evolved the complete explanation of all the electromagnetic phenomena observed before him, but predicted many hitherto unknown. The results of his researches may be summarized in the statement that an electric current in a linear circuit of any form is equivalent in its action, whether on magnets or other circuits, to a magnetic shell bounded by the circuit, whose strength at every point is constant and proportional to the strength of the current. By his beautiful theory of molecular currents, he gave a theoretical explanation of that connection between electricity and magnetism which had been the dream of previous investigators. If we except the discovery of the laws of the induction of electric currents made about ten years later by Faraday, no advance in the science of electricity can compare for completeness and brilliancy with the work of Ampère. Our admiration is equally great whether we contemplate the clearness and power of his mathematical investigations, the aptness and skill of his experiments, or the wonderful rapidity with which he elaborated his discovery when he had once found the clue.

Recent
progress
of
electro-
dynam-
ics.

In 1821 Faraday, who was destined a little later to do so much for the science of electricity, discovered electromagnetic rotation (*Quarterly Journal*, xii.), having succeeded in causing a horizontal wire carrying a current to rotate continuously across the vertical lines of a field of magnetic force. The experiment was very soon repeated in a variety of forms by De la Rive, Barlow, Ritchie, Sturgeon, and others; and Davy (*Phil. Trans.*), in 1823, observed that, when two wires connected with the pole of a battery were dipped into a cup of mercury placed on the pole of a powerful magnet, the fluid metal rotated in opposite directions about the two electrodes. The rotation of a magnet about a fixed current and about its own axis was at once looked for, and observed by Faraday and others. The deflection of the voltaic arc by the magnet had been observed by Davy in 1821 (*Phil. Trans.*); and in 1840 Walker observed the rotation of the luminous discharge in a vacuum tube. For many beautiful experiments on the

influence of the magnet on the strata, &c., in vacuum tubes, we are indebted to Plücker, De la Rive, Grove, Gassiot, and others who followed them.

One of the first machines in which a continuous motion was produced by means of the repulsions and attractions between electromagnets and fixed magnets or electro-magnets was invented by Ritchie (*Phil. Trans.*, 1833). The artifice in such machines consists in reversing the polarity of one of the electromagnets when the machine is near the position of equilibrium. For a general theory of these machines, showing the reasons why they are not useful as economic motive powers, see Jacobi (*Mémoire sur l'Application de l'Électro-magnétisme au Mouvement des Machines*, Potsdam, 1835), and Joule (*Mech. Mag.*, xxxvi.). Electro-magnetic engines have, however, found a restricted use in scientific workshops, such as Froment's, in driving telegraphic apparatus, &c.

In 1820 Arago (*Ann. de Chim. et de Phys.*, t. xv.) and Davy (*Annals of Philosophy*, 1821) discovered independently the power of the electric current to magnetize iron and steel. Savary (*Ann. de Chim. et de Phys.*, t. xxxiv., 1827) made some very curious experiments on the alternate directions of magnetization of needles placed at different distances from a wire conveying the discharge of a Leyden jar. The dependence of the intensity of magnetization on the strength of the current was investigated by Lenz and Jacobi (*Pogg. Ann.*, xlvii., 1839), and Joule found that magnetization did not increase proportionately with the current, but reached a maximum (Sturgeon's *Ann. of El.* iv. 1839). The farther development of this subject, which really belongs to magnetism, has been carried on by Weber, Müller, Von Waltenhofen, Dub, Wiedemann, Quintus Icilius, Riecke, Stoletow, Rowland, and others. The use of a core of soft iron, magnetized by a helix surrounding it, has become universal in all kinds of electrical apparatus. Electromagnets of great power have in this way been constructed and used in electrical researches by Brewster, Sturgeon, Henry, Faraday, and others.

The most illustrious among the successors of Ampère was Wilhelm Weber. He greatly improved the construction of the galvanometer, and invented the electro-dynamometer. To these instruments he applied the mirror scale and telescope method of reading, which had been suggested by Poggendorff, and used by himself and Gauss in magnetic measurements about 1833. In 1846 he proceeded with his improved apparatus to test the fundamental laws of Ampère. The result of his researches was to establish the truth of Ampère's principles, as far as experiments with closed circuits could do so, with a degree of accuracy far beyond anything attainable with the simple apparatus of the original discoverer. The experiments of Weber must be looked upon as the true experimental evidence for the theory of Ampère, and as such they form one of the corner-stones of electrical science.

While experiment was thus busy, theory was not idle. In 1845 Grassmann published (*Pogg. Ann.*, lxiv.) his *Neue Theorie der Electrodynamik*, in which he gives an elementary law different from that of Ampère, but leading to the same results for closed circuits. In the same year F. E. Neumann published yet another law. In 1846 Weber announced his famous hypothesis connecting electrostatic and electrodynamical phenomena. Much has been written on the subject by Carl Neumann, Riemann, Stefan, Clausius, and others. Very important are three memoirs by Helmholtz, in *Crelle's Journal* (1870-2-4), in which a general view is taken of the whole question, and the works of his predecessors are critically handled. We shall have occasion, in the body of the article, to refer to the dynamical theory of Clerk Maxwell, which promises to effect a revolution in this part of electrical science.

Electro-
magnetic
engine

Magnet-
ization
by
electric
current

Recent
progress
of
electro-
dynam-
ics.

Theor.
of
electro-
dynam-
ics.

By his discovery of thermo-electricity in 1822 (*Pogg. Ann.*, vi.), Seebeck opened up a new department. He found that when two different metals are joined in circuit there will be an electric current in the circuit if the junctions are not at the same temperature; he arranged the metals in a thermo-electric series, just as Volta and his followers had arranged them in a contact series. Cumming (*Annals of Phil.*, 1823) found that the order of the metals was not the same at different temperatures. This phenomenon has been called thermo-electric inversion. In 1834 Peltier discovered that if a current be sent round a circuit of two metals in the direction in which the thermo-electromotive force would naturally send it, then the hot junction is cooled, and the cool junction heated. This effect, which is reversible, and varies as the strength of the current, is called the Peltier effect. Sir W. Thomson made many experiments on thermo-electricity, and applied to the experimental results the laws of the dynamical theory of heat. His reasonings led him to predict a new thermo-electric phenomenon, the actual existence of which he afterwards verified by an elaborate series of very beautiful experiments (*Phil. Trans.*, 1856). He has given a general theory of the thermo-electric properties of matter, taking into account the effect of structure, &c. His experimental researches have been ably continued by Professor Tait, who, guided by theoretical considerations to the conjecture that the curves in what Thomson called the "thermo-electric diagram"¹ must be straight lines, made an extended series of experiments, and showed that they were in general very approximately either straight lines or made up of pieces of straight lines. Our knowledge of thermo-electricity has been advanced by Becquerel, Magnus, Matthiessen, Lœroux, Avenarius, and others. Thermo-electric batteries of considerable power have been constructed by Markus Noë, and Clamond, and employed more or less in the arts.

In 1824 Arago (*Ann. de Chim. et de Phys.*, t. xxvii. &c.) made a remarkable discovery, which led ultimately to results of the greatest importance. He found that when a magnetic needle is suspended over a rotating copper disc the needle tends to follow the motion of the disc. This phenomenon, which has been called the "magnetism of rotation," excited great interest; Barlow (*Phil. Trans.*, 1825), Herschel, Seebeck (*Pogg. Ann.*, vii., 1826), and Babbage (*Phil. Trans.*, 1825) made elaborate researches on the subject; and Poisson (*Mém. de l'Acad.*, vii., 1826) attempted to give a theoretical explanation in his memoir on magnetism in motion. The true explanation was not arrived at until Faraday took up the subject a little later. We may mention, here, however, the experiments of Plücker, Matteucci, and Foucault on the damping of the motions of masses of metal between the poles of electromagnets. The damping of a compass needle suspended over a copper plate, observed by Seebeck (*l.c.*), has been taken advantage of in the construction of galvanometers.

In 1831 Faraday began, with the discovery of the induction of electric currents, that brilliant series of experimental researches which has rendered his name immortal. The first experiment which he describes was made with two helices of copper wire wound side by side on a block of wood, and insulated from each other by intervening layers of twine. One of these helices was connected with a galvanometer, and the other with a battery of a hundred plates, and it was found that on making and breaking the battery circuit a slight sudden current passed through the galvanometer in opposite directions in the two cases. He also discovered that the mere approach or removal of a circuit carrying a current would induce a current

in a neighbouring closed circuit, and that the motion of magnets produces similar effects. To express in a concise manner his discoveries, Faraday invented his famous conception of the lines of magnetic force, or lines the direction of which at any point of their course coincides with that of the magnetic force at that point. His discovery can be thus stated:—Whenever the number of lines of force passing through a closed circuit is altered, there is an electromotive force tending to drive a current through the circuit, whose direction is such that it would itself produce lines of force passing through the circuit in the opposite direction. Nothing in the whole history of science is more remarkable than the unerring sagacity which enabled Faraday to disentangle, by purely experimental means, the laws of such a complicated phenomenon as the induction of electric currents. The wonder is only increased when we look to his papers, and find the first dated November 1831,² and another January 1832, in which he shows that he is in complete possession of all the general principles that are yet known on the subject. Faraday very soon was able to show that the current developed by induction had all the properties of the voltaic current, and he made an elaborate comparison of all the different kinds of electricity known,—statical, dynamical or voltaic, magneto-, thermo-, and animal electricity,—showing that they were identical so far as experiment could show. In 1833 Lenz made a series of important researches (*Pogg. Ann.*, xxxi., 1834, xxxiv., 1835), which, among other results, led him to his celebrated law by means of which the direction of the induced current can be predicted from the theory of Ampère, the rule being that the direction of the induced current is always such that its electromagnetic action tends to oppose the motion which produces it. This law leads to the same results as the principles of Faraday. The researches of Ritchie and Henry about this time, and of Dove a little later, are also of importance. In 1845 F. E. Neumann did for magneto-electric induction what Ampère did for electro-dynamics, by developing from the experimental laws of Lenz the mathematical theory of the subject (*Abh. der Berl. Akad. der Wissenschaft.*, 1845–7). He discovered a function which has been called the "potential" (of one linear current on another or on itself), from which he deduced a theory of induction completely in accordance with experiment. About the same time Weber deduced the mathematical laws of induction from his elementary law of electrical action, which, as we have already seen, he applied to explain electrostatic and electromagnetic action. In 1846 Weber, applying his improved instruments, arrived at accurate verifications of the laws of induction, which by this time had been developed mathematically by Neumann and himself. In 1849 Kirchhoff determined experimentally in a certain case the absolute value of the current induced by one circuit in another; and in the same year Edlund made a series of careful experiments on the currents of self and mutual induction, which led to the firmer establishment of the received theories. Helmholtz gave the mathematical theory of the course of induced currents in various cases, and made a series of valuable experiments in verification of his theory (*Pogg. Ann.*, lxxxi., 1851). Worthy of mention here are also the experiments and reasonings of Felici in 1852. In the *Philosophical Magazine* for 1855, Sir W. Thomson investigated mathematically the discharge of a Leyden jar through a linear conductor, and predicted that under certain circumstances the discharge would consist of a series of decaying oscillations. This oscillatory discharge was observed in 1857 by Feddersen (*Pogg. Ann.*, cviii.) The law of Weber has been applied

¹ A mode of representing the phenomena of thermo-electricity which has been greatly developed and improved by Tait.

² The first experiment seems to have been actually made on the 29th August 1831. See Bence Jones's *Life of Faraday*, vol. ii. p. 1.

by Kirchhoff to the case of conductors in three dimensions. The most important of all the recent contributions to this part of electrical science is the theory of Clerk Maxwell, which aims at deducing the phenomena of the electromagnetic field from purely dynamical principles with the aid of the fewest possible hypotheses (*Phil. Trans.*, 1864; *Electricity and Magnetism*, 1873). He has established the general equations which determine the state of the electric field, and he has by means of these equations constructed an electromagnetic theory of light, which is full of suggestions for the philosopher, whether speculative or experimental. The theory of Helmholtz, and his valuable criticisms on the works of those that have laboured in this department, are to be found in three memoirs already alluded to.

Magneto-electric machines.

Magneto-electricity has been largely applied in the arts. One of the first machines for producing electricity by induction was made by Pixii. It consisted of a fixed horseshoe armature wound with copper wire, in front of which revolved about a vertical axis a horseshoe magnet. The machine was furnished with a commutator for delivering the alternating currents in a common direction. By means of this machine Faraday and Hachette decomposed water and collected the disengaged gases separately. Many variations of this type of machine were constructed by Ritchie, Saxton, Clark, Von Ettingshausen, Stöhrer, Dove, Wheatstone, and others. In 1857 Siemens effected a great improvement by inventing the form of armature which bears his name. The next improvement was to replace the fixed magnets by electromagnets, the current for which was furnished by a small auxiliary machine. Wilde's machine (1867) is of this kind. Siemens, Wheatstone, and others suggested that the fixed electromagnet should be fed by a coil placed on the armature itself, so that starting from the residual magnetism of the armature the machine goes on increasing its action up to a certain point. Ladd's machine (1867) is constructed on this principle. The most recent of these machines is that of Gramme, the peculiarity of which is that the coil of the armature is divided up into a series of coils arranged round an axis, the object being to produce a continuous instead of a fluctuating current. It has been proposed of late to employ electromagnetic machines in lighting streets and workshops, and the experiment has been tried with some success. They have been employed for some time back in lighthouse work. The most important inductive apparatus for the physicist is the induction coil or inductorium, which has been brought to great perfection in the workshop of Ruhmkorff. Poggendorff (*Annalen*, 1855) suggested several improvements in this kind of apparatus. Fizeau, who added the condenser (1853), Foucault, who designed the interrupter which bears his name (1855), and Ritchie, who devised the plan of dividing the coil into sections by insulating partitions, have all aided in bringing the instrument to perfection. Very powerful machines of this kind have been constructed. A large one in the Polytechnic Institution, London, gives a 29-inch spark, and one recently constructed by Apps for Mr Spottiswoode gives a spark of 42 inches. The mathematical theory of magneto-electric machines has been treated by Maxwell (*Proc. Roy. Soc.*, 1867). He has also given a theory of the action of the condenser in the inductorium (*Phil. Mag.*, 1868). Two papers by Strutt (now Lord Rayleigh) in *Phil. Mag.*, 1869-70, are very interesting in connection with the same subject.

Ohm's law.

In the year 1827 Dr G. S. Ohm rendered a great service to the science of electricity by publishing his mathematical theory of the galvanic circuit (*Die Galvanische Kette mathematisch bearbeitet*). Before his time the quantitative circumstances of the electric current had been indicated

in a very vague way by the use of the terms "intensity" and "quantity," to which no accurately defined meaning was attached. Ohm's service consisted in introducing and defining the accurate notions—electromotive force, current strength, and resistance. He indicated the connection of these with experiment, and stated his famous law that the electromotive force divided by the resistance is equal to the strength of the current. The theory on which Ohm based his law may be and has been disputed, but the law itself and the applications which Ohm and others have made of it are in the fullest agreement with all known facts. The merit of Ohm really consists in having satisfactorily analysed a great group of phenomena which had up to his time baffled all those who attempted the task. How great his service was is easily seen when we remark the progress of those who adopted his ideas as compared with those who for a time hesitated to do so. Ohm was guided in his mathematical work by analogy with the problem of the flux of heat, and introduced for the first time into the theory of the pile, the equivalent of the modern word *potential*. Ohm's word was *electroscopic force* or *tension* (*Spannung*), and he showed that the fall of the potential is uniform along a homogeneous linear conductor. He considered that the potential was analogous to the temperature, and the flow of electricity to the flow of heat, so that the former just as much as the latter obeys the law of continuity. Ohm verified his theoretical conclusions with thermo-electric piles, and he observed, as Erman (*Gilb. Ann.*, 1801) had done before him, the differences of potential at different points of the circuit. Davy, Pouillet, and Becquerel laboured at the experimental verification of Ohm's law, and a great body of evidence was given by Fechner in his *Maassbestimmungen über die Galvanische Kette* (1831). The law of the fall of potential was verified by the elder Kohlrausch, who employed in his researches Volta's condenser and Dellmann's electrometer (*Pogg. Ann.*, lxxv., 1848). Later researches of a similar nature were made by Gaugain and Branly. Among recent investigations bearing on Ohm's law, the most remarkable is the verification for electrolytes by Kohlrausch (the younger) and Nippoldt. They principally used alternating currents in their researches, which were furnished by a "sine inductor," the measuring instrument employed being the electro-dynamometer of Weber. In the report of the British Association for 1876 an account is given of some experiments,¹ in which the testing of this law seems to have been carried to the limit of experimental resources. It must now be allowed to rank with the law of gravitation and the elementary laws of static electricity as a *law of nature* in the strictest sense. Many remarkable applications of Ohm's law have been made of late, in particular to linear conductors by Ohm, Poggendorff, and especially Kirchhoff (*Pogg. Ann.*, 1845-7-8). The works of Helmholtz, Smaasen, and Kirchhoff on conduction in three dimensions must also be mentioned. Very important, on account of the experimental results with which they deal, are the calculations of Du Bois Reymond (*Pogg.*, lxxi., 1845) and Riemann (*Werke*, Leipsic, 1876) on Nobili's rings, and of Kirchhoff (*Pogg.*, lxvii., 1848), W. R. Smith (*Proc. Roy. Soc. Edin.*, 1869-70), Quincke, Stefan, Adams, and others on conduction in plates. Theoretical applications to the varying currents in submarine cables of great interest have been made by Thomson (*Phil. Mag.*, 1856) and Kirchhoff (*Pogg. Ann.*, 1857), while practical researches of the greatest importance to telegraphy have been made on this and kindred subjects by Faraday, Wheatstone, Guillemin, Varley, Jenkin, and others.

Great improvements in galvanometers and galvanometry

¹ Suggested mainly by Prof. Clerk Maxwell, and carried out by the present writer.

re- have been made in our time. One of the first to use an electro-magnetic instrument for measuring or indicating currents was Schweigger, who in 1820 invented the "multiplier." Nobili used (1825) the astatic "multiplier" with two needles, which is sometimes named after him. Becquerel (1837) used the electromagnetic balance, which was employed in an improved form by Lenz and Jacobi. Pouillet invented the sine and tangent compasses (1837). The defects of the latter instrument were pointed out by Poggenдорff, and remedies suggested by him as well as Wheatstone and others. Weber effected great improvements in the construction and use of galvanometers, adapted them for the measurement of transient currents, and elaborated the method of oscillations which had been much used by Fechner. In 1849 Helmholtz invented the tangent compass with two coils which bears his name. Great improvements in delicacy and promptness of action have been made by Sir William Thomson in galvanometers destined for the measurement of resistance, and for indicating the feeble currents of submarine cables.

The measurement of resistance has been carried to great perfection, chiefly owing to the labours of those who have re- busied themselves in perfecting the electric telegraph. Among such the highest place must be assigned to Sir Charles Wheatstone; his memoirs in the *Philosophical Transactions* (1843) gave a great impulse to this department of our science. He invented the rheostat, which underwent several modifications, but is now superseded by the resistance box which was first used by Siemens. The earlier methods of Ohm, Wheatstone, and others for measuring resistance were defective, because they depended on the constancy of the battery which furnished the current. These defects are completely obviated in the more modern "null methods," which may be divided into two classes—those which depend on the use of the differential galvanometer introduced by Becquerel, and those which are modifications of the Wheatstone's bridge method, invented by Christie and brought into use by Wheatstone. As examples of the latter, we may mention the methods of Thomson, and of Matthiessen and Hockin, for measuring small resistances, and Thomson's method for measuring the resistance of the galvanometer (see Maxwell's *Electricity and Magnetism*, pp. 404, 410). Many determinations of the specific resistances of metals and alloys have been made by Davy, Ohm, Becquerel, Matthiessen, and others. To Matthiessen in particular science is indebted for great improvements in method and a large body of valuable results in this department. The metals have been arranged in a series according to their conducting powers; and this series is found to be nearly the same for electricity as for heat. The conductivity of metals decreases as the temperature increases, the rate of decrease being nearly the same for most pure metals, but much smaller and more variable for alloys, which, on the other hand, have in general a large specific resistance. The earlier attempts to measure the resistance of electrolytes were not satisfactory, owing to insufficient allowance for polarization. In later times this difficulty has been overcome or avoided, and concordant results have been obtained by Beetz, Paalzow, Kohlrausch, Nippoldt, and Grotthuss. The three last, using the electro-dynamometer and sine inductor, have made elaborate researches, establishing among many other interesting results that the conductivity of electrolytes increases with the temperature (*Pogg. Ann.*, 1869-74).

The measurement of the electromotive force and that of internal resistance of batteries in action are problems which, in their most general form, are inextricably connected. It is easy to measure with considerable accuracy the electromotive force of an open battery. We have merely to

connect its poles with a Thomson's electrometer, and compare the deflection thus obtained with that due to some standard electromotive force. Another very satisfactory method is Latimer Clarke's modification of Poggenдорff's compensation method (see Maxwell, 413). It is likewise not difficult to measure by a variety of methods, the most satisfactory being that of Mance (Maxwell, 411), the internal resistance of a battery when it is only traversed by a feeble current. But the measurement of the electromotive force and internal resistance of a battery working a strong current has hardly as yet been achieved with success; not that we undervalue the ingenious and important methods of Paalzow, Von Waltenhofen, Beetz (Wiedemann, i. § 181), and Siemens (*Pogg. Ann.*, 1874). The concordant results of the last two are indeed very remarkable. Still all these methods are more or less affected by the fact that the electromotive force of a battery depends on the current which it is sending (see Beetz in *Pogg. Ann.*, cxlii.).

The "crown of cups" of Volta was the parent of a Batteries great many other arrangements for the production of voltaic electricity. These had for their end either compactness or diminution of the internal resistance by enlarging the plates; we may mention the batteries of Cruickshank (1801), Wollaston (1815), and Hare (1822). In 1830 Sturgeon introduced the capital improvement of amalgamating the zinc plates. In 1840 Smee used platinum or silver plates instead of copper; by platinizing these he avoided to a considerable extent polarization by adhering hydrogen. In 1836 Daniell invented the two-fluid battery which bears his name. This battery is the best constant battery hitherto invented, and is, under various modifications, largely used in practical and scientific work. In the same year Grove invented his well-known battery, which surpasses Daniell's in smallness of internal resistance and in electromotive force, although, on the other hand, it is more troublesome to manage and is unsuited for long-continued action. Cooper, in 1840, replaced the expensive platinum plates of Grove's battery by carbon. This modification was introduced in a practical form into the battery of Bunsen (1842), which is much used on the Continent, and combines to a certain extent the advantages of Grove and Daniell. Among the more recent of one-fluid batteries may be mentioned the bichromate battery of Bunsen and the Léclanché cell. It is impossible here even to allude to all the forms of battery that have been invented. We may, however, in passing notice the gravitation batteries of Meidinger and Varley, and the large tray cell of Sir William Thomson.

Following up the discoveries of Nicholson, Carlisle, Electro-Davy, and others, Faraday took up the investigation of the chemical decompositions effected by the electric current. In 1833 he announced his great law of electro-chemical equivalents, which made an epoch in the history of this part of electricity. He recognized and for the first time thoroughly explained the secondary actions which had hitherto masked the essential features of the phenomenon. Faraday's discovery gave a new measure of the current, and he invented an instrument called the voltameter, which was much used by those who followed out his discoveries. Space fails us to notice in detail the labours of those who verified and developed Faraday's discovery. De la Rive, Becquerel, Soret, Buff, Beetz, Hittorf, Matteucci, Daniell, Miller, and many others have worked in this field.

Many theories of electrolysis have been given. That of Theories Grotthuss (1805) has been held under various modifi- of electro-lysis cations by many physicists; but none of these theories have done more than give us a convenient mode of representing experimental results. Clausius (*Pogg. Ann.*,

ci., 1857) has published a remarkable molecular theory of electrolysis, which is free from some of the objections to the views of Grotthuss and his followers.

Polarization.

The advances made in the experimental study of electrolysis reacted on the theory of the galvanic battery. It was now recognized that the cause of the inconstancy of batteries is the opposing electromotive force due to the existence of the products of decomposition at the plates of the battery. Gautherot, in 1802, observed the polarization current from electrodes which had been used for electrolysis. Ritter confirmed his discovery, and constructed on the new principle his secondary pile. Ohm also experimented on this subject. Fechner and Poggendorff suspected the existence of a transition resistance (*Uebergangswiderstand*) at the places where the chemical products were evolved. But the experiments of Lenz, Beetz, and others soon showed that a *vera causa* existed in the electromotive force of polarization amply sufficient to explain their results. The influence of the strength of the current, the size and nature of the plates, time, &c., on polarization have been investigated by many physicists, among whom are prominent Beetz and Poggendorff. Determinations of the electromotive force of polarization have been made by Daniell, Wheatstone, Poggendorff, and Beetz, and recently by Tait and others. Among recent labours on polarization are to be mentioned those of Helmholtz and his pupils. We must not omit to notice here the gas battery of Grove, and the powerful secondary piles which have recently been constructed by Planté. We refer those interested in these and kindred subjects to the exhaustive accounts in Wiedemann's *Galvanismus*. Justice to all contributors to our knowledge is impossible in our limited space.

Contact and chemical theories of the pile.

This is perhaps the place to mention the great battle that raged so long between the upholders of the two rival theories of the action of the pile. Volta and his immediate successors held that the current was due to the electromotive force of contact between the dissimilar metals in the circuit, the function of the electrolyte being simply to transmit the electricity, there being no contact force between metals and liquids. The upholders of the chemical theory sought for the origin of the current in the chemical affinity between the zinc and the acid or their equivalents in the battery, and, in the first instance at least, denied the existence of the contact force of Volta. It was soon shown, however, on the one hand, that there *was* a contact force between metals and liquids, and, on the other, that an electric current could be generated without a heterogeneous metallic circuit at all.

Later holders of both theories modified their views as experiment established the necessity for so doing. Ohm and Fechner and other Continental philosophers inclined to a modified contact theory, and Sir William Thomson at present lends his weighty authority to that side. On the other side are the great names of Faraday, Becquerel, and De la Rive. The contact theorists devoted their attention more to the electrostatic phenomena of the pile, while the chemical theorists studied with great minuteness the phenomena of electrolysis, so that both theories have rendered good service to science. Now-a-days most physicists probably recognize too well the defects of both theories to think it worth while to attack either, and take refuge more or less in eclecticism.

Application of the principle of the conservation of energy.

There was one point which the older adherents of the contact theory overlooked, the importance of which was more or less dimly perceived by their chemical opponents. This was, in modern language, the question, where does the energy come from which appears as kinetic energy in the moving parts of electromagnetic engines, as heat in the conducting wires, through which a current is being driven, and so

forth? It was not until the dynamical theory of heat had been perfected that the first answer to this question was given. Joule (*Phil. Mag.*, 1841) had arrived experimentally at Joule's law, the law which regulates the generation of heat in conductors by the electric current, and his law was verified by Lenz and Becquerel, both for metals and electrolytes. Reasoning from Joule's law on the case where the whole of the energy appears in the form of heat, Thomson (*Phil. Mag.*, 1851) established the important theorem that the electromotive force of an electro-chemical apparatus is, in absolute measure, equal to the mechanical equivalent of the chemical action on one electro-chemical equivalent of the substance. Calculations of the electromotive force of a Daniell's cell, from the results of Joule, Andrews, and Favre and Silbermann, have given numbers agreeing with the direct measurements of Bosscha. The total amount of the electromotive force in the circuit having been thus satisfactorily determined, the question between the rival theories is reduced to the determination of the seat of this force—At which of the junctions does it act?

Besides his great services in other branches of electricity, Faraday did much to advance electrostatics. His experimental investigations on electrostatic induction are of great interest, and his discovery of the effect of the medium between the electrified bodies opened out a new aspect of the phenomenon quite unsuspected by those who held too closely to the theories of action at a distance. He introduced the term specific inductive capacity, and measured the capacity of several solid substances, showing that in these it was much greater than that of air. He conceived that his results were at variance with any theory of action at a distance, and gave a theory of his own, which accounted for all his facts, and which guided him in his investigations. Matteucci and Siemens adopted the views of Faraday, and the latter introduced refined methods for measuring specific inductive capacities. Such measurements have been made in later times by Barclay and Gibson for paraffin, and by Silow for certain fluids. The most remarkable result thus obtained, however, are those of Boltzmann, who succeeded not only in detecting but in actually measuring the differences between the specific inductive capacities of different gases. Faraday had looked in vain for such differences, and concluded that the specific inductive capacity was the same for all gases. The phenomenon of the residual discharge was recognized and experimented on by Faraday. Kohlrausch, Gaugain, Wüllner, and others have also experimented on it; and quite recently Mr Hopkinson has obtained some very interesting results regarding the superposition of residual discharges. These results are analogous to the curious phenomena of "elastic recovery" observed by Kohlrausch.

Sir W. Snow Harris was a very able experimenter, and did much to improve electrostatic apparatus. He used the electrical balance and the bifilar suspension balance invented by himself. On the strength of his results he questioned the soundness of the views of Coulomb. The work of Harris on the influence of the surrounding medium on the electric spark is of great importance. Faraday made a series of beautiful experiments on this subject, and arrived at a body of results which still form a good portion of the *established* facts on this subject. Very important in this connection are the measurements of Sir W. Thomson of the electromotive force required to produce a spark in air between two conductors, which he has found to be disproportionately smaller for large distances than for small.

The luminous phenomena attending the electric discharge, especially in vacuum tubes such as those of Geissler, are exceedingly beautiful, and have of late formed a favourite subject of experimental study. Many interesting results have been obtained, the significance of which we may

not yet rightly comprehend. Among the older labours in this field we may mention those of Plücker and Hittorf, De la Rive, Riess, Gassiot, and Varley. But even as we write our knowledge of the subject is extending, and we refrain from referring to more modern results; for historical sketching—a difficult task in any case—is unsafe in an open field like this, where some apparently insignificant fact may contain the germ of a great discovery. We may here mention the experiments of Wheatstone on the velocity of electricity, valuable less for the results he obtained than for the ingenious application of the rotating mirror, then used for the first time, which has since been applied with much success in the study of the electric discharge.

One of the greatest names in electrical science is that of Riess. In his classical research on the heating of wires by the discharge from a battery of Leyden jars, he did for electricity of high potential what Joule did for the voltaic current. The electro-thermometer which he used in these researches was an improvement on the older instruments of Kinnersley and Harris. Riess repeated and extended the experiments of Coulomb, and effected many improvements in the apparatus for electrostatical experiments. His *Reibungs-electricität* is a work of great value, and was for long the best book of reference open to the experimental student. Happily we have now another in the recently published work of M. Mascart.

Sir William Thomson revolutionized experimental electricity by introducing instruments of precision. Chief among these are his quadrant and absolute electrometers. His portable electrometer and water-dropping apparatus are instruments of great value to the meteorologist in the study of atmospheric electricity, a science which he has done much in other ways to forward. Besides this, we owe to him many valuable suggestions for electrical apparatus and experimental methods, some of which have been carried out by his pupils.

The theory of statical electricity has made great progress since Poisson's time. Among its successful cultivators we may mention Murphy (*Electricity*, 1833), and Plana (1845). The latter went over much the same ground as Poisson, extending his results. It was, however, by Green (*Essay on The Application of Mathematical Analysis to the Theories of Electricity and Magnetism*, 1828; or *Mathematical Papers*, edited by N. M. Ferrers), a self-taught mathematician, that the greatest advances were made in the mathematical theory of electricity. "His researches," as Sir William Thomson has observed, "have led to the elementary proposition which must constitute the legitimate foundation of every perfect mathematical structure that is to be made from the materials furnished in the experimental laws of Coulomb. Not only do they afford a natural and complete explanation of the beautiful quantitative experiments which have been so interesting at all times to practical electricians, but they suggest to the mathematician the simplest and most powerful methods of dealing with problems which, if attacked by the mere force of the old analysis, must have remained for ever unsolved." One of the simplest applications of these theorems was to perfect the theory of the Leyden phial, a result which (if we except the peculiar action of the insulating solid medium, since discovered by Faraday) we owe to his genius. He has also shown how an infinite number of forms of conductors may be invented, so that the distribution of electricity in equilibrium on each may be expressible in finite algebraical terms,—an immense stride in the science, when we consider that the distribution of electricity on a single spherical conductor, an uninfluenced ellipsoidal conductor, and two spheres mutually influencing one another, were the only cases solved by Poisson, and indeed the only cases conceived to be solvable by mathematical writers. The work of Green, which con-

tained these fine researches, though published in 1828, had escaped the notice not only of foreign, but even of British mathematicians; and it is a singular fact in the history of science that all his general theorems were rediscovered by Sir William Thomson, Charles and Sturm, and Gauss (see *Reprint of Thomson's papers*). Sir William Thomson, however, pushed his researches much further than his fellow-labourers. He showed that the experimental results of Sir William Snow Harris, which their author had supposed to be adverse to the theory of Coulomb, were really in strict accordance with that theory in all cases where they were sufficiently simple to be submitted to calculation. He was guided in his earlier investigations by an analogy between the problems involved in steady flux of heat and the equilibrium of electricity on conductors. He showed in 1845 how the peculiar electric polarization discovered by Faraday in dielectrics, or solid insulators subjected to electric force, is to be taken into account in the theory of the Leyden jar, so as to supply the deficiency in Green's investigations. We also owe to Sir William Thomson new synthetical methods of great elegance and power. The theory of electric images, and the method of electric inversion founded thereon, constitute the greatest advance in the mathematical theory of electrostatics since the famous memoir of Green. These he has applied in the happiest manner to the demonstration of propositions which had hitherto required the resources of the higher analysis, and he has also found by means of them the distribution on a spherical bowl, a case of great interest in the theory of partially closed conductors, which had never been attacked or even dreamt of as solvable before. The work of Professor Clerk Maxwell on *Electricity and Magnetism*, which appeared in 1873, has already exerted great influence on the study of electricity both in England and on the Continent. In it are fully given his valuable theory of the action of the dielectric medium. He regards the electrical forces as the result of stress in the medium, and calculates the stress components which will give the observed forces, and at the same time account for the equilibrium of the medium. The striking discovery recently made by Mr Kerr of Glasgow, of the effect on polarized light exerted by a piece of glass under the action of strong electric force, is of great importance in connection with Maxwell's theory, and realizes a cherished expectation of Faraday, of whom Maxwell is the professed exponent. We must allude here once more to Maxwell's electromagnetic theory of light, the touchstone of which is the proposition that in transparent media, whose magnetic inductive capacity is very nearly equal to that of air, the dielectric capacity is equal to the square of the index of refraction for light of infinite wave length. Although, as perhaps was to be expected, owing to disturbing influences such as heterogeneity, this proposition has not been found in good agreement with experiment in the case of solids, yet for liquids (Silow, *Pogg. Ann.*, clv. clviii.) and gases (Boltzmann, *Ibid.* clv.) the agreement is so good as to lead us to think that the theory contains a great part of the whole truth.

In the earlier stages of the science several units were introduced for the measurement of quantities dealt with in electricity. As examples of these we may mention the wire of Jacobi, and the mercury column of Siemens, a metre long, with a section of a square millimetre, which at given temperatures furnished units of resistance; the Daniell's cell, which furnished the unit of electromotive force, the chemical unit of current intensity, &c. All these units were perfectly arbitrary, and there was no connection of any kind between them. The introduction of a rational system of unitation, based on the fundamental

Absolute
units.

units of time, mass, and length, was one of the greatest steps of our time. The impulse came from the famous memoir of Gauss, *Intensitas Vis Magnetice Terrestris ad Mensuram absolutam revocata*, 1832. In conjunction with Weber, he introduced his principles into the measurement of the earth's magnetic force. To Weber belongs the credit of doing a similar service for electricity. He not only devised three different systems of such units—the electro-dynamical, the electrostatic, and the electromagnetic—but he carried out a series of measurements which practically introduced the last two systems. The fundamental research in this subject is to determine in electromagnetic measure the resistance of some wire from which, by comparison, the electromagnetic unit of resistance can be constructed. Measurements of this kind were made by Kirchhoff in 1849; more carefully in two different ways by Weber in 1851; by the committee of the British Association in 1863, &c.; by Kohlrausch in 1870; and by Lorenz in 1873. Accounts of these important researches will be found in Wiedemann and Maxwell, and in the collected reports of the British Association on "Electrical Standards." The ratio of the electrostatic to the electromagnetic unit of electric quantity is a velocity (according to Maxwell's electromagnetic theory of light it is the velocity of light), the experimental determination of which is of the greatest theoretical and practical importance. Such determinations have been made by Weber and Kohlrausch in 1856, by Maxwell in 1868, and by Thomson in 1869. The results are not so concordant as might be desired, but the research is a very difficult one.

For convenience in practice the British Association committee have recommended certain multiples of the absolute unit, to which they have given names—*e.g.*, the Ohm, the Volt, the Farad, &c. These have become current to a great extent among practical electricians in this country. For practical purposes, an empirical standard of electromotive force has been introduced by Latimer Clark, whose value in volts is given as 1.457. It is very important, in order to be able to reduce chemical to absolute measure, to know accurately the electro-chemical equivalent of water. Values for this have been found by Weber (1840), Bunsen (1843), Casselman (1843), and Joule (1851). Kohlrausch (1873) made a careful determination of the electro-chemical equivalent of silver, from which the electro-chemical equivalent of water can be calculated.

GENERAL SKETCH OF PHENOMENA.

Fundamental experiment.

If a piece of glass and a piece of sealing-wax be each rubbed with a dry woollen cloth, it will be found that both the glass and the wax have acquired the property of attracting indiscriminately any small light body in the neighbourhood; and it will be further observed, in many cases, that the small bodies, after adhering for a little to the glass or wax, will be again repelled.

These actions have at first sight a likeness to the attractions and repulsions of magnetic bodies, but they are sufficiently distinguished from these—1st, By their origin,—being excited by friction and other causes in a great variety of bodies, whereas magnetic action is powerfully exhibited and communicated only by certain varieties of iron and iron ore, by nickel and cobalt, and by certain arrangements which we shall have to mention by-and-by; 2d, By the nature of the bodies acted on; for these may be, in the case of excited glass or wax, light particles of any substance, whereas the only bodies powerfully acted on magnetically are either magnets or their equivalents, or iron, nickel, and cobalt; and 3d, By the fact that every magnet has two poles possessing opposite properties, whereas an electrified body may have similar properties in every part of its surface.

If the experiment were carefully tried it would be found that a piece of glass excited as above repels another piece of glass similarly excited, but attracts an excited piece of wax. A convenient way of exhibiting these actions, which also brings under our notice another fact of fundamental importance, is as follows. Two gilt balls of elder pith are fastened to the ends of a light needle of shellac, which is balanced horizontally on a point carried on a vertical stand (fig. 1). To the stand a stop is fixed for convenience, to prevent the needle from spinning more than half round. If we touch the ball A with a piece of excited glass, and B with a piece of excited sealing-wax, and touch a ball C, fastened to a shellac stem, with a piece of excited glass, then C will chase A away till it is brought up by the stop, while it will, on the other hand, attract B. If, again, C be touched with a piece of excited wax, it will attract A and repel B.

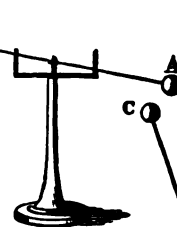


Fig. 1.

Pieces of glass or wax excited in this way are said to be *electrified*, and the balls which by contact have acquired properties similar to those of the originally electrified bodies are said to be *electrified by conduction*.

It appears from the above experiment that the electrifications of glass and sealing-wax, when rubbed with woollen, have opposite properties, which they communicate to bodies brought into contact with them. A body which has similar electrification to a piece of glass rubbed with woollen is said to be vitreously or positively electrified; a body with similar electrification to a piece of sealing-wax rubbed with woollen is said to be resinously or negatively electrified. The result of the above experiment may then be summarized thus:—

Bodies similarly electrified, whether positively or negatively, repel each other.

Bodies oppositely electrified attract each other.

We have seen that a pith ball becomes, by contact with a positively electrified piece of glass, itself positively electrified. If we take two pith balls, electrify one of them positively, and then touch both simultaneously by a piece of thin wire, suspended by white silk, and test them with the electroscopic needle described above, they will be found both positively electrified; each will repel A and attract B, though less powerfully than the originally electrified ball did, before the connection between them was made. The success of the experiment will be found independent of the length or shape of the wire, and will be equally good with silver, gold, iron, lead, or any other metal. But, if we use a thread of glass or shellac to connect the balls, the electrification of the first ball will be found unaltered, and the second will remain neutral—that is, it will not attract or repel another neutral ball, and will equally attract both balls, A and B, of the electroscopic needle. The difference in the power of transmitting electrical properties from one body to another, or of aiding in electrification by *conduction*, leads us to divide all substances into two classes—conductors, which do very readily, and non-conductors, which do not, or do not very readily, transmit electrification from one body to another. If we connect an electrified conductor by means of another conductor to a very large conducting body, such as the earth, it will be found that so much electrification has been carried away from the small body that it is left sensibly neutral. If, accordingly, we wish a conducting body to preserve its electrification unaltered, we must support it on some non-conducting substance. When thus supported the body is said to be *insulated*, the non-con-

ducting support being called the *insulator*, a name which has on that account been given to non-conductors generally.

We have remarked above that a neutral pith ball attracts equally the positive and negative balls of the electroscopic needle; this leads us to remark, more explicitly than we have hitherto done, that an electrified body in general and in the first instance attracts a neutral or unelectrified body. The explanation of this action is that the originally neutral body in presence of the electrified body becomes itself electrified for the time. It is said to be electrified by *induction*, and it is very easy to show, by using large bodies, not only that the originally neutral body is actually electrified, but that it is oppositely electrified in different parts. Thus (fig. 2) A and B are two bodies suitably insulated and placed one above the other. If B be originally neutral, and A be positively electrified, then the lower end of B will be negatively, and the upper end positively electrified; as may be easily shown by exploring with a small positively electrified pith ball suspended by a dry white silk thread; the little ball will be attracted towards the lower end of B, and repelled from the upper. If we remove the body A, or, which (as we have seen) amounts to the same thing, connect it with the earth, and so "*discharge*" its electrification, we shall find that all traces of electrical action in B have disappeared—i.e., the small positively electrified pith ball will be *attracted* everywhere; and, if we discharge it too, it will neither be attracted nor repelled anywhere.

Provisional Theory.

Before going further into detail, it will be convenient to give a working theory of electrical phenomena, so far as we have considered them. The use of such a theory at the present stage is to enable us to co-ordinate and classify the results of experiment, and to furnish a few leading principles under which we may group results which appear to be due to a common cause. Such a theory is invaluable as a *memoria technica* for experimental results, and is useful in suggesting directions for experimental inquiry; but in framing it we must be careful to make it contain as little as possible beyond the results of actual experiment, and in using it we must be on our guard against allowing it to prepossess our minds as to what may be the ultimate explanation of the phenomena we are considering.

Following the caution of Coulomb and the example of Sir William Thomson, we shall avoid the use of the term *electrical fluid*, and substitute instead the more succinct and less misleading word *electricity*. We suppose that a body which exhibits electrical properties (as above defined) has associated with its mass a certain quantity of something which, without attempting further definition, we shall call *electricity*. Of our right to use the word *quantity* here we shall give experimental justification by-and-by, and then the question of the appropriate unit will (*vide infra*, "electric quantity") be discussed. We may suppose that elec-

tricity is distributed throughout the whole mass of a body, and speak of electrical "*volume density*," meaning the quantity of electricity in an element of volume divided by the element of volume. We shall also speak of an *element of electricity*, meaning the electricity in an element or very small portion of a body. In certain cases we shall find that electricity resides on the surface of a body; electrical "*surface density*" then means quantity of electricity on an element of surface divided by the element of surface, and element of electricity the electricity on an element of surface.

For shortness, we shall denote positive or vitreous electricity by the mathematical sign +, and resinous or negative electricity by the sign -, remarking that the choice of the signs is arbitrary, and reserving for the present the question of how far we may associate with these signs the corresponding mathematical ideas.

We shall assume that every element of electricity repels every other element of the same sign, and attracts every other element of opposite sign. The precise law of this force will be investigated further on.

This force considered as acting on any element of electricity we shall call an electric force. In perfectly conducting substances electricity moves with perfect freedom under any electromotive force, however small. In perfect non-conducting substances electricity will not move under any electromotive force, however great. Any case in nature lies somewhere between these extremes, but into questions of gradation, &c., we do not enter for the present.

When the forces due to other electrical elements acting on the electricity in any element of a body have a resultant, that resultant acts on the element itself, and is called the ponderomotive force, to distinguish it from the electromotive¹ (or electric) force which tends to move + electricity in one direction and - electricity in the opposite direction.

When a body is neutral, we shall assume that it contains *equal and equally distributed* quantities of + and - electricity, and we shall further suppose those to be practically unlimited in amount. A + electrified body is then to be conceived as a body which has excess of + electricity and a - electrified body as one which has excess of - electricity. Communication of + electricity to a body is in accordance with this to be regarded as equivalent to the abstraction of an equal amount of - electricity, and conversely.

It is easy to see that the above assumptions will explain in a general way the phenomena already described. Thus the + electricity of the electrified pith ball C acting on the + electricity of the ball A of the electroscopic needle repels it, and this force by our assumption is equally exerted on the matter of A, therefore A tends to move away from C, and will do so as long as it is free to move. The action on the - electrified ball B is similarly explained. Conduction and discharge to earth may be explained in a similar manner.

The attraction of an electrified body (+ let us suppose) A on a neutral insulated body B is thus explained. The + electricity on A (fig. 3) attracts the - electricity in B and repels the + electricity, so that, though there is still on the whole as much + electricity as - electricity, yet the distribution is no longer the same, for, the electricity being free to move, the - electricity under the attraction approaches A until the non-conducting air

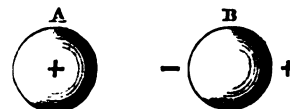


Fig. 3.

¹ It might be well to use the term "electric force" here, for "electromotive force" is afterwards used to mean the line integral of a force (see below, p. 24).

and the attraction of the separated +electricity on B stops it, and the +electricity recedes in similar fashion. When electrical equilibrium has been attained the action of the + electricity of A on the - electricity of B will exceed its action on the + electricity of B, which is on the whole more distant,¹ the electromotive force on the electricity of B will be on the whole attractive, and hence the ponderomotive force on B, will be also attractive.

The above explanation involves of course the general explanation of *electrification by induction*.

Experimental investigation of Electrical Quantity, Distribution, and Force.

Electroscopes and electrometers.

In what follows we shall suppose that we have an instrument which will serve as an electroscope and to some extent as an electrometer; that is, which shall tell us readily whether a body brought into communication with it is + or - electrified or not at all, and also enable us to tell when one body is more strongly electrified + or - than another.

The gold-leaf electroscope of Bennet or the dry pile electroscope of Bohnenberger will meet these requirements, and have been much used in electrical researches. We shall, however, suppose that we are using the rudimentary form of Thomson's electrometer constructed by Elliot Brothers for lecture-room experiments, which is now much used in England, and answers very well. For a description of these and other electroscopes and electrometers, see article **ELECTROMETER**.

We shall also assume for the present that we have the means of producing and communicating to any body as much of either kind of electrification as we please, and pass on to consider the data of experiment regarding the distribution of statical electricity in conducting bodies. We are thus at the very outset brought face to face with the idea of electric quantity.

Electric Quantity.

We have to explain how the introduction of the term quantity into electrical science is justified by experiment, and how we can multiply and subdivide quantities of electricity. Although it is no doubt possible to introduce the notion of quantity independently of the *measure* of electric force, yet the most convenient and *practical measure* of quantity depends on the measurement of force, and the absolute electrostatic unit of quantity is stated in this way. We are naturally led, therefore, to combine with the study of quantity and distribution the experimental study of the laws of electric force.

We shall have occasion to allude to two leading experimental methods that have been used in investigating the present subject. These might be called the old method and the new.

The old method, which did so much for electrical science in the master hand of Coulomb, depended on the use of the torsion balance and proof plane, both invented by Coulomb himself. This method was used by Reiss and others up to Faraday's time.

Coulomb's torsion balance.

Michell, about Coulomb's time or a little before, first suggested the idea of measuring small forces by the torsion of a wire. He proposed to apply the method to measure the attraction of gravitation between two bodies of moderate size, thus finding the mean density of the earth, and the method was actually carried out by Cavendish; but Coulomb was in all probability unaware of Michell's suggestion. He made careful preliminary experiments (the first of the kind) on the torsion of wires, and found that the couple

required to twist a straight wire through a given angle varies as the angle of torsion multiplied by the fourth power of the diameter of the wire directly, and as the length of the wire inversely (*Mém. de l'Acad.*, 1784).

The balance used by Coulomb in most of his experiments is represented in figure 4.

ABDC is a cylinder of glass 1 foot in diameter and 1 foot high. This cylinder is closed by a glass lid pierced centrally and eccentrically by two openings, each about 20 lines wide. Into the middle opening is cemented a glass tube 2 feet high, to the upper end of which is fitted a torsion head; the separate parts of the head are shown larger at the side of the figure. H is a collar cemented to the glass tube; MO a metal disc, divided on the edge into 360 degrees; this disc is fastened to a tube N, which slips into the collar H. K is a button whose neck turns easily in a hole in MO; to the lower part of the button is fastened a small clamp, which seizes the wire of the balance. I is an arm with a small projecting piece which slips over the edge of the disc MO. This piece has a fiducial mark on it, which enables us to read off the position of the arm on the graduated edge of MO. The horizontal arm *bd* consists of a silk thread or fine straw covered with sealing wax terminated by a thread of shellac at *b* about 18 lines long, which carries a pith ball 2 or 3 lines in diameter. At the other end of the arm is a vertical disc of oiled paper, which serves as a counterpoise to the pith ball, as a damper to the oscillations, and as an index by means of which the position of the horizontal arm can be read off on a graduation carried round the glass cylinder. The eccentric hole in the cover of the balance allows the introduction of the fixed ball *a*; this is carried on a shellac stem fastened to a clamp P, which by means of fiducial marks can be placed in a fixed position on the cover. The wire in Coulomb's balance was of silver, about 30 cm. long. Its diameter was .0035 cm., and it weighed about .003 gm. He found by the method of oscillations that a couple equivalent to the weight of .17 milligramme, acting at the end of an arm a decimetre long, would keep the wire twisted through 360°.

FIG. 4.—Torsion Balance.

Besides this form of balance Coulomb used others, some more delicate for electroscopic purposes, and others less so, but of larger dimensions, into which he could introduce electrified bodies of considerable size.

Faraday used Coulomb's balance, and Snow Harris used the bifilar balance, which is a modification of Coulomb's. In the second volume of his *Experimental Researches*, however, Faraday gives a general method of experimenting, which to a great extent has superseded the older method. This may be called the "cage method;" it depends for its success on the use of some delicate instrument for measuring differences of potential; this was supplied by the quadrant electrometer of Sir William Thomson, which has thus completely revolutionized the whole system of electrostatic measurement.

Faraday's experiment was as follows (*Exp. Res.*, vol. ii. p. 279):—

Let A (fig. 5) be an insulated hollow conductor with an opening to allow admission to the interior. Faraday used a pewter ice pail, 10½ in. high and 7 in. in diameter. Connect the outside of A with one electrode of an electrometer E, which may for most purposes be the rudimentary form of Thomson's electrometer mentioned above. Connect the other electrode of the electrometer with the earth. If now we introduce a positively electrified body, say a brass ball C,

¹ It is here tacitly assumed that the attraction between two elements of electricity decreases as the distance between them increases.

² A cylinder of wire gauze will answer equally well, and allows the experimenter to see better what he is doing. Such a cylinder we shall call for shortness an "electric cage."

suspended by a white silk string, we shall find that the electrometer needle is deflected through a certain angle, the spot of light going a certain distance to the right, say, of the scale. It will be found that, provided the ball C is more than a certain depth (about 3 in. in Faraday's experiment) below the mouth of the pail, no further motion of the ball, right or left, up or down, will affect the indications of the electrometer. It will also be found that the same indications will be got to whatever point of the *outside* of the pail the electrometer wire is attached. If we diminish or increase the + electrification of C, the electrometer deflection will diminish or increase accordingly. If we introduce a negatively electrified ball C', the deflection will be to the left, and everything else as before. If C gives a certain positive (right) deflection, and C' an equal (left) deflection, then if we introduce C and C' together, the deflection will be zero. If C and C' be both + electrified and give equal + deflections, then introduced together they will give a double + deflection, and if three such balls, all giving equal + deflections, be introduced together, they will give a treble + deflection.

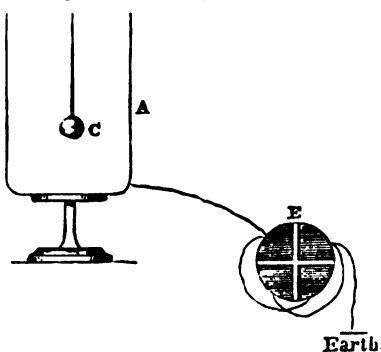


Fig. 5.

It is obvious that this experiment of Faraday's not only gives a very ready test of the electrical state of bodies, but at once suggests the notion of electrical quantity, and a theoretically possible electrostatic unit. Suppose, in fact, we take for our test the deflection of a Thomson's electrometer of given sensibility, then we might specify as a unit of electrical quantity the quantity of + electricity on or in a brass ball of given size, which will produce with a given cage a certain given deflection of the electrometer.

To make this definition useful we must have the means of transferring a given charge from one body to another, and charging a body with any multiple or submultiple of our unit, and of charging a body with any multiple or submultiple of the unit of negative electricity, which we may define as the quantity of - electricity which will just annul the action of the unit of + electricity in the electric cage.

All these requirements may be satisfied by suitably modifying Faraday's experiment. We saw that we might move the ball about in the middle of our electric cage without affecting the electrometer deflection; we find, moreover, that when we withdraw the electrified ball without touching the cage, the needle returns to zero. If, however, before withdrawing the ball we cause it to touch the inside of the cage, the electrometer deflection remains the same as before, and after the ball has been removed the deflection is still the same, while if we examine the ball, we find that all traces of electrification have disappeared.

To transfer a given quantity of electricity.—If we provide ourselves with two cages, a large one G, and a small one H, and take a ball C, electrified positively with unit quantity as above defined, then testing C in cage G, in connection with the electrometer, we get a certain deflection D. If now we transfer the electrification of C to H, by the process just described, and then put H inside G, we shall get the same deflection D as before. It appears, therefore, that we can transfer electrification from one body to another without loss; we thus fulfil one of our requirements, and give an additional justification of the use of the word quantity in the present case.

To get any multiple or submultiple of the electric unit.—We may repeat the process above performed on the small cage H by touching its inside with the ball C, again electrified to unit quantity. All the electrification will pass to H

as before, and if we now test H in G we shall get a deflection 2 D. We can in this way get any multiple we please of the unit charge. If we take the electrified brass ball C and touch it by a perfectly equal neutral ball C', on introducing C into G we shall get deflection $\frac{1}{2}$ D; if we touch C again by C', previously rendered neutral, we shall get deflection $\frac{1}{4}$ D, and so on; if we had touched C *simultaneously*, as in fig. 6, with two equal neutral balls, we should have got deflection $\frac{1}{2}$ D, and so on. We can thus get any submultiple of our unit charge.



Fig. 6.

To get a given multiple and submultiple of the negative unit.—This is possible when we can get a quantity of - electricity, which will just destroy the action of a given quantity of + electricity in the electric cage. If we introduce our given quantity of + electricity into the cage H, without allowing the conductor carrying it to touch the cage and at the same time put the outside of the cage in communication with the ground, then if we remove the conductor with the given quantity of + electricity and put it in G, it will give the same + deflection as before, while H tested in the same way will give a negative deflection exactly equal to the former, and if both be introduced together there will be no deflection. We can, therefore, in this way get a - quantity equal and opposite to a given + quantity.¹

Electrical Distribution.

Experiments had been made before Coulomb's time to determine what effect the nature of a body has on electrical distribution. Gray and White concluded, from an experiment with two cubes of oak, one hollow and the other solid, "that it was the surface of the cubes only which attracted." Le Monnier² showed that a sheet of lead gave a better spark when extended than when rolled together. These experiments point to the conclusion that electrical distribution in conducting bodies depends merely on the shape of the bounding surface.

We may make experiments confirmatory of this conclusion with the electric cage. If we electrify a brass sphere A, and then touch it with another sphere B, and test the electrification of B in the cage, we shall find that the amount of electricity taken by B is always the same, whatever its material may be, so long as the radius of its external surface is the same. Experiment is unable to detect any difference in this respect between a solid sphere of lead and the thinnest soap-bubble of the same radius. Coulomb took a large cylinder of wood, in which he made several holes four lines in diameter and four lines deep. Having electrified the cylinder and insulated it, he examined its electrical condition by means of the proof-plane. This instrument, so much used by Coulomb, consisted merely of a small disc of gilt paper (in this case a line and a half in diameter) fastened to the end of a needle of shellac. The disc is applied to any point of a body whose electrification we wish to test so as to be in the tangent plane to the surface of the body. Assuming for a moment, what we shall by-and-by prove, that electricity resides on the surface of bodies, it is natural to suppose that the proof-plane, when placed as described, will form part of the bounding surface, and will therefore take up as much electricity as was originally on the part of the surface which it

¹ The substance of the above and a good deal of what follows is taken from Maxwell's *Electricity and Magnetism*, vol. I. We recommend the student to read his remarks on quantity, § 35, venturing to suggest, as an illustration of the transmission of energy by action at a distance, the case of two bar magnets, in which the energy of vibration is transmitted and retransmitted periodically. See Tait's *Recent Advances in Physical Science*, p. 179.

² Mascart t. i. p. 90.

covers. If now we remove the proof-plane in the direction of the normal, keeping it, as nearly as possible, parallel to the surface, the electricity will not leave it; but we shall carry safely away all that it had when in contact with the surface of the body. We may return to the consideration of the proof-plane when we come to study mathematically the laws of electrical distribution.

In the experiment with which we are now concerned, Coulomb used a very delicate balance (a force of $\frac{1}{1000}$ of a milligramme was sufficient to keep the wire twisted through 90°). When the proof-plane was applied to any point of the external surface of the wooden cylinder, and then introduced into the torsion balance, it repelled the electrified ball of the balance with great force. When it was carefully introduced into one of the holes, made to touch the bottom, and then carefully withdrawn so as not to touch the edge of the hole, it produced no appreciable effect on the balance.

Hollow sphere experiment.

Coulomb varied this experiment as follows. He insulated and electrified a hollow sphere of metal (fig. 7), and by carefully introducing a proof-plane through a small opening tested the electrical condition of the interior surface. He found no sensible trace of electricity inside, except very near the edge of the small opening. Hence we conclude that if the sphere had been closed entirely there would have been no electrification inside. Many experiments have been made to illustrate the proposition that electricity resides entirely on the surface of conductors. Franklin put a long chain inside a metal teapot, which he insulated and electrified. When he seized the chain by a hook at the end of a glass rod and pulled it out of the teapot he observed that a pair of pith balls, suspended side by side from the teapot, collapsed more and more as the chain was drawn out, and he concluded that the electrification of the teapot, being now spread over a greater surface, had become weaker.



Fig. 7.

Franklin's experiment.

Biot's experiment.

The following experiment of Biot's has become classical. A spherical conductor A (fig. 8) is supported on an insulating stem D. Band C are two hollow hemispheres fastened to insulating handles E and F. When these are fitted together they form a sphere somewhat larger than A, with a small hole in it through which the stem D can pass. If we electrify A very strongly, so that when put in the electric cage it powerfully deflects the electrometer, and then close B and C over A, and make either B or C touch it, then separate B and C, and test A, B, and C in the cage, we shall find that all the electricity has gone from A and spread itself over B and C.

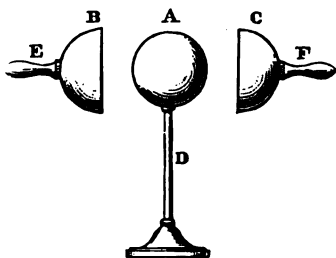


Fig. 8.

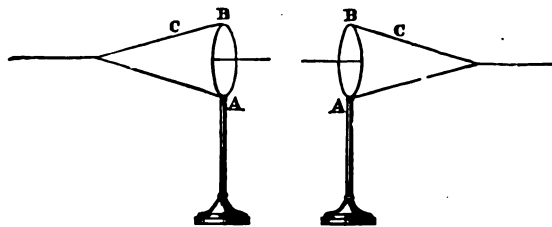


Fig. 9

involving the same principle. AB (fig. 9) is a wire ring supported on an insulating stand; C is a conical muslin bag fitted to the ring with two strings fastened to the vertex of the cone, giving the experimenter the means of quickly turning the bag inside out. If the bag be electrified in the first position in the figure and tested with the proof-plane and electric cage, it will be found that the outside only is electrified. If we turn the bag inside out and test it, we shall find as before that what is now the outside, and was formerly the inside, is alone electrified. The electricity has thus passed through the bag so as to be on the outside in both cases.

Before leaving for a time the question of the distribution of electricity on conductors, it may be well to warn the student to accept with due reserve the proposition that electricity resides entirely on the surface of conductors, and to remind him that such a proposition is from the nature of the case incapable of direct experimental proof, for we cannot operate in the substance of a mass of metal. Some of the experiments we have quoted bear more directly on the question than others. Coulomb's experiment, for instance, shows, strictly speaking, merely that electricity avoids cavities and re-entrant angles. Again, in Faraday's experiment with the linen bag, we have not clearly defined what we mean by the outside of the body. The proposition has on the whole been suggested rather than proved. Its meaning will become clearer as we go more and more into the theory of distribution,¹ and we shall meet with it by-and-by as one of the first propositions in the mathematical theory.

Laws of Electric Force.

Before proceeding to give an account of Coulomb's quantitative experiments on electrical distribution, we shall describe shortly the methods by which he arrived at the laws of electric force, and did for electricity what Newton did for astronomy, i.e., laid the foundation for a mathematical theory of the subject based on the hypothesis of *action at a distance*.

In this research Coulomb used the form of balance which we described above. The clamp holding the fixed ball of the balance is so adjusted that the centre of the ball falls in a horizontal line through zero of the graduation on the glass cylinder and the prolongation of the suspending wire; the torsion button is turned till its arm is at zero; the disc, button and all, is then turned till the disc on the arm and the centre of the movable ball are in a line with the zero of the lower graduation. The fixed ball, which had been removed to allow of the last adjustment, being replaced, and the movable ball having come to rest in contact with it, both are electrified by means of a small metal ball carried on an insulating stem of shellac. The balls repel each other, and the movable ball takes up a certain position of equilibrium; the corresponding angle is read off. The torsion button is then turned through an angle which is noted, so as to bring the balls nearer together. The new position of the beam is again read off; this may be repeated a third time. We are then in possession of data from which we can draw conclusions as to the law of electrical force at different distances.

Let us assume that the force between two elements of positive electricity (supposed collected at two points, technically speaking, "regarded as physical points") varies inversely as the square of the distance between them. It will be shown in the mathematical theory that two spheres *uniformly*² electrified, as we shall at present

¹ One additional caution may be useful, viz., not to confound this proposition with another of fundamental importance, of which we can give direct experimental proof of the most conclusive nature "that there is no electrical action inside a hollow conductor containing no charged bodies."

² This condition is not absolutely satisfied in any experiment; it is approximately satisfied in Coulomb's experiment.

Experiment determining the elements of the force.

assume the two balls in the balance to be, repel each other, as if their electricity were collected at their centres.

Let ϵ be the angle of equilibrium in any case, τ the angle of torsion. O (fig. 10) is the centre of the beam, F and M the centres of the fixed and movable ball (we suppose $OF=OM$); OK is perpendicular to FM. Then $FM^2 \propto \sin^2 \frac{\epsilon}{2}$.

Hence moment of the force on M about

$$O \propto \frac{\cos \frac{\epsilon}{2}}{\sin^2 \frac{\epsilon}{2}}, \text{ and the torsional couple } \propto \tau + \epsilon.$$

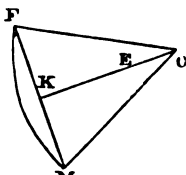


Fig. 10.

Hence in the three cases the value of $(\tau + \epsilon) \sin \frac{\epsilon}{2} \tan \frac{\epsilon}{2} = A$ (say) must be the same, if the law of the inverse square agree with the experiments.

Coulomb made many experiments of the kind we have described. The following is the result which he has given of one such:—

	Observed.	Calculated.	Difference.
τ	ϵ	ϵ	
0	36° 0'	36° 0'	...
126°	18 0	18 6	6'
567	8 30	9 4	34

The third column is obtained from the two preceding. A is calculated by putting $\tau = 0$ and $\epsilon = 36^\circ$ in the formula

$$(\tau + \epsilon) \sin \frac{\epsilon}{2} \tan \frac{\epsilon}{2} = A.$$

Then using this value of A and the observed value of τ , the formula is employed to find ϵ in the two second cases. The agreement between the observed and calculated values of ϵ is the test of the truth of the law we have assumed. The agreement in the second line is as good as can be expected when possible errors of experiment are considered. It will be seen, moreover, that the calculated is in excess of the observed value, which is what we ought to expect, owing to the loss of electricity which goes on during the time consumed in the experiment. That there is such a loss may be proved experimentally by simply leaving the movable ball to itself after any of the three operations; it will be seen to move slowly towards the fixed ball. We shall return hereafter to this loss of electricity, with regard to the exact nature of which authorities are not quite agreed.¹ In the third line the agreement is less good, but here the proximity of the balls renders the supposition of uniformity no longer even approximately allowable. The mutual repulsion tends to drive the electricity on each ball farther from the other ball, and thus the action between the balls is as if the electricity on each were collected at points beyond the centre, so that the observed repulsion must be less than that calculated on the supposition of uniformity of distribution.

Coulomb also made experiments with the torsion balance to test whether the law of the inverse square applies to the attraction as well as to the repulsion of electrified bodies. His experiments confirmed the law; but the difficulty of operating is much greater in this case than in the former. He therefore adopted another method of experimenting. A small conducting disc was fixed nor-

mally on the end of a small shellac needle, which was hung up, so as to be horizontal, on a fibre of raw silk attached to a horizontal scale. An insulated conducting globe was set up with its centre in the same vertical plane as the scale, and in the same horizontal plane as the centre of the small disc. The globe and disc were oppositely electrified, and the period of oscillation of the needle was found by observing the duration of 15 swings. The time of oscillation follows the pendulum law, and varies inversely as the square root of the force acting on the needle, hence the duration of 15 oscillations will vary inversely as the square root of the force, i.e. directly as the distance between the centres of the globe and disc, if the law of the inverse square hold. Coulomb's experiment gave the following results:—

Distance of centres of globe and disc.	Duration of 15 oscillations.	Ratio of distance to duration.
9	20	2.22
18	41	2.28
24	60	2.50

The numbers in the third column ought to be all equal. The deviation from equality are not greater than can fairly be explained by loss of electricity and errors of observation.

Coulomb also investigated, both by means of the torsion balance and by the method of oscillations, the relation between electric force and quantity.

He electrified the two balls of the torsion balance by simultaneous contact with another ball, and observed the angle of equilibrium; he then halved the quantity on the fixed ball by touching it with an equal neutral ball, and reduced the torsion till the angle of equilibrium, and, in consequence, the distance between the balls was the same as before; he found the torsional couple in the second case to be somewhat less than half what it was in the first. He therefore concluded that the force between two elements of electricity varies as the product of the quantities.

Coulomb's experiments were repeated, and his results confirmed by Riess,² and by Marié-Davy.³ Experiments which, when properly interpreted, lead to the same results, were made by Snow Harris,⁴ and by Egen.⁵

We have then arrived at this general law of electric force:—

If two quantities q, q' of electricity be supposed collected at two points, whose distance is d , the force between them acts in the straight line joining the points and $\propto \frac{qq'}{d^2}$.

So far, this law might be merely an approximation to the truth. Later on, however, it will be seen to be logically deducible from experiments which in delicacy infinitely surpass those just described. The law of Coulomb is in fact established as certainly as the law of gravitation itself.⁶

By means of the law now given the unit of electrical quantity can be defined in a satisfactory and practical manner. This unit we now state to be that quantity of positive electricity which, when collected into a point, repels with unit of force an equal quantity similarly collected into a point at unit distance from the former.

If we take centimetre, gramme, and second as our units of length, mass, and time, the unit force will be that force which in a second generates in a gramme of matter a velocity of a centimetre per second.

¹ This is only one of the many experimental difficulties which beset the use of the torsion balance, one of the most difficult of all instruments to use successfully. To appreciate the skill and sagacity of Coulomb in this and other matters, the student must read more detailed accounts (Riess and Mascart, or *Mémoires de l'Acad.*, about 1785) of his labours than we can give here. He will be richly repaid for his trouble. Nothing is better calculated to rouse the failing enthusiasm of the tyro in experimental electricity than a perusal of the works of Coulomb, unless it be to read the *Experimental Researches of Faraday*.

² *Reibungselectricität*, Bd. I. p. 94.

³ Mascart, i. p. 67.

⁴ *Phil. Trans.*, 1834 and 1836. In connection with which we call the attention of the student to the classical paper of Sir W. Thomson, *Reprint of Papers on Electrostatics and Magnetism*, p. 15 sqq.

⁵ Riess, Bd. I. p. 94.

⁶ We suppose, of course, that we are dealing always with one and the same dielectric throughout.

The law of electric force between two quantities q and q' now becomes

$$\text{Force} = \frac{qq'}{d^2}.$$

The unit of quantity which we have just defined is called the electrostatic unit, in contradistinction to the electromagnetic unit which we shall define hereafter.

Since the dimension of unit of force is $[LMT^{-2}]$, where L, M, T symbolize units of length, mass, and time, we have for the dimension of unit of electrical quantity $[Q]$

$$[Q] = [LF] = [L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}].$$

Quantitative Results concerning Distribution.

It has already been indicated that electricity in equilibrium resides on the surface of conducting bodies. We must now review shortly the experimental method by which this surface distribution has been more closely investigated. We shall state here some of the general principles arrived at, and one or two of the results, reserving others for quotation when we come to the mathematical theory of electrical distribution.

The most important experiments are due to Coulomb. He used the proof-plane and the torsion balance. Riess, who afterwards made similar experiments, used methods similar to those of Coulomb.

Allusion has already been made to the use of the proof-plane, and it has been stated that when applied to any part of the surface of an electrified body, it brings away just as much electricity as originally occupied the part of the surface which it covers. If, therefore, we electrify the movable ball of the torsion balance in the same sense as the body we are to examine, and note the repulsion caused by the proof-plane when introduced in place of the fixed ball after having touched in succession two parts of the surface of the body, we can, from the indications of the balance, calculate the ratio of the quantities of electricity on the plane in the two cases, and hence the ratio of the electrical densities at the two points of the surface. We suppose, of course, that the proof-plane is small enough to allow us to assume that the electrical density is sensibly uniform over the small area covered by it. In some of his experiments Riess used a small sphere (about two lines in diameter) instead of the small disc of the proof-plane as Coulomb used it. The sphere in such cases ought to be very small, and even then, except in the case of plane surfaces, its use is objectionable, unless the object be merely to determine, by twice touching the same point of the same conductor, the ratio of the whole charges on the conductor at two different times. The fundamental requisite is that the testing body shall, when applied, alter the *form* of the testing body as little as possible,¹ and this requisite is best satisfied by a small disc, and the better the smaller the disc is. The theoretically correct procedure would be to have a small portion of the actual surface of the body movable. If we could remove such a piece so as to break contact with all neighbouring portions simultaneously, then we should, by testing the electrification of this in the balance, get a perfect measure of the mean electric surface density on the removed portion. We shall see that Coulomb did employ a method like this.

¹ It is evident from what we have advanced here that the use of the proof-plane to determine the electric density at points of a surface where the curvature is very great, *e.g.*, at edges or conical points is inadmissible. If we attempt to determine the electrical density at the vertex of a cone by applying a proof-sphere there, as Riess did, we shall very evidently get a result much under the mark, owing to the blunting of the point when the sphere is *in situ*. We should, on the other hand, for an opposite reason, get too large a result by applying a proof-plane edgewise to a point of a surface where the curvature is continuous.

There are various ways of using the torsion balance in researches on distribution. We may either electrify the movable ball independently (as above described), or we may electrify it each time by contact with the proof-plane when it is inserted into the balance. It must be noticed that the repulsion of the movable ball is in the first case proportional to the charge on the proof-plane, but in the second to the square of the charge, so that the indications must be reduced differently.

In measuring we may either bring the movable ball to a fixed position, in which case the whole torsion required to keep it in this position is proportional to the charge on the proof-plane (or to its square, if the second of the above modes of operation be adopted), or we may simply observe the angle of equilibrium and calculate the quantity from that. It is supposed, for simplicity of explanation in all that follows, that the former of the two alternatives is adopted, and that the movable ball is always independently charged.

The gradual loss of electricity experienced more or less by every insulated conductor has already been alluded to. This loss forms one of the greatest difficulties to be encountered in such experiments as we are now describing. If we apply the proof-plane to a part of a conductor and take the balance reading, giving a torsion τ_1 say, and repeat the observation, after time t , we shall get a different torsion τ_2 , owing to the loss of electricity in the interval. This loss, partly if not mainly due to the insulating supports, depends on a great many circumstances, some of which are entirely beyond even the observation of the experimenter. We may admit, however, what experiment confirms within certain small limits, that the rate of loss of electricity is proportional to the charge, and we shall call $\frac{\tau_1 - \tau_2}{t}$ (the loss per unit of time on hypothesis of uniformity) the coefficient of dissipation (δ). This coefficient, although, as we have implied, tolerably constant for one experiment, will vary very much from experiment to experiment, and from day to day; it depends above all on the weather.

Supposing we have determined this coefficient by such an observation as the above, then we can calculate the torsion τ' , which we should have observed had we touched the body at any interval t' after the first experiment; for we have, provided t' be small,

$$\tau' = \tau_1 - \delta t' = \tau_2 + \delta(t - t').$$

In particular, if $t' = \frac{1}{2}t$, we have

$$\tau' = \frac{1}{2}(\tau_1 + \tau_2).$$

Coulomb used this principle in comparing the electric densities at two points A and A' of the same conductor. He touched the two points a number of times in succession, first A , then A' , then A again, and so on, observing the corresponding torsions $\tau_1, \tau_1', \tau_2, \tau_2', \&c.$, the intervals between the operations being very nearly equal. He thus got for the ratio of the densities at A and A' the values $\frac{\tau_1 + \tau_2}{2\tau_1}$

$\frac{2\tau_2}{\tau_1 + \tau_2}, \frac{\tau_2 + \tau_3}{2\tau_2}, \&c.$ These values ought to be all equal: the mean of them was taken as the best result.

In certain cases, where the rapidity of the electric dissipation was too great to allow the above method to be applied, Riess used the method of paired proof-planes. For a description of this, and for some elaborate calculations on the subject of electrical dissipation, the reader is referred to Riess's work.

The cage method is well adapted for experiments on distribution. The proof-plane, proof-sphere, or paired proof-planes may all be used in conjunction with it. If the cage be fairly well insulated, and a tolerably delicate Thomson's electrometer be used, so that the cage may

be made large, and the surface density on its outside therefore small, there will be little loss of the external charge; and the method has this advantage, that dissipation from the proof-plane inside the cage does not affect the result of the measurement in hand, it being indifferent, *qua* effect on the electrometer, whether the electricity inside the cage be on the proof-plane, in the air, or elsewhere, provided merely it be inside. The state of the cage as to electrified air, &c., is easily tested by the electrometer at any time.

Coulomb's Results.—If we electrify a sphere, and test the electrical density at two points of its surface. experiment will show, as would be expected from the symmetry of the body, that the density at the two points is the same. If we test the electric density at any point of a sphere, and then halve its charge by division with an equal neutral sphere, and test the electric density again, we shall find it half what it was before. The electric density at any point is therefore proportional to the whole charge on the sphere, or to the *mean density*, meaning by that the whole charge divided by the whole surface of the sphere.

If, instead of a sphere, we operate with an ellipsoid generated by the revolution of an ellipse about its major axis, we shall find that the electric density is not uniform as in the case of the sphere, but greater at the sharp ends of the major axis than at the equator, and the ratio of the densities increases indefinitely as we make the ellipsoid sharper and sharper. This leads us to state a principle of great importance in the theory of electrical distribution, viz., that the electrical density is very great at any pointed part of a conductor.

If we determine the ratio of the densities at two points of an ellipsoid,¹ diminish the charge, and redetermine the same ratio, we shall find that, although the actual densities are diminished, the ratio remains the same; and if we determine the density at any point of the ellipsoid, and then halve its charge by touching it with an equal and similar ellipsoid (they must be placed with their axes in the same straight line, and made to touch at the poles),² and redetermine the density at the same point as before, we shall find that the density in the second case is half that in the first. We have in fact, in general, the important proposition that—

The density at any point of a conductor is proportional to the whole charge on the conductor, or, what is the same, to the mean density.

The following case given by Coulomb is interesting; it shows the tendency of electricity towards the projecting parts, ends, or points of bodies. The conductor was a cylinder with hemispherical ends,—the length of the cylinder being 30 inches, its diameter 2 inches. Coulomb gives the following results:—

Distance from end.	Density.
5 in.	1.00
2	1.25
1	1.80
0	2.30

The density at the end is thus more than twice that at the middle.

Other results, taken from Coulomb's unpublished papers, may be found in Biot,³ Mascart, or Riess. His results for a circular disc we shall quote further on.

¹ We suppose in all these experiments that we are dealing with a single body, sufficiently distant not only from all electrified bodies but from all neutral conductors to be undisturbed by them. This condition is essential.

² It would not do to make the pole of one touch the equator of the other, or to place them otherwise unsymmetrically.

³ *Traité de Physique.*

Riess made a series of experiments on cubes, cones, &c.; but as these are not of theoretical interest, the calculation in such cases being beyond the powers of analysis at present, and as the use of the proof-plane or sphere with bodies where edges and points occur is not free from objection, we content ourselves with referring to Riess's work for an account of the results.

Coulomb made a series of experiments on bodies of different forms, which he built up out of spheres of different sizes, or out of spheres and cylinders. These are of very great interest, partly on account of the close agreement of some of the results with the deductions subsequently made by Poisson from the mathematical theory, and partly on account of the clearness with which they convey to the mind the general principles of electric distribution. His method in most cases was to build up the conductor and electrify it with all the different parts in contact, and then after separating the parts widely, to determine the *mean density* or the whole amount of electricity on each part by the proof-plane or otherwise.

For spheres in contact he found the following results,— $S, Q, \sigma; S', Q', \sigma'$ denoting the surface, quantity of electricity, and mean surface density for the two spheres respectively.

$\frac{S'}{S}$	$\frac{Q'}{Q}$	$\frac{\sigma'}{\sigma}$
3.36	3.8	1.09
14.80	11.1	1.33
62.00	37.6	1.65

From this it appears that although the whole amount of electricity on the large sphere is greater than that on the small, yet the mean density for the smaller sphere is greater than for the larger. The above result also affords an experimental illustration of the action of the earth in discharging a conductor connected with it. Comparing the conductor to the small sphere and the earth to the large sphere of 62 times the superficial area of the small one, if we start with charge Q on small sphere and then put the two in contact, the charge on the small sphere will be reduced to $\frac{1}{38.6} Q$, so that the mean density is diminished in the ratio 1 : 38.6. This ratio increases indefinitely as the ratio $\frac{S'}{S}$ increases. These results are in satisfactory agreement with Poisson's calculations. Coulomb was led by his observations to assign 2 as the limit of the ratio of the mean densities when the ratio of the diameters of the spheres is infinitely great; the mathematical theory gives $\frac{\pi^2}{6}$ or 1.65.

Coulomb also determined the density at the apex or smaller end of the body formed by two unequal spheres in contact. The following are his results, the mean density of the larger sphere being unity:—

Ratio of radii.	Density at apex.	
	Observed.	Calculated.
1	1.27	1.32
2	1.55	1.83
4	2.35	2.48
8	3.18	3.09
8	4.00	4.21

When two equal spheres are placed in contact the distribution will of course be the same in each; Coulomb found that, from the point of contact up to a point on the surface of either sphere distant from it by about 20°, no trace of electricity could be observed; at 30°, 60°, 90°,

180° respectively, the electric density had the relative values .20, .77, .96, 1.00. When the spheres are unequal the distribution is no longer alike on each. On the small sphere it is less uniform, and the density at the point of the small sphere diametrically opposite the point of contact is greater than anywhere else on the body. The distribution on the larger sphere is more uniform than on the smaller, and the more unequal the spheres are the more uniform is the distribution on the larger, and the smaller the unelectrified part in the neighbourhood of the point of contact.

The following results of Coulomb are useful illustrations of distribution on elongated and pointed bodies :—

Three equal spheres (2 in. diameter) in contact, with their centres in the same straight line: the mean densities were 1.34, 1.00, 1.34 on the spheres 1, 2, and 3 respectively.

Six equal spheres as before: mean densities on 1, 2, and 3 = 1.56, 1.05, 1.00.

Twelve equal spheres: mean densities on 1, 2, and 6 = 1.70, 1.14, 1.00.

Twenty-four equal spheres: mean densities on 1, 2, and 12 = 1.75, 1.07, 1.00.

Large (8 in. diameter) sphere with four small (2 in.) spheres applied to it, all the centres in line: the mean density on large sphere being 1, that on the small one next it was .60 that on the extreme small one 2.08.

Large sphere 1, and twenty-four (2 to 25) small ones: mean densities on 1, 2, 13, 24, 25 = 1.00, .60, 1.28, 1.46, 2.17.

MATHEMATICAL THEORY OF ELECTRICAL EQUILIBRIUM.

We take as the basis of our theory the assumptions already laid down under the head Provisional Theory, and in addition the precise elementary law of electrical action established by Coulomb. We shall also suppose that we have only perfect conductors and perfect non-conductors to deal with, the medium being in all cases the same, viz., air. When we have to deal with electrified non-conductors we shall suppose the electrification to be rigid, i.e. incapable of disturbance by any electric force we have to consider.

In our mathematical outline we have in view the requirements of the physical more than the mathematical student, and shall pass over many points of great interest and importance to the latter, for full treatment of which we must refer him to original sources, such as the classical papers of Green, the papers of Sir William Thomson, and the works of Gauss. Of peculiar interest mathematically is the elegant and powerful memoir of the last—*Allgemeine Lehrsätze in Beziehung auf die im verkehrten Verhältnisse des Quadrats der Entfernung wirkenden Anziehungs- und Abstossungskräfte*, in which will be found detailed discussions of the continuity of the integrals used in the potential theory, &c. The works of Green and Thomson are too well known in this country to require farther remark.

Defini-
tions.

When, in what follows, we speak of the *electric field*, we mean simply a portion of space which we are considering with reference to its electrical properties; it will be found conducive to clearness to regard that space as *bounded*. In general the natural boundary would be the walls of the experimenting room; but, for mathematical purposes, we shall, unless the contrary is stated, suppose our field to be bounded by a sphere of radius so great that the action at a point on its circumference due to an electrified body in the field is infinitely small.

The *resultant force* at a point in the electric field is the force which would be exerted on a unit of + electricity placed there without disturbing the electrical distribution elsewhere. It is plain that the resultant force has a definite magnitude and direction at every point in the field, and consequently is in modern mathematical language a *vector*. A curve drawn in the field such that its tangent at every point is in the direction of the resultant force at that point is called a *line of force*. We can draw such a line through every point of space, and if we suspend at any point a

small conducting needle, it is obvious, from what we have already laid down about induction, that it will take up a position very nearly parallel to the line of force; so that if we start from any point and carry the centre of the needle always in the direction in which the needle points we should trace out a line of force.

The *potential* at any point is the work done by a unit of + electricity in passing from that point to the infinitely distant boundary of the electric field, the electric distribution being supposed undisturbed. It is usual to call the infinitely distant boundary a place of zero potential. Zero is to be understood in the sense of "point or position from which we reckon."¹

Consider two points P, Q, infinitely near each other ^{Force} in the field, and draw a curve from P passing through Q to ∞. Then, if F be the component parallel to ^V PQ of the resultant force at P, we have by our definition

$$F \cdot PQ = V_P - V_Q;$$

or in differential notation

$$F ds = -dV,$$

hence

$$F = -\frac{dV}{ds} \quad \dots \dots (1),$$

and

$$V = \int_P^\infty F ds = \int_P^\infty (X dx + Y dy + Z dz) \quad \dots (2),$$

where V denotes the potential at P, and X, Y, Z the components parallel to the co-ordinate axes of the resultant electric force. We clearly have as particular cases of (1)

$$X = -\frac{dV}{dx} \quad Y = -\frac{dV}{dy} \quad Z = -\frac{dV}{dz} \quad \dots (3).$$

We may remark that, in all cases which we shall consider at present, the work done in passing from any point to any other point is the same whatever the intermediate path of our exploring unit. Hence V as above defined is a single valued function, and the formulae (3) gives the components of resultant force when V is known.

The work done by a unit of + electricity in passing by any path from P to Q is called the *electromotive force* from P to Q; it is obviously equal to the difference of the potentials at the two points. Thus

$$V_P - V_Q = \int_P^Q (X dx + Y dy + Z dz) \quad \dots (4).$$

is the electromotive force from P to Q.

Suppose we concentrate *m* units of electricity at any point P, and ^{Expt} require the potential due to this at a point Q, distant D from P. ^{sion} Applying (2), and, since any path to ∞ may be chosen, taking the ^{of V} integral along the production of PQ to ∞, we get ^{term}

$$V = \int_D^\infty \frac{m}{r^2} dr = \frac{m}{D} \quad \dots \dots (5). \quad \text{defn integ}$$

If we have any number of discrete points with charges *m*₁, *m*₂, *m*₃,... at distances D₁, D₂, D₃,... from Q, since the work done by the exploring unit under the action of the whole is got by adding up the work done under the action of each part separately, we clearly have

$$V = \frac{m_1}{D_1} + \frac{m_2}{D_2} + \&c. = \sum \frac{m}{D} \quad \dots (6).$$

From this we may pass to the case of a continuous volume distribution. If *ρ* be the volume density at the point ξηζ, and V the potential at xyz, we have

$$V = \iiint \frac{\rho d\xi d\eta d\zeta}{D} \quad \dots \dots (7),$$

where D denotes $\sqrt{(\xi-x)^2 + (\eta-y)^2 + (\zeta-z)^2}$, and the integral is to be extended over every part of the field where there is any charge,—or, which is the same thing, over the whole field, on the understanding that *ρ* = 0 where there is no charge.

If, as will generally be the case, the electricity is distributed on a surface in such a way that on an element *dS* of surface there is a quantity *σdS* of electricity, where *σ* is a finite surface density, then

$$V = \iint \frac{\sigma dS}{D} \quad \dots \dots (8),$$

where D has the same meaning as before, and the integral is extended all over the electrified surface or surfaces.

¹ It may be well here to warn the reader that measurement of potential is relative, just as much as measurement of distance is, and to caution him against the fallacious idea of absolute zero of potential.

of V. We may make here the important remark that, so long as ρ or σ is not infinite, the integrals in (7) and (8) are finite and continuous. This depends on the fact, which we cannot stop to prove, that the part of the potential at P, contributed by an infinitely small portion of electricity surrounding P, is infinitely small.

In practice, therefore, the electric potential is always continuous; for although we may in theory speak of discrete points and electrified lines where finite electrification is condensed into infinitely small space, yet no such cases ever occur in nature. It may also be shown for any electrical system of finite extent, that, as the distance of P from O, any fixed point at a finite distance from the system is increased indefinitely, the potential at P approaches more and more nearly the value $\frac{M}{D}$, where M is the algebraical sum of all the electricity in the system, and D the distance of P from O. Hence at any point infinitely distant from O, $V=0$.

We next proceed to prove the following proposition, which will form the basis of the subsequent theory:—

The surface integral of electric induction taken all over the surface inclosing any space is equal to 4π times the algebraical sum of all the electricity in that space.

By the electric induction across any element of the surface (taken so small that the resultant force at every point of it may be regarded as uniform) is meant the product of the area of the element into the component of the resultant force in the direction of the normal to the element which is drawn outwards with respect to the inclosed space. Thus dS being an element of surface, ϵ the angle between the positive direction of the resultant force R and the outward normal ν , and E the sum of all the electricity in the inclosed space, the proposition in symbols is—

$$\iint R \cos \epsilon dS = 4\pi E \quad (9).$$

We shall prove it in the manner most naturally suggested by the theory of electrical elements acting at a distance, by first showing that it is true for a single element e either outside or inside the surface. Let us suppose e to be at a point P, fig. 11, within S, which for greater generality we may suppose to be a re-entrant surface. Draw a small cone of vertical solid angle $d\omega$ at P, and let it cut the surface in the elements QR, Q'R', Q''R''; let the outward normals to these be QM, Q'M', Q''M''. The elements of the surface integral contributed by QR, Q'R', Q''R'' are obviously $\frac{QR \cos \epsilon}{PR^2}$, &c.; but $QR = \frac{d\omega PR^2}{\cos \epsilon}$,

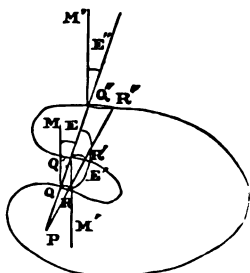


Fig. 11.

$QR' = -\frac{d\omega PR'^2}{\cos \epsilon'}$, and $Q''R'' = \frac{d\omega PR''^2}{\cos \epsilon''}$; hence the three elements of the integral become $+cd\omega$, $-cd\omega$, $+cd\omega$; and the sum is $cd\omega$. Adding now the contributions from all the little cones which fill up the solid angle of 4π about P, we get

$$\iint R \cos \epsilon dS = e \iint d\omega = 4\pi e.$$

Had the point P been outside, the numbers of emergences and entrances would have been equal, the contribution of each cone zero, and on the whole

$$\iint R \cos \epsilon dS = 0.$$

Combining these results, we see that the proposition is true for a single element. Hence, by summation for all the elements, we can at once extend it to any electrical system; for all the elements external to S give zero, and all the internal elements give $4\pi \Sigma e = 4\pi E$.

Let us apply the above proposition to the space enclosed by the infinitely small parallelepiped whose centre is at xyz , and the co-ordinates of whose angles are $x \pm \frac{1}{2}dx$, $y \pm \frac{1}{2}dy$, $z \pm \frac{1}{2}dz$. The contributions to the surface integral from the two faces perpendicular to the x -axis are $-\left(\frac{dV}{dx} + \frac{dx}{2} \frac{d^2V}{dx^2}\right) dydz$ and $\left(\frac{dV}{dx} - \frac{dx}{2} \frac{d^2V}{dx^2}\right) dydz$.

Adding these and the four parts from the remaining sides, and equating to $4\pi \rho dx dy dz$, which is the $4\pi E$ in this case, we have

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} + 4\pi \rho = 0,$$

or, as it is usually abbreviated,

$$\nabla^2 V + 4\pi \rho = 0 \quad (10).$$

Equation (10), originally found by Laplace for the case $\rho=0$, and extended by Poisson, has been called the characteristic equation of the potential. It may be applied at any point where ρ is finite and the electric force continuous. It might be shown by examining the integrals representing X, Y, Z , and $\frac{dV}{dx}$, &c., that the electric force is continuous wherever there is finite volume density. Equation (10) may be looked on either as an equation to determine the potential when ρ is given, or as an equation to determine ρ when V is given. We shall have occasion to use it in both ways.

The characteristic equation cannot be applied in the form (10) when the resultant force is discontinuous. This will be found to be the case at a surface on which electricity is distributed with finite surface density. Let us consider the values of the resultant force at two points, P and Q, infinitely near each other, but on opposite sides of such a surface. Resolve the resultant force tangentially and normally to the surface. If we consider the part of the force which arises from an infinitely small circular disc, whose radius, though infinitely small, is yet infinitely great compared with the distance between P and Q, we see that infinitely little is contributed to the tangential component at P or Q by this disc, while it can be readily shown that the part of the normal component arising therefrom is $2\pi\sigma$, directed from the disc in each case, when σ is the surface density. Hence, since the part of the resultant force arising from all the rest of the electrified system obviously is not discontinuous between P and Q, the tangential component is continuous when we pass through an electrified surface, but the normal component is suddenly altered by $4\pi\sigma$.

For a thorough investigation of these points the reader is referred to Gauss or Green. We can arrive very readily at the amount of the discontinuity of the normal force by applying (9) to the cylinder formed by carrying an infinitely short generating line round the element dS , so that one end of the cylinder is on one side of dS and the other on the other, the lateral dimensions being infinitely small, but still infinitely greater than the longitudinal. The only part of the integral which is of the order of dS is the part arising from the two ends; hence if N, N' be the value of the normal components on the two sides of S , we clearly get

$$(N - N') dS = 4\pi \sigma dS, \text{ or } N - N' = 4\pi \sigma.$$

If V_1, V_2 denote the potentials on the two sides of S , and ν_1, ν_2 the normals to dS , drawn towards these sides respectively, then we may obviously write our equation

$$\frac{dV_1}{d\nu_1} + \frac{dV_2}{d\nu_2} + 4\pi \sigma = 0 \quad (11).$$

Written in this form the equation has been called the surface characteristic equation of the potential. It may be looked upon as a characteristic surface condition, which must be fulfilled by the values of V on the two sides of an electrified surface on which the surface density σ is given, and where, in consequence, there is discontinuity in the first differential coefficients of V ; or it may be looked on as an equation to determine σ when V_1 and V_2 are given.

We have seen that we can draw through every point of Level the electric field a line of force. A surface constructed so that the potential at every point of it has the same value is called an *equipotential or level surface*. We can obviously draw such a surface passing through every point of the field. It is clear, too, that the line of force at every point must be perpendicular to the level surface passing through that point. For since no work is done on a unit of + electricity in passing from one point of a level surface to a neighbouring point, there can be no component of the resultant force tangential to the surface; in other words, the direction of the resultant force is perpendicular to the surface. This is expressed otherwise by saying that the lines of force are orthogonal trajectories to the level surface.

If we take a small portion of a level surface, and draw through every point of the boundary a line of force, we shall thus generate a tubular surface which will cut orthogonally every level surface which it meets. Such a surface is called a *tube of force*.

Let a tube of force cut two level surfaces in the elements dS and dS' . Apply to the space contained by the part of the

tube between the surfaces our fundamental equation (9). We thus get, since there is no normal component perpendicular to the generating lines of the tube,

$$RdS - R'dS' = 0, \quad \dots \quad (12),$$

provided the tube does not cut through electrified matter between the two surfaces. Here R and R' denote the resultant force at dS and $d'S'$, which are supposed so small that the force may be considered uniform all over each of them. It appears then that *the product of the resultant force into the area of the normal section of a tube of force is constant for the same tube so long as it does not cut through electrified matter; or what amounts to the same, the resultant force at any point of a tube of force varies inversely as the normal section of the tube at that point.*

Important property of tubes of force
 $RdS = R'dS'$

If we divide up any level surface into a series of small elements, such that the product RdS is constant for each element and equal to unity, and draw tubes of force through each small element, then the electric induction through any finite area of the surface is equal to the number of tubes of force which pass through that area; for if n be that number, we have, summing over the whole of the area—

$$\Sigma RdS = n \quad \dots \quad (13),$$

the left hand side of which is the electric induction through the finite area. It is clear, from the constancy of the product RdS for each tube of force, that if this is true for one level surface it will be true for every other cut by the tubes of force. It is evident that the proposition is true for any surface, whether a level surface or not, as may be seen by projecting the area considered by lines of force on a level surface, and applying to the cylinder thus formed the surface integral of electric induction, it being remarked as obvious that the same number of tubes of force pass through the area as through the projection. This enables us to state the proposition involved in equation (9) in the following manner:—

Charge measured by tubes of force.

The excess of the number of tubes of forces which leave a closed surface over the number which enter is equal to 4π times the algebraical sum of all the electricity within the surface.

(N.B.—The positive direction of a line of force is that direction in which a unit of + electricity would tend to move along it.) This proposition enables us to measure the charge of a body by means of the lines¹ of force. We have only to draw a surface inclosing the body, and very near to it, and count the lines of force entering and leaving the surface. If the number of the latter, diminished by the number of the former, be divided by 4π , the result is the charge on the body.

If we apply (13) to a portion of an equipotential surface so small that R may be considered uniform over the whole of it, we may write

$$R = \frac{n}{dS} \quad \dots \quad (14);$$

Resultant force measured by lines of force.

or in words:—*The resultant force at any point is equal to the number of lines of force per unit of area of level surface at that point, meaning thereby the number of lines of force which would pass through a unit of area of level surface if the force were uniform throughout, and equal to its value at the point considered.*

We are now able to express by means of the lines of force the resultant force at any point of the field, and the charge in any element of space. The electrical language thus constructed was invented by Faraday, who continually used it in his electrical researches. In the hands of Sir William Thomson, and particularly of Professor Clerk Maxwell, this language has become capable of representing, not

¹ Here we drop the distinction between line and tube of force. We shall hereafter suppose the lines of force to be always drawn so as to form unit tubes, and shall speak of these tubes as lines of force, thereby following the usual custom.

only qualitatively but also quantitatively, with mathematical accuracy, the state of the electric field. It has the additional advantages of being well fitted for the use of the practical electrician, and of lending itself very readily to graphical representation.

It will be convenient, before passing to electrical applications, to state here another general property of the potential which follows from our fundamental proposition.

The potential cannot have a maximum or minimum value at a point where there is no electricity.

Maximum or minimum potential impossible in free space.

For if a maximum value were possible, we could draw round the point a surface at every point of which the potential was decreasing outwards; consequently at every point of this surface the normal component of the resultant force in the outward direction would be positive, and a positive number of lines of force would leave the surface. But this is impossible, since, by our hypothesis, there is no electricity within. Similarly there could be no minimum value.

From this it follows at once that *if the potential have the same value at every point of the boundary of a space in which there is no electrified body, then the potential is constant throughout that space, and equal to the value at the boundary.* For if the potential at any point within had any value greater or less than the value at the boundary, this would be a case of maximum or minimum potential at a point in free space, which we have seen to be impossible.

Case of space bounded by its surface

In order that there may be electrical equilibrium in a perfect conductor, it is necessary that the resultant electric force should be zero at every point of its substance. For if it were not so at any point the positive electricity there would move in the direction of the resultant force and the negative electricity in the opposite direction, which is inconsistent with our supposition of equilibrium. This condition must be satisfied at any point of the conductor, however near the surface. At the surface there must be no tangential component of resultant force, otherwise electricity would move along the surface. In other words, the resultant force at the surface must be normal; its magnitude is not otherwise restricted;² for by our hypothesis electricity cannot penetrate into the non-conducting medium.

These conditions are clearly sufficient. We may sum them up in the following single statement:—

If the electricity in any conductor is in equilibrium, the potential must have the same value at every point in its substance.

Condition of electrical equilibrium

For if the potential be constant, its differential coefficients are zero, so that inside the conductor the resultant force vanishes. Also the surface of the conductor is a level surface, and therefore the resultant force is everywhere normal to it. This constant value of the potential we shall henceforth speak of as the *potential of the conductor*.

Since the potential is constant at every point in the substance of a charged conductor, we have at every point $\nabla^2 V = 0$, and hence by the equation of Poisson $\rho = 0$; that is, there is no electricity in the substance of the conductor. We thus get, as a theoretical conclusion from our hypothesis, the result already suggested by experiment, that electricity resides wholly on the surface of conductors.

If we apply the surface characteristic equation to any point of the surface of a conductor, we get

$$\sigma = -\frac{1}{4\pi} \frac{dV}{dn} = \frac{R}{4\pi}, \quad \dots \quad (15),$$

which gives the surface density in terms of the resultant force and reciprocally.

We may put this into the language of the lines of force by saying that *the charge on any portion of the surface of a conductor is equal to the number of lines of force issuing from it divided by 4π .*

Since the surface of a conductor in electric equilibrium

² Of course in practice there is an upper limit, at which disruptive discharge occurs.

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is always a level surface, it follows, from what we have already proved about a space bounded by a surface of constant potential, that, *inside a hollow conductor the potential is constant, provided there be no electrified bodies within.* This is true, no matter how we electrify the conductor or what electrified bodies there may be *outside.* Hence, if we inclose any conductor A completely within another B, then electrify B and put A in metallic communication with it, A will not become charged either + or -; for, A being at the same potential as B, electricity will not tend to flow from the one to the other. This is in reality Biot's¹ experiment with the hemispheres, to which we have already alluded; only the point of view is slightly changed. The most striking experiment ever made in illustration of the present principle is that described by Faraday in his *Experimental Researches.* He constructed a hollow cube (12 feet in the edge) of conducting matter, and insulated it in the lecture-room of the Royal Institution. We quote in his own words the part of his description which bears on the present question:—

"1172. I put a delicate gold-leaf electrometer within the cube, and then charged the whole by an outside communication, very strongly for some time² together; but neither during the charge or after the discharge did the electrometer or air within show the least sign of electricity. . . . I went into the cube and lived in it, and using all other tests of electrical states, I could not find the least influence upon them, though all the time the outside of the cube was powerfully charged, and large sparks and brushes were darting off from every point of its outer surface."

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The proposition that the potential is constant inside a hollow conductor containing no electrified bodies may be regarded as one of the most firmly established in the whole of experimental science. The experiments on which it rests are of extreme delicacy. It is of the greatest theoretical importance; for we can deduce from it the law of the inverse square. Taking the particular case of a spherical shell, uninfluenced by other bodies, on which of course the electrical distribution must from symmetry be uniform, it can be demonstrated mathematically that, if we assume the action between two elements of electricity to be a function of the distance between them, then that function must be the inverse square, in order that the potential may be constant throughout the interior. A demonstration of this proposition was given by Cavendish, who saw its importance; a more elaborate proof was afterwards given by Laplace; for a very elegant and simple demonstration we refer the mathematical reader to Clerk Maxwell's *Electricity*, vol. i. § 74. This must be regarded as by far the most satisfactory evidence for the law of the inverse square; for the delicacy of the tests involved infinitely surpasses that of the measurements made with the torsion balance; and now that we have instruments of greatly increased sensitiveness, like Thomson's quadrant electrometer, the experimental evidence might be still further strengthened.

general
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In the problem to determine the distribution of electricity in a given system of conductors, the data are in most cases either the charge or the potential for each conductor. If the conductor is insulated it can neither give nor lose electricity, its charge is therefore given. If, on the other hand, it be connected with some inexhaustible source of electricity at a constant potential, its potential is given. Such a source the earth is assumed to be; and we shall henceforth take the potential of the earth as zero, and reckon the potential of all other bodies with reference to it. If all our electrical experiments were con-

ducted in a space inclosed by a perfectly conducting envelope, the potential of this envelope would be the natural zero of our reckoning.

It will be useful to analyse more closely the distribution on a system of conductors, in order to see how far the above data really determine the solution of the electrical problem. For this purpose the following proposition is useful. If e_1, e_2, \dots, e_n be the charges at the points 1, 2, . . . n of any system, and V the potential at P, and if V' be the potential at P due to e_1', e_2', \dots, e_n' at 1, 2, . . . n, then the potential at P due to $e_1 + e_1', e_2 + e_2', \dots$ at 1, 2, . . . is $V + V'$. This principle follows at once from the definition of the potential as a sum formed by the mere addition of parts due to all the single elements of the system.

Principle
of elec-
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sition.

Applied to a system of conductors in equilibrium, it may evidently be stated thus: If E_1, E_2, \dots, E_n and V_1, V_2, \dots, V_n be the respective charges and potentials for the conductors 1, 2, 3 . . . n in a state of equilibrium, E_1', E_2', \dots, E_n' and V_1', V_2', \dots, V_n' corresponding charges and potentials for another state of equilibrium, then $E_1 + E_1', \dots, E_n + E_n', V_1 + V_1', \dots, V_n + V_n'$ will be corresponding charges and potentials for a third state of equilibrium.

Suppose that in the system of conductors 1, 2, 3, . . . n the conductor 1 is kept at potential 1 and all the others at potential zero, then it can be shown that there is one and only one distribution of electricity fulfilling these conditions. Mathematically stated, the problem is to determine a function V , which shall satisfy the equation $\nabla^2 V = 0$ throughout the space unoccupied by conductors, and have the values 1, 0, 0, . . . 0 was respectively at each point of the surfaces of 1, 2, . . . n respectively.

Consider the integral

$$I = \iiint \left\{ \frac{\partial V}{\partial x}^2 + \frac{\partial V}{\partial y}^2 + \frac{\partial V}{\partial z}^2 \right\} dx dy dz \dots (16),$$

where the integration is extended all over the space unoccupied by conductors. If we consider all the values which this integral may have, consistent with the boundary conditions $V = 1, V = 0, \dots$ &c. at the surfaces of 1, 2, . . . &c., it is obvious that there must be a minimum value; for the integral is essentially positive, and cannot become less than zero.

$$\text{Now } \delta I = 2 \iiint \left(\frac{dV}{dx} \frac{\delta V}{dx} + \text{&c.} \right) dx dy dz \\ - 2 \iiint \delta V \nabla^2 V dx dy dz \dots (17)$$

by partial integration. The surface terms vanish, since $\delta V = 0$ at every surface. Hence $\nabla^2 V = 0$ is the condition for a maximum or minimum value of I , and since we know that a minimum value exists, there must be a solution of this equation. It can, moreover, be shown, by a method which we shall apply below to the more general problem, that there is only one solution of $\nabla^2 V = 0$ consistent with the given conditions, and this will of course be that which makes I a minimum. If our mathematical methods were powerful enough to determine V , we might proceed to find the surface density for each conductor by means of the formula $\sigma = -\frac{1}{4\pi} \frac{dV}{dn}$; then the charges on the conductors could be found

by means of the integral $-\frac{1}{4\pi} \iint \frac{dV}{dn} dS$. In very few cases indeed could we actually find these charges; we have, however, demonstrated their existence and shown that our problem is definite.

Let these charges on 1, 2, . . . n be called q_1, q_2, \dots, q_n . Corresponding to the data 0, 1, 0, . . . 0 for the potentials of 1, 2, . . . n, we should get a series of charges $q_{21}, q_{22}, \dots, q_{2n}$, and so on; $q_{11}, q_{22}, q_{33}, \dots$ are called the coefficients of self-induction or capacity for the conductors 1, 2, 3, . . .; q_{12}, q_{13}, \dots , are called the coefficients of induction of 1 on 2, 1 on 3, &c. It is obvious that these coefficients depend solely on the form and relative position of the conductors. It follows, from the principle of the superposition, that, if 1, 2, . . . n be at the potentials $V_1, 0, 0, \dots, 0$, then the charges on them will be $q_1 V_1, q_{12} V_1, \dots, q_n V_n$. We

Coeffi-
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and in-
duction.

¹ The experiment was first made by Cavendish. There is an account of it in his hitherto unpublished papers.

² Faraday was looking for what he called the absolute charge of matter; incidentally the experiment illustrates the point we are discussing.

may construct then a series of states of equilibrium represented thus:—

Potential,	V_1	0	0	...	0
Charge,	$q_{11}V_1$	$q_{12}V_1$	$q_{13}V_1$...	$q_{1n}V_1$
Potential,	0	V_2	0	...	0
Charge,	$q_{21}V_2$	$q_{22}V_2$	$q_{23}V_2$...	$q_{2n}V_2$

and so on. Superposing all these, we get a system in equilibrium, in which the potentials are V_1, V_2, \dots, V_n and the charges

$$\left. \begin{aligned} E_1 &= q_{11}V_1 + q_{12}V_2 + \dots + q_{1n}V_n \\ E_2 &= q_{21}V_1 + q_{22}V_2 + \dots + q_{2n}V_n \\ &\text{&c.} \end{aligned} \right\} \dots (18).$$

It appears therefore that the $2n$ quantities E_1 , &c., V_1 , &c., are connected by n linear equations; so that when n of them are given, the rest can be determined in terms of these in a definite manner.

Returning then to our general problem, we see that, when either the charge or the potential is given for each conductor, the electrical problem is determinate, and a solution is given by the linear equations of (18). The potential at any point of the field can be written down very easily. Suppose in fact v_1 to be the value at the point P of the function V which we determined in solving the case where the potentials 1, 0, 0, ... 0 are given for 1, 2, ... n , v_2 the corresponding function for the case 0, 1, 0, ... 0, and so on. Then the potential at P in the general case is obviously

$$V = V_1v_1 + V_2v_2 + \dots + V_nv_n \dots (19),$$

where v_1, v_2, \dots, v_n are all known functions, and V_1, V_2, \dots, V_n are all either given, or determined in terms of given quantities by the equations (18).

It is very easy to show that there is no other solution of the problem than the one we have found.

Suppose in fact that V' is a function different from V , which satisfies all the conditions of the problem. Consider the function $U = V - V'$, since V and V' both satisfy the equation $\nabla^2 V = 0$, we have $\nabla^2 U = 0$. Also at surfaces where V is given $U = 0$. At surfaces where V is not given, we have $U = \text{constant} - \text{constant} = \text{constant}$; and, since in this case the charge will be given, we shall have

$$\iint \frac{dV}{d\nu} dS = \iint \frac{dV'}{d\nu} dS; \text{ and therefore } \iint \frac{dU}{d\nu} dS = 0.$$

Now we have

$$\begin{aligned} &\iiint \left\{ \left(\frac{dV}{dx} - \frac{dV'}{dx} \right)^2 + \text{&c.} \right\} dx dy dz \\ &\iiint \left\{ \left(\frac{dU}{dx} \right)^2 + \left(\frac{dU}{dy} \right)^2 + \left(\frac{dU}{dz} \right)^2 \right\} dx dy dz \\ &= \iint \frac{dU}{d\nu} U dS - \iiint U \nabla^2 U dx dy dz. \end{aligned}$$

The first term vanishes for all the surfaces,—for some because $U = 0$, for others because U is constant and $\iint \frac{dU}{d\nu} dS = 0$; and the second term vanishes because $\nabla^2 U = 0$.

Hence the integral on the left hand must vanish, and that too element by element, since every element is positive. Hence we must have

$$\frac{dV}{dx} = \frac{dV'}{dx}, \quad \frac{dV}{dy} = \frac{dV'}{dy}, \quad \frac{dV}{dz} = \frac{dV'}{dz}.$$

Hence V and V' can only differ by a constant. But such difference is precluded by the boundary conditions. Hence the functions are identical; in other words, there is but one solution to the problem we have proposed.

It is very easy to show, by methods of which we have already had an example, that the value of V thus found makes the integral

$$\frac{1}{8\pi} \iiint \left(\left(\frac{dV}{dx} \right)^2 + \left(\frac{dV}{dy} \right)^2 + \left(\frac{dV}{dz} \right)^2 \right) dx dy dz$$

a minimum. Now, we shall show directly that this inte-

gral represents the potential energy of the system. It follows, therefore, that the distribution which we have found is in *stable* equilibrium.

If we solve the equations (18), we shall get

$$\left. \begin{aligned} V_1 &= p_{11}E_1 + p_{12}E_2 + \dots + p_{1n}E_n \\ V_2 &= p_{21}E_1 + p_{22}E_2 + \dots + p_{2n}E_n \\ &\text{&c.} \end{aligned} \right\} \dots (20).$$

A set of equations which we might obviously have arrived at by first principles. The physical meaning of the coefficients $p_{11}, p_{12},$ and p_{1n} is very obvious; they are the potentials, corresponding to a state of equilibrium, in which the charges on 1, 2, 3, ... n are 1, 0, 0, ... 0, and so on. $p_{11}, p_{12},$ &c., are called coefficients of potential; and, *mutatis mutandis*, all the remarks already made about $q_{11}, q_{12},$ &c., apply to them. Many interesting and important theorems have been proved about these coefficients, for which we refer the reader to Maxwell (*Electricity*, vol. i. chap. 2), whose treatment of the subject we have in the main been following. One of these, of great importance, we shall prove here, because it leads us to state a very important general theorem, which we shall have occasion to use again.

The mutual potential energy of two electrical systems, A and B, is the work done in removing the two systems to an infinite distance from each other, the internal arrangement of each system being supposed unaltered during the process. It is clear that we may suppose either that A is fixed and B moves off to infinity, or that B is fixed and A moves; the work done in both cases is, by Newton's third law of motion, the same. This is sometimes expressed by saying that *the potential of A on B is the same as that of B on A*.

In fact, the expression for the mutual potential energy is

$$\sum \frac{qq'}{D} \dots (21),$$

where q is any element of electricity belonging to A, and q' any element belonging to B, and D is the distance between them, the summation being extended so as to include every pair of elements. We may arrange (21) as follows:—

$$q'_1 \frac{q}{D} + q'_2 \frac{q}{D} + \text{&c.},$$

each group belonging to a point in B, or, as we may write it,

$$q'_1 V_1 + q'_2 V_2 + \text{&c.}, \text{ or } \sum q' V.$$

We may also arrange (21) in the form

$$q_1 \frac{q'}{D} + q_2 \frac{q'}{D} + \text{&c.},$$

each group belonging to a point in A. Hence we have the following equalities:—

$$\sum q' V = \sum \frac{qq'}{D} = \sum q V' \dots (22).$$

The first and last of these expressions are called respectively the potential of A on B, and the potential of B on A, and this equality explains the statement made above.

The two systems A and B may be different states of equilibrium of the same system, if we choose. In this case we may still farther modify the expression in (22), and write

$$V_1 \sum q' + V_2 \sum q' + \text{&c.} = V'_1 \sum q + V'_2 \sum q + \text{&c.} \quad (\text{See Gauss, } l.c.)$$

So that we may state the proposition thus:—If $E_1, E_2, \dots, E_n, V_1, V_2, \dots, V_n$, and $E'_1, E'_2, \dots, E'_n, V'_1, V'_2, \dots, V'_n$ be the respective charges and potentials of the conductors in two different states of equilibrium, then we have

$$\sum E' V = \sum E V' \dots (23).$$

If we take for the two states of the system

$$\frac{E}{V} \parallel \begin{vmatrix} q_{11} & q_{12} & q_{13} & \dots & q_{1n} \\ 1 & 0 & 0 & \dots & 0 \end{vmatrix}$$

$$\text{and } \frac{E'}{V'} \parallel \begin{vmatrix} q_{21} & q_{22} & q_{23} & \dots & q_{2n} \\ 0 & 1 & 0 & \dots & 0 \end{vmatrix}$$

equation (23) becomes

$$q_{21} = q_{12} \dots (24),$$

or, in words, *the coefficient of induction of 1 on 2 is equal to that of 2 on 1.*

There is one more general theorem on electrical distribution which, from its great practical importance, deserves a place here. Suppose we take a hollow conductor of any form, place any electrical system inside it, and connect the conductor with the earth, then equilibrium will be established, in such a way that the potential of every portion of the conductor is zero. Now, the potential being zero at all infinitely distant points, we may regard the outside space as inclosed by a surface of zero potential; hence the potential at every point in this space must be the same, and there can be no electrical action anywhere outside.

Again, removing the internal system, let us place any system outside the conductor, and, besides, charge it to any desired extent, keeping it insulated this time. Then the outer and inner surfaces of the conductor will be level surfaces; and, since there is no electricity inside the inner surface, the potential in the interior will be constant. Hence the external system, in a state of equilibrium, exerts no action whatever within. Now we may evidently, without mutual disturbance, superpose such an internal and external system as we have described, and still get a system in equilibrium. It is, moreover, clear that we can in this way satisfy the most general conditions that can be assigned. Hence, since we know that there can be only one solution of the problem of electrical equilibrium, the synthetical one thus obtained represents the actual state of affairs. When, therefore, a hollow conductor with any external and internal systems is in equilibrium, *the equilibrium of the internal is independent of that of the external system.*

Moreover, if we draw any surface in the substance of the hollow conductor, no lines of force cross it in one direction or the other; therefore the whole amount of electricity within must be zero; in other words, *the charge on the internal surface of the conductor is equal and opposite to the algebraical sum of the charges on all the bodies within.*

These propositions contain the principle of what are called electrical screens, i.e. sheets of metal used to defend electrical instruments, &c., from external influences. On the practical efficiency of gratings in this way, see Maxwell (§ 203); on the application to the theory of lightning conductors, see a paper by him in the reports of the British Association for 1876.

If we take the simple case where there is no external system, but only a charge on the hollow conductor, we get a complete explanation of Faraday's ice-pail experiment.

The potential energy of a system of charged conductors is the work required to bring them from a neutral state to the charges and potentials which they have at any time. The state of zero potential energy here contemplated is of course that in which there is an equal amount of + and - electricity everywhere in the system, or, as we might put it, the state in which there is no electrical separation. Now if Q denote the potential energy of the system, we have with the notation of (21)

$$Q = \sum \frac{qq'}{D} \dots \dots \dots (25),$$

the summation including every pair of elements in the system. If the system be in equilibrium, then, reasoning as above, it is obvious that $\sum EV$ is just twice $\sum \frac{qq'}{D}$, inasmuch as each pair of elements will come in twice. Hence we get

$$Q = \frac{1}{2} \sum EV \dots \dots \dots (26).$$

This is an expression of the greatest importance. We can give it various forms; by means of (18) and (20) we get

$$Q = \frac{1}{2} \sum \sum q_r V_r = \frac{1}{2} \sum \sum p_r E_r \dots \dots (27).$$

So that Q is a homogeneous quadratic function of the potentials or of the charges. If, therefore, we increase the potentials of *all* the conductors, or the charges of *all* the conductors in any ratio, we increase thereby the potential energy in the duplicate of that ratio.

We can by a transformation, which is a particular case of a theorem of Green's, obtain a very remarkable volume integral for the potential energy of an electrical system.

Let V denote the potential at any point in the field. Consider the integral

$$\frac{1}{8\pi} \iiint \left(\left| \frac{dV}{dx} \right|^2 + \left| \frac{dV}{dy} \right|^2 + \left| \frac{dV}{dz} \right|^2 \right) dx dy dz,$$

where the integration is to be extended throughout the whole of the space unoccupied by conductors. We have by partial integration

$$\iiint \frac{dV}{dx} dx dy dz = \iint V \frac{dV}{dx} dy dz - \iiint V \frac{d^2 V}{dx^2} dx dy dz,$$

and two similar equations. Hence

$$\begin{aligned} \frac{1}{8\pi} \iiint \left(\left| \frac{dV}{dx} \right|^2 + \left| \frac{dV}{dy} \right|^2 + \left| \frac{dV}{dz} \right|^2 \right) dx dy dz \\ = \frac{1}{8\pi} \iint V \frac{dV}{dv} dS - \frac{1}{8\pi} \iiint V \nabla^2 V dx dy dz, \end{aligned}$$

where the surface integration extends over the surface of all the conductors, and it is to be noticed that dv is drawn from the conductor into the insulating medium. If ρ and σ be volume and surface densities,

$$\sigma = -\frac{1}{4\pi} \frac{dV}{dv}, \text{ and } \rho = -\frac{1}{4\pi} \nabla^2 V.$$

Thus we get

$$\begin{aligned} \frac{1}{8\pi} \iiint \left(\left| \frac{dV}{dx} \right|^2 + \left| \frac{dV}{dy} \right|^2 + \left| \frac{dV}{dz} \right|^2 \right) dx dy dz \\ = \frac{1}{2} \iint V \sigma dS + \frac{1}{2} \iiint V \rho dx dy dz \dots \dots (28). \end{aligned}$$

This result includes a more general case than our present one; for it shows that the potential energy of an electrical system is given by the integral on the left hand side in all cases, whether there is equilibrium or not. It is not even restricted to the case of perfect conductors and perfect non-conductors, for a slight modification of our preliminary statements would include that case as well. At present, however, we have $\rho = 0$ everywhere, and V constant at the surface and in the substance of each conductor, so that the right hand side is simply the expression $\frac{1}{2} \sum EV$ which we have already found for the potential energy; we may therefore write

$$\begin{aligned} Q = \frac{1}{8\pi} \iiint \left(\left| \frac{dV}{dx} \right|^2 + \left| \frac{dV}{dy} \right|^2 + \left| \frac{dV}{dz} \right|^2 \right) dx dy dz \\ = \frac{1}{8\pi} \iiint R^2 dv \dots \dots \dots (29), \end{aligned}$$

R being the resultant force at any point of the field, and dv the element of volume. It is clear that we may if we like extend the integration over the *whole* field, since in the substance of any conductor $R = 0$.

When we know the potential energy of an electrical system it is very easy to find the force which resists or tends to produce any change of configuration. Two particular cases are of common occurrence and of considerable interest. First, let the charges on all the conductors be kept constant. Let the variable which is altered by the supposed change of configuration be ϕ , and let Φ be the corresponding force¹ tending to increase ϕ . Then, since no energy is supplied from without, if we suppose the displacement made infinitely slowly, so that no kinetic energy is generated, we have

¹ Or generalized force component, i.e., the amount of work per unit of ϕ done in increasing ϕ .

Force tending to produce any change of configuration.

$$\phi \delta \phi + \delta Q = 0 \quad \dots \dots \dots (30).$$

$$\text{or} \quad \phi = -\frac{\delta Q}{\delta \phi} \quad \dots \dots \dots (31).$$

Referring to the second of the expressions in (27), we see that this may be written

$$\phi = -\frac{1}{2} \sum_{r=1}^{r=n} \sum_{s=1}^{s=n} E_r E_s \frac{dp_{rs}}{d\phi}.$$

From this it is evident that in similarly electrified states of the same system the force tending to produce a given displacement varies as the square of the electrification. It is important to remark that in the present case the system tends to move so that its potential energy is *decreased*.

Secondly, let us suppose that the potentials of the different conductors are kept constant during any displacement, energy being supplied from without.

We shall suppose the change made in two steps. First, we shall suppose the given displacement to take place while the charges remain constant. On this supposition the force exerted will, to the first order of small quantities, be the same as that exerted when we suppose the potential not to vary; hence

$$\phi \delta \phi + \frac{1}{2} \sum E \delta V = 0.$$

Next, supply energy from without so that the potentials become again V_1, V_2, \dots and the charges $E_1 + \delta E_1, E_2 + \delta E_2, \dots$. The final result will be the same, to first order of small quantities, as if the two changes had been made simultaneously. Now, applying the theorem of mutual potential energy to the two states of our system,

$$\frac{E_1}{V_1} \left\| \frac{E_1}{V_1 + \delta V_1} \right\| \frac{E_2}{V_2 + \delta V_2} \dots \text{and} \frac{E_1}{V_1} \left\| \frac{E_1 + \delta E_1}{V_1} \right\| \frac{E_2 + \delta E_2}{V_2} \dots,$$

we have $\sum (E_1 + \delta E_1)(V_1 + \delta V_1) = \sum (EV),$

hence $\sum E \delta V = -\sum V \delta E \quad \dots \dots \dots (32);$

therefore $\phi = -\frac{1}{2} \sum E \frac{dV}{d\phi} = -\frac{1}{2} \sum V \frac{dE}{d\phi} = \frac{dQ}{d\phi} (V \text{ const.}) \quad \dots \dots (33).$

By (27) this may be written

$$\phi = \frac{1}{2} \sum_{r=1}^{r=n} \sum_{s=1}^{s=n} V_r V_s \frac{dq_{rs}}{d\phi}.$$

The energy supplied from without is

$$\begin{aligned} & \frac{1}{2} \{ \sum (E + \delta E)V - \sum E(V + \delta V) \} \\ & = \frac{1}{2} \sum E \delta V - \frac{1}{2} \sum E \delta V = -\sum E \delta V = 2\phi \delta \phi = 2\delta Q, \text{ by (32).} \end{aligned}$$

In other words, when the potentials of a system are kept constant by supply of energy from without, the system tends to move so as to increase the potential energy of electrical separation, and the amount of energy supplied from without is double this increase. If we suspend side by side two balls, each connected with the positive pole of a battery, the other pole of which is connected with the ground, the balls will tend to separate, and in separating they will gain with reference to gravity a certain amount δQ of potential energy; the charges on the balls will also increase to an extent representing an increase of electrical potential energy δQ , and the batteries will be drawn upon for an amount of $2\delta Q$.

Cases where problem has been solved. Ellipsoid. The problem of electrical equilibrium has been completely solved in very few cases. We proceed to give a short sketch of what has been done in this way, which may indicate to the reader what is known on this head.

We can deduce the distribution and potential in the case of an ellipsoid from known propositions about the attractions of ellipsoidal shells of gravitating matter.

Consider an ellipsoidal shell, the axes of whose bounding surfaces are (a, b, c) $(a + da, b + db, c + dc)$, where $\frac{da}{a} = \frac{db}{b} = \frac{dc}{c} = \mu$. The potential of such a shell at any internal point is constant, and the equipotential surfaces for external space are ellipsoids confocal with (a, b, c) . (See Thomson and Tait, §§ 519 *seq.*) Hence if we distribute electricity on an ellipsoid (a, b, c) such that its density at every point is proportional to the thickness of the shell formed by the similar ellipsoids (a, b, c) $(a + da, b + db, c + dc)$, the distribution will be in equilibrium. Thus if $\sigma = A\theta$, where θ is the thickness at any point and ρ the volume density of the shell; then the quantity of electricity on any element dS is A times the mass of the corresponding element of the shell; and if Q be the whole quantity of electricity on the ellipsoid, $Q = A$ times the whole mass of the shell.

The mass of the shell is $\frac{4}{3}\pi\rho d(abc) = 4\pi\rho abc\rho$, therefore $Q = A4\pi\rho abc\rho$. Also $\theta = \mu\rho$ where ρ is the perpendicular from the centre of the ellipsoid on the tangent plane. Whence we get

$$\sigma = \frac{Q\rho}{4\pi abc} \quad \dots \dots \dots (34);$$

that is, the density at any point varies directly as the distance of the tangent plane at that point from the centre.

Returning again to our ellipsoidal shell, we know that the resultant force at any external point P due to this shell is to that due to a "confocal shell" passing through the point in the ratio of the masses. Let the volume density in the two be ρ , and let the perpendicular on the tangent plane at P to the confocal $(\sqrt{a^2 + \lambda}, \sqrt{b^2 + \lambda}, \sqrt{c^2 + \lambda})$ through P be ω . Then the thickness of the shell at P is $\mu\omega$, and the force at P due to the shell $4\pi\rho\mu\omega$. Hence the force due to the original shell is

$$-\frac{dV}{d\nu} = 4\pi\rho\mu\omega \frac{abc}{\sqrt{(a^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda)}} \quad \dots \dots (a),$$

$d\nu$ being an element of the normal at P . Now if x, y, z be the co-ordinates of P , we have, by differentiation of

$$\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} + \frac{z^2}{c^2 + \lambda} = 1,$$

$$\frac{2xdx}{a^2 + \lambda} + \frac{2ydy}{b^2 + \lambda} + \frac{2zdz}{c^2 + \lambda} = \left\{ \frac{x^2}{(a^2 + \lambda)^2} + \frac{y^2}{(b^2 + \lambda)^2} + \frac{z^2}{(c^2 + \lambda)^2} \right\} d\lambda.$$

Suppose we take dx, dy, dz in the direction of the normal, then $dx = d\nu \frac{\omega x}{a^2 + \lambda}$, &c., and the last equation reduces to

$$d\lambda = 2\omega d\nu.$$

Hence from (a) we get

$$-dV = \frac{2\pi\rho\mu abc d\lambda}{\sqrt{(a^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda)}}.$$

Integrating this from λ to ∞ , and remembering that the potential vanishes at an infinite distance, we get

$$V = 2\pi\omega abc \int_{\lambda}^{\infty} \frac{d\lambda}{\sqrt{(a^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda)}} \quad \dots \dots (B).$$

We pass from this to the electrical case by putting for $4\pi\rho\mu abc$, which is the mass of the shell, Q , which represents the quantity of electricity on the ellipsoid. We thus get

$$V = \frac{Q}{2} \int_{\lambda}^{\infty} \frac{d\lambda}{\sqrt{(a^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda)}} \quad \dots \dots (35),^1$$

which gives the potential due to a charge Q on an isolated ellipsoid abc at any point on the confocal $(\sqrt{a^2 + \lambda}, \sqrt{b^2 + \lambda}, \sqrt{c^2 + \lambda})$. It is obvious that, of the three confocals at P , that is meant which belongs to the same family as (a, b, c) , e.g., if (a, b, c) be an ellipsoid, as opposed to a hyperboloid of one or two sheets, then $(\sqrt{a^2 + \lambda}, \sqrt{b^2 + \lambda}, \sqrt{c^2 + \lambda})$ must be an ellipsoid.

If we put $\lambda = 0$, we get the value of the potential V_0 at the surface. Now $\frac{Q}{V_0}$ is what we have defined above as the capacity of the ellipsoid; we get therefore in the reciprocal of the integral

$$\frac{1}{2} \int_0^{\infty} \frac{d\lambda}{\sqrt{(a^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda)}} \quad \dots \dots (36),$$

an expression for the capacity of an isolated ellipsoid.

In the particular case of an ellipsoid of revolution, the Plane- above integral, which is in general an elliptic integral, can be found in finite terms. In the case of a planetary ellipsoid, $a = b > c$; and we find for the capacity

$$\frac{\sqrt{a^2 - c^2}}{\frac{1}{2}\pi - \epsilon} \quad \dots \dots \dots (37),$$

where ϵ is the least angle whose tangent is $\frac{c}{\sqrt{a^2 - c^2}}$.

If we make $c = 0$, then $\epsilon = 0$; and the planetary ellipsoid Circular reduces to a circular disc, the capacity for which is there- disc.

fore $\frac{2a}{\pi}$, that is, $\frac{1}{1.571}$ that of a sphere of the same radius

¹ This demonstration was suggested by that given by Thomson (*Reprint of Papers*, p. 10) to establish a slightly different formula.

(for the capacity of a sphere is obviously equal to its radius). Cavendish had arrived by experiment at the value $\frac{1}{1.57}$ (see Thomson's *Reprint*, p. 180), a very remarkable result for his time. It is very easy, by taking the limit of the right hand side of (34), to find the expression for the density at a distance r from the centre of the disc; it is

$$\sigma = \frac{Q}{4\pi a\sqrt{a^2 - r^2}} - \frac{V}{2\pi^2\sqrt{a^2 - r^2}} \quad (38).$$

In the case of an ovary ellipsoid, $a = b < c$; and the capacity is

$$\log \left(\frac{2\sqrt{c^2 - a^2}}{c + \sqrt{c^2 - a^2}} \right) \quad (39);$$

from which several limiting cases may be deduced.

Formula (34), applied to a very elongated ovary ellipsoid, shows us that the density at the pointed ends is very great compared with that at the equator. The ratio of the densities in fact increases indefinitely with the ratio of the longest to the shortest dimension. We have in such an infinitely elongated ellipsoid an excellent type of a pointed conductor.

The effect of a point or an edge on a conductor may be very easily shown by drawing a series of level surfaces, the first of which is the surface of the conductor itself, which has, say, an edge on it. The consecutive surfaces have sharpness of curvature corresponding to the edge, which gets less and less as we recede from the conductor. The level surfaces at an infinite distance are spheres. Tracing, then, any tube of force from an infinite distance, where the sections of all are equal, inwards towards the discontinuity, we see that the section becomes narrower as the curvature of the level surfaces sharpens, and at a mathematical edge the section is infinitely small, and therefore the force is infinitely great. At a mathematical point this is doubly true. At such places the force tending to drive the electricity into the insulating medium becomes infinite. In practice the medium gives way, and disruptive discharge of some kind occurs.

We can find the distribution on a spherical conductor influenced by given forces, such for instance as would arise from rigidly electrified bodies in the neighbourhood.

The method of procedure would be as follows:—Let U be the potential of the rigidly electrified system alone at any point of the sphere. Then the problem is to determine a function V , which shall satisfy the equation $\nabla^2 V$ at every point of space, and have the value $C - U$ at the surface of the sphere, where C is a constant to be determined by the conditions of the problem. Expand $C - U$ in series of surface harmonics, and let the result be

$$C - U = \gamma_0 + \gamma_1 + \gamma_2 + \dots \text{ \&c.} \quad (a).$$

Then the value of V is

$$V = \gamma_0 + \gamma_1 \frac{r}{a} + \gamma_2 \frac{r^2}{a^2} + \dots \text{ inside the sphere } \quad (b),$$

$$\text{and } V = \gamma_0 \frac{a}{r} + \gamma_1 \frac{a^2}{r^2} + \gamma_2 \frac{a^3}{r^3} + \dots \text{ outside } \quad (c).$$

For these evidently satisfy Laplace's equation, have the given value (a) at the surface of the sphere, and are finite and continuous everywhere. From (b) and (c), by means of the surface characteristic equation, we can deduce an expression for the density at any point of the sphere, and for the whole charge. If the latter is given we have a condition to determine C ; if, on the other hand, the value of the potential of the sphere were given, then this would be the value of C .

The case of two mutually influencing spheres was treated by Poisson in the famous memoir which really began the mathematical theory of electricity. We regret that we cannot afford space for more than a mere sketch of his methods.

Consider the potentials due to the distributions on each sphere. Let a and b be the radii of the two spheres, r and r' the distances

of any point P from their respective centres, and μ and μ' the cosines of the angles r and r' make with the line joining the centres of the spheres. Since the distributions are evidently symmetrical about the central line, we can obviously expand the potentials due to each distribution in zonal harmonics relative to the corresponding sphere. Hence, if $4\pi a\phi\left(\mu, \frac{r}{a}\right)$ denote potential due to sphere a at any point inside it, we have

$$4\pi a\phi\left(\mu, \frac{r}{a}\right) = A_0 + A_1 Q_1 \frac{r}{a} + A_2 Q_2 \frac{r^2}{a^2} + \dots \quad (a).$$

The potential at any external point is

$$A_0 \frac{a}{r} + A_1 Q_1 \frac{a^2}{r^2} + A_2 Q_2 \frac{a^3}{r^3} + \dots \quad (b),$$

which may be written $4\pi \frac{a^2}{r} \phi\left(\mu, \frac{a}{r}\right)$.

Similarly we have for the other sphere

$$4\pi b\phi\left(\mu', \frac{r'}{b}\right) = B_0 + B_1 Q_1' \frac{r'}{b} + B_2 Q_2' \frac{r'^2}{b^2} + \dots \quad (c)$$

for the potential at any internal, and $4\pi \frac{b^2}{r'} \phi\left(\mu', \frac{b}{r'}\right)$ for the potential at any external point.

The whole potential, then, will be given by

$$V = 4\pi \frac{a^2}{r} \phi\left(\mu, \frac{a}{r}\right) + 4\pi \frac{b^2}{r'} \phi\left(\mu', \frac{b}{r'}\right)$$

at any point external to both spheres.

Also $V = 4\pi a\phi\left(\mu, \frac{r}{a}\right) + 4\pi \frac{b^2}{r'} \phi\left(\mu', \frac{b}{r'}\right)$ inside a ; and

$$V = 4\pi \frac{a^2}{r} \phi\left(\mu, \frac{a}{r}\right) + 4\pi b\phi\left(\mu', \frac{r'}{b}\right) \text{ inside } b.$$

Now, the conditions of the problem require that the values of V in the two last cases shall be constant. Our functions are, therefore, to be determined by the equations

$$\left. \begin{aligned} a\phi\left(\mu, \frac{r}{a}\right) + \frac{b^2}{r'} \phi\left(\mu', \frac{b}{r'}\right) &= h \\ \frac{a^2}{r} \phi\left(\mu, \frac{a}{r}\right) + b\phi\left(\mu', \frac{r'}{b}\right) &= g \end{aligned} \right\} \quad (d),$$

which are to be satisfied with obvious restrictions on r and r' in each case. Reverting, however, to the expressions (a), (b), (c), &c., we see that we need not solve the problem in the general form thus suggested; for it will be sufficient if we determine the constants $A_0, A_1, \&c., B_0, B_1, \&c.$ Now, if we make $\mu=1, \mu'=1$,—that is, consider only points on the central line,—then $Q_1=1, Q_2=1, \&c., Q_1'=1, Q_2'=1, \&c.$ $A_0, A_1, \&c., B_0, B_1, \&c.$ are the coefficients

of $\frac{a}{r}, \frac{a^2}{r^2}, \&c.$ and $\frac{b}{r'}, \frac{b^2}{r'^2}, \&c.$ in the expressions for the potentials inside the spheres a and b . Hence, if $f\left(\frac{r}{a}\right)$ and

$F\left(\frac{r'}{b}\right)$ denote the values of $\phi\left(\mu, \frac{r}{a}\right), \phi\left(\mu', \frac{r'}{b}\right)$, when $\mu=1$ and $\mu'=1$, we need only solve the equations

$$\left. \begin{aligned} af\left(\frac{r}{a}\right) + \frac{b^2}{c-r} F\left(\frac{b}{c-r}\right) &= h \\ \frac{a^2}{c-r} f\left(\frac{a}{c-r}\right) + bF\left(\frac{r'}{b}\right) &= g \end{aligned} \right\} \quad (e),$$

where we have replaced r and r' by their values $c-r'$ and $c-r$, c being the distance between the centres of a and b . Poisson then eliminates the function F , by choosing a new variable ξ , such that $r' = \frac{b^2}{c-\xi}$, and remarks that we may give to ξ any value between $+a$ and $-a$, and therefore we may write r for ξ ; we thus have the same variable in both the equations, and $F\left(\frac{b}{c-r}\right)$ which occurs in both may be eliminated. The result is

$$af\left(\frac{r}{a}\right) + \frac{a^2 b}{c^2 - b^2 - cr} f\left(\frac{ac - ar}{c^2 - b^2 - cr}\right) = h - \frac{gb}{c-r} \quad (f).$$

This is the functional equation on which depends the solution of the problem of two mutually influencing spheres.

Poisson treats very fully the case of two spheres in contact; for which case, taking $a=1$, the above equation becomes

$$f(r) - \frac{b}{b+(1+b)(1-r)} f\left(\frac{1+b-r}{b+(1+b)(1-r)}\right) = h - \frac{gb}{1+b-r} \quad (g).$$

¹ We are, of course, assuming acquaintance with the properties of spherical harmonics.

He finds a solution,

$$f(r) = \frac{bh}{(1+b)(1-r)} \int_0^1 \frac{t^{-\frac{1}{1+b}} - 1}{1-t} e^{\frac{br}{(1+b)(1-r)}} dt. \quad (\theta).$$

It is then easy to find $F(r)$, and write down the general expressions for the potential. Poisson goes on to show that the density at the point of contact of the spheres is zero. He finds, for the mean density on the two spheres 1 and b respectively,

$$A = \frac{bh}{1+b} \int_0^1 \frac{t^{-\frac{1}{1+b}} - 1}{1-t} dt,$$

this being, in fact, the value of $f(0)$,

$$\text{and } B = \frac{h}{b(1+b)} \int_0^1 \frac{t^{-\frac{1}{1+b}} - 1}{1-t} dt.$$

He shows that the calculation of the ratio β of A to B may be reduced to the calculation of the first of these integrals only. For the difference $4\pi b^2 B - 4\pi A$ between the charges on 1 and b he finds the elegant expression

$$\frac{4\pi^2 bh}{1+b} \cos \frac{\pi}{1+b},$$

from which it follows that the whole charge is always greater on the sphere of greater radius. He then calculates the value of β for various values of b , and its limit for $b=0$, and next the ratio of the densities at the two points diametrically opposite the point of contact, and finds for the mean density on each of two equal spheres in contact $A = h \log 2$. He also calculates for this last case the ratio of the greatest to the mean density. In the case of two unequal spheres, the ratio of the greatest density on the smaller to the mean density on the larger is found for various values of b . He then passes on to investigate the densities for various values of μ .

Plana
and
Roche.

All these results are compared with the measurements of Coulomb, and found in satisfactory accordance with them. In his first memoir, Poisson considers the case where the distance between the spheres is great compared with the radii; and in a subsequent memoir he considers the case of two spheres at any distance.

Plana (*Sur la distribution de l'électricité à la surface des deux Sphères*, Turin, 1845) extended the calculations of Poisson, using much the same methods. He also calculated approximately the mean densities in the case of several spheres in contact, and arrived at results which agreed satisfactorily with the experiments of Coulomb. For a table of his results, see the end of the first volume of Riess's *Reibungselectricität*. An account of the work of Roche, who also followed in the footsteps of Poisson, will be found in Mascart, t. i. p. 290 *seqq.*

Synthe-
tical me-
thod of
Green.

The researches of Green led him to a very valuable synthetical method, by means of which we can construct an infinite number of cases where we can find the electrical distribution. Suppose that we take *any distribution whatever* of electricity, for which we know the potential at any point, and consequently the level surfaces. Take any level surface, or parts of level surfaces, inclosing the whole of the electricity, and suppose these level surfaces to become actual conducting sheets of metal. Suppose the electrical distribution inside to be rigid, and connect the sheets of metal with the earth, so as to reduce them to potential zero. The sheets will become charged in such a way that the whole potential at every point in them and external to them is zero. Let now U be the potential at any external point due to inside distribution, and V that due to the charge on the sheets, then we have everywhere on or outside the sheets, $U + V = 0$, or $V = -U$. Now U is constant at every point of each sheet; hence V is so also. Hence the distribution to which V is due is an equilibrium distribution *per se*. Removing now our internal distribution, and changing the sign of that on the sheets, we have a distribution of electricity in equilibrium on a

set of conductors of known form, the potential of which at any external point is $V = U$, where U is known. Also the potential V is clearly constant inside every conductor. Hence, applying the characteristic surface equation, we get for the density at any point of any of our conductors the expression

$$\sigma = -\frac{1}{4\pi} \frac{dU}{dn}.$$

We might make this a little more general, and state our result thus:—*If we distribute on a level surface or surfaces of any electrical system, completely inclosing that system, electricity with surface density at every point* $\sigma = -\frac{k}{4\pi} \frac{dU}{dn}$, *this distribution will of itself be in equilibrium, and the potential at any external point will be kU .*

We have given a physical demonstration of this important theorem. The mathematical reader will easily see the application to this case of the general reasoning about the solution of $\nabla^2 V = 0$, of which we have already given examples. For a simple but interesting case of this general theorem, see Thomson and Tait's *Natural Philosophy*, vol. i. § 508.

To Sir William Thomson we owe the elegant and powerful methods of "Electric Images" and "Electric Inversion." By means of these he arrived, by the use of simple geometrical reasoning, at results which before had required the higher analysis. We shall endeavour to illustrate these by two simple examples. We do not follow the methods of the author (for which, see his papers), but take advantage of what we have already laid down.

Let A be any point outside a sphere (fig. 12) of radius a , and centre C . Let $AC = f$, and take B in CA such that $CB \cdot CA = a^2$, or $CB = \frac{a^2}{f}$; then it is easily proved that, if P be any point on the sphere,

$$\frac{BP}{AP} = \frac{a}{f}.$$

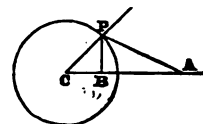


Fig. 12.

Hence if E be any quantity of electricity, we have

$$\frac{E}{AP} - \frac{\frac{a}{f} E}{BP} = 0.$$

Therefore, if we place a quantity E of electricity at A , and a quantity $-\frac{a}{f} E$ at B , the sphere will be a level surface of these two, that, namely, for which the potential is zero. Another level surface of the system is evidently an infinitely small sphere surrounding A . Hence it follows, from the theorem of Green which we have just discussed, that a distribution of electricity on the sphere, the density of which is given by $\sigma = \frac{R}{4\pi}$ (where R is the resultant force due to E and $-\frac{a}{f} E$ at any point of the sphere), together with a quantity E at A , gives a system in equilibrium, the potential due to which at any point outside the sphere is the same as that of E at A , and $-\frac{a}{f} E$ at B .

It appears, therefore, that the action of the electricity induced on the uninsulated sphere by the electrified point A is equivalent at all external points to the action of $-\frac{a}{f} E$ at B . The electrified point B is called by Sir William Thomson the electrical image of A in the sphere. It is obvious that the whole charge on the sphere is $-\frac{a}{f} E$, and we can very easily find the density at any point.

In fact, resolving along CP , which we know to be the direction of resultant force, the forces due to A and B , we get

$$R = \frac{E}{AP^2} \cos CPA - \frac{\frac{a}{f} E}{BP^2} \cos CPB$$

$$-\frac{E}{AP^2} \left(\frac{a^2 + AP^2 - f^2}{2aAP} \right) - \frac{fE}{aAP^2} \left(\frac{f^2 + AP^2 - a^2}{2fAP} \right) - \frac{(f^2 - a^2)E}{aAP^2}.$$

$$\therefore \sigma = -\frac{(f^2 - a^2)E}{4\pi aAP^2} \quad (40).$$

We might have any number of external points and find the image of each. We should thus get a system which might be called the image of the external system. The distribution induced in an uninsulated sphere by such an external system could easily be found by adding up the effect of each external element found by means of its image. Similar methods might also be applied to an internal system. The solution can be generalized without difficulty to the case where either the charge or potential of the sphere is given.

Suppose the charge Q given; superpose on the distribution found above a uniform distribution of amount $Q + \frac{a}{f}E$. This will produce a constant potential $\frac{Q}{a} + \frac{E}{f}$ all over the sphere, and therefore will not disturb the equilibrium. We have thus got the required distribution of the given charge Q under the influence of A . The density of any point is given by

$$\sigma = \frac{Q}{4\pi a^2} + \frac{E}{4\pi af} - \frac{(f^2 - a^2)E}{4\pi aAP^2} \quad (41).$$

So far the method of images is simply a synthetical method for obtaining distributions on a sphere. But Sir William Thomson has shown us how to convert it into an instrument for transforming any electrical problem into a variety of others.

If P be any point (fig. 13), O a fixed point, and P' be taken in OP such that $OP \cdot OP' = a^2$, then P' is called the inverse of P with respect to O , which is called the origin of inversion, or simply the origin; a is the radius of inversion. We may¹ thus invert any locus of points into another locus of points, which we may call the inverse of the former.

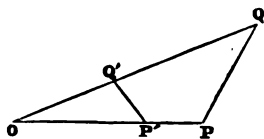


Fig. 13.

Let P, Q and P', Q' be any two points and their inverses. Let us suppose that there is a charge E at Q , and a charge E' at Q' , which is the image of E in a sphere with radius a and centre O ; so that $E' = \frac{a}{OQ}E$. Let V and V' be the respective potentials of E and E' at P and P' . Then we have obviously

$$\frac{V'}{V} = \frac{a}{r'} - \frac{r}{a},$$

where $OP = r, OP' = r'$. It is very easy to show that, if $ds, dS, d\sigma, \sigma, \rho$, be elements of length, surface, and volume, and surface and volume densities, and the same symbols with dashes the inverses of these, then we have

$$\left. \begin{aligned} \frac{ds}{ds} &= \frac{a^2}{r^2} = \frac{r'^2}{a^2}; \quad \frac{dS'}{dS} = \frac{a^4}{r^4} \text{ \&c.} \\ \text{and } \frac{\sigma'}{\sigma} &= \frac{r^3}{a^3} = \frac{a^3}{r'^3}; \quad \frac{\rho'}{\rho} = \frac{r^3}{a^3} = \frac{a^3}{r'^3} \\ \text{also } \frac{E'}{E} &= \frac{r}{a} = \frac{a}{r'}; \quad \frac{V'}{V} = \frac{r}{a} = \frac{a}{r'} \end{aligned} \right\} \quad (42).$$

By means of these equations it is easy to invert any electrical system. Take, for example, the case of any conductor in electrical equilibrium; then, since its potential is everywhere constant, it inverts into a surface distribution, the potential at any point of which distant r' from the origin is by (42) $\frac{a}{r'}C$, where C is the constant potential of the conductor. The surface density at any point of the system is found from that of the corresponding point on the conductor by the equation

¹ For the general properties of curves and their inverses, the reader may consult Salmon's *Solid Geometry*. He will have no difficulty in proving for himself such as we shall require here.

$$\sigma' = \frac{a^3}{r^3} \sigma.$$

Again, if we consider the system thus found, it is obvious that, if we place a quantity $-aC$ of electricity at the origin, this will make the potential at every point of the system zero, and we have a solution of the case of an uninsulated conductor, whose surface is the inverse of that of the given conductor, under the influence of an electrified point.

As an example of the use of this method, let us invert the uniform distribution on a sphere with respect to an origin on its circumference, the radius of inversion being the diameter of the sphere. The sphere inverts into an infinite plane, touching at the other end A of the diameter through the origin. Let C be the potential on the sphere so that $\sigma = \frac{a^2 C}{2\pi d}$, where d is the diameter. Hence the density at any point P on an infinite plane influenced by a quantity $-Cd$ of electricity placed at a point O distant d from it is given by

$$\sigma' = \frac{a^2 C}{2\pi r^2}.$$

Again, inverting points inside the sphere, for which the potential is constant, we get the potential due to the distribution on the infinite plane, at points on the other side from the inducing point, the result being

$$V' = \frac{dC}{r},$$

which is the same as that due to dC at O . Hence the potential at a point on the same side as O is that due to a quantity dC placed at O' , where $O'A = OA$. O' is in fact the image of O . If we write Q for $-Cd$, then we get

$$\left. \begin{aligned} \sigma' &= \frac{Qd}{2\pi r^2} \\ V' &= \frac{Q}{r} \end{aligned} \right\} \quad (43).$$

These results might of course have been deduced as particular cases of a sphere and point.

Many beautiful applications of these methods will be found in the *Reprint* of Sir William Thomson's papers and in Maxwell's *Electricity and Magnetism*. Two of these are of especial importance. Adopting the method of successive influences given by Murphy (*Electricity*, 1833, p. 93), and conjoining with it the method of images, Sir William Thomson treated the problem of two spheres. For his results, see *Reprint*, pp. 86-97. At the end of that paper two valuable tables are given—I. "Showing the quantities of electricity on two equal spherical conductors of radius r , and the mutual force between them, when charged to potentials u and v respectively;" II. "Giving the potentials and force when the charges D and E are given." The ratio of u to v in the first case and of D to E in the second is also given, for which at a given distance there is neither attraction nor repulsion. An interesting experiment on this curious phenomenon is described in Riess, Bd. i. § 186. For an application of dipolar co-ordinates to the problem of two spheres, see Maxwell.

Thomson also applied his methods to determine the distribution on spherical bowls of different apertures. See *Reprint*, p. 178 *sqq.* His numerical results on p. 186 are extremely interesting, as affording a picture of the effect of gradually closing a conductor, and are of great value in giving the experimenter an idea as to what aperture he may allow himself in a vessel which he desires should be for practical purposes electrically closed. It would lead us too far to discuss here the analytical method of conjugate functions, and the allied geometrical method of inversion in two dimensions. A full account of these, with important applications, will be found in Maxwell, vol. i. § 182 *sqq.*

We shall conclude our applications with a brief notice of a few of the ordinary electrostatic instruments, referring the reader for an account of some others to the article **ELECTROMETER**.

If two plates be placed parallel to each other, and one

Parallel plates. of them raised to potential V , while the other is connected with the earth, then there will be certain charges E and F on the two plates. If p and r be the coefficients of self-induction for A and B , and q the coefficient of mutual induction, then in the present case

$$E = pV, \quad F = qV,$$

and the energy of the distribution is obviously

$$Q = \frac{1}{2}EV - \frac{1}{2}pV^2,$$

so that the work done by completely discharging the condenser $\propto V^2$. If we suppose the plates very large compared with the distance between them, then we may treat the case, for all points not very near the edge, as if the plates were infinite.

In this case the lines of force are straight, and the number of lines of force which leave any area on A is equal to that of those which enter the opposite area on B . Hence the surface densities on the plates are equal and opposite in sign. Also we clearly have

$$\sigma = \frac{R}{4\pi} = \frac{V}{4\pi d} \dots \dots \dots (44).$$

For the number of lines of force which cross any unit of area parallel to the plates is constant, and therefore the resultant force is constant at every point between the plates.

Principle of accumulators. It appears, therefore, from (44) that if we make the distance between our plates very small, the density on the inner surface will be very great, and the whole charge on A very great. An apparatus of this kind for collecting large quantities of electricity at a moderate potential is called an accumulator or condenser. One of the first instruments of this kind was Franklin's pane, which consisted of two sheets of tinfoil pasted opposite each other on the two sides of a pane of glass. There is of course a practical limit to the increase of capacity in such arrangements, because a spark will pass when the insulating medium is too thin. The greater dielectric strength of glass makes it more convenient than air for an insulating medium, and we shall see by-and-by that it has other advantages as well. When the plate A is of finite size there will in general be a distribution of electricity on the back comparable with the charge which A would hold at potential V if B were absent. When the distance between the plates is small, by far the greater portion of the capacity is due to the presence of B . Advantage of this principle has been taken

Condensing electroscope. in the condensing electroscope of Volta, which is an ordinary gold-leaf apparatus, except that the knob is replaced by a circular disc on which is placed another disc fitted with an insulating handle; the discs are covered with a thin coat of varnish which serves as an insulating medium. If we connect with either disc, say the lower, a source of electricity of feeble potential V , and connect the upper disc at the same time with the earth, then a large quantity of electricity at potential V collects on the lower disc. Now remove all connections, and lift away the upper disc. The capacity of the lower disc is thereby enormously diminished. Therefore, since the charge is unaltered, its potential must rise correspondingly; and the gold leaves may diverge very vigorously, although a simple connection with the lower disc alone would scarcely have moved them. This instrument is of great use in all cases where we have an unlimited supply of electricity at feeble potential. Sir William Thomson has devised an accumulator of measurable capacity, called the Guard Ring Accumulator, which is a modification of the arrangement we are discussing.

Guard ring accumulator.

AB (fig. 14) is a flat cylindrical metal box, the upper end of which is truly plane, and has a circular aperture, into which fits, without touching, a plane disc C , which is supported on the bottom of the box by insulating supports, so that its upper surface is in the same plane with the lid of the box. DE is a metal disc which can be moved by a screw through measured distances, always remaining

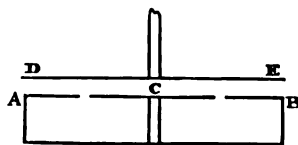


Fig. 14.

parallel to AB . When desired, C can be put in communication with AB . It may then be regarded as forming part of an infinite plate, so that if AB be at potential V , and DE at potential zero, then the surface density on C will be equal to $\frac{V}{4\pi d}$, where d is the distance between the plates; and if A be the area of C the whole amount of electricity on C is $\frac{AV}{4\pi d}$. If now we break the connection between C and the box and discharge the box, we are left with a known quantity of electricity on C , viz. $\frac{AV}{4\pi d}$.

The most usual and for many purposes the most convenient form of accumulator is the Leyden jar. This is merely a glass jar (fig. 15) coated to a certain height outside and inside with tinfoil. The mouth of the jar is stopped with a cork or wooden disc, which serves the double purpose of keeping dirt and moisture from the uncovered glass inside, and of carrying a wire in metallic connection with the inside coating, which passes up through the stopper and ends in a metal knob. If the glass of the jar be very thin, we may find the distribution on the two coatings by neglecting the curvature; the electric density on the inner surface of the two coatings will then be the same as in the case of parallel plates. If, therefore, the inner coating be at potential V , and the outer at potential zero, the density on the inner coating will be $\frac{V}{4\pi d}$, and that on the outer $-\frac{V}{4\pi d}$. In the particular case we are considering the inner coating forms very nearly a closed conductor, so that there will be very little electricity on its inner surface, and there will also be very little on the wire and knob compared with the amount on the surface of the inner coating which is next the glass. We may therefore put for the whole electricity on the inner coating $\frac{SV}{4\pi d}$, where S is the extent of its surface. The capacity C of the jar is then given by



Fig. 15.

$C = \frac{S}{4\pi d} \dots \dots \dots (45).$

Green calculated to a first approximation the effect of the curvature on the capacity, and found that, if R and R' be the greatest and least radii of curvature of the inner coating at any point, then the densities on the inner and outer coatings are given by

$$\frac{V}{4\pi d} \left\{ 1 \pm \frac{d}{2} \left(\frac{1}{R} + \frac{1}{R'} \right) \right\} \dots \dots \dots (46),$$

and consequently the capacity of the inner coating by

$$\frac{1}{4\pi} \left\{ \iint \frac{dS}{d} + \frac{1}{2} \iint \left(\frac{1}{R} + \frac{1}{R'} \right) dS \right\} \dots \dots (47).$$

In any case, C being a constant, we have charge $E = CV$ and energy $Q = \frac{1}{2}CV^2$. Hence if we connect the inner coatings of n similar jars, and charge them to potential V , all the outer coatings being at the same time connected with the earth, we have, E and Q representing the whole charge and energy,

$$\left. \begin{aligned} E &= nCV \\ Q &= \frac{n}{2} CV^2 \end{aligned} \right\} \dots \dots \dots (48).$$

If we discharge such a battery of n jars into another of n' similar jars, by connecting the knobs together, and the outer coatings to earth in each case, we have, U being the common potential after discharge,

$$\left. \begin{aligned} nCV &= nCU + n'CU \\ \text{and } U &= \frac{n}{n+n'} V \end{aligned} \right\} \dots \dots \dots (49).$$

There is therefore a loss of energy represented by

$$\frac{1}{2}nCV^2 - \frac{1}{2}(n+n')CU^2, \quad \text{that is} \quad \frac{nn'}{2(n+n')} CV^2 \dots \dots \dots (50).$$

In other words, an $\frac{n'}{n+n'}$ th part of the potential energy is lost. When a battery of jars is discharged through a circuit in which there is a fine wire of large resistance, the greater part of the potential energy lost in the discharge appears as heat in the fine wire. Riess made elaborate experiments on the heating of wires by the discharge in this way, and the results of his experiments are in agreement with the formulæ which we have just given. (See Heating Effects.)

series. We may also arrange a battery of jars by first charging each separately to potential V in the usual way, and then connecting them in series, so that the outer coating of each jar is in metallic connection with the inner coating of the next. In such an arrangement of jars, it is obvious that in passing from the outer coating of the last at potential zero to the inner coating of the first, the potential will rise to nV . When we come to discharge such a series, the electromotive force to begin with is nV , so that for any purpose in which great initial electromotive force is required this combination has great advantages over n jars abreast. The "striking distance," for instance, i.e., the greatest distance at which the discharge by spark will just take place through air, is much greater. On the other hand, the quantity of electricity which passes is less, being only CV instead of nCV ; the whole loss of potential energy in a complete discharge is, however, the same.

cascade. The case which we have been discussing must be carefully distinguished from that of a series of jars charged by "cascade," where n uncharged jars are connected up in succession as in last case, and the first charged by connection with the electric machine to potential V , while the outer coating of the last of the series is connected to earth, and the rest of the jars insulated. The whole electromotive force in this case is clearly only V , and, if all the jars be similar, the potential difference between the coatings in each is $\frac{V}{n}$; the charge on the inner coating of the first is therefore $\frac{CV}{n}$ and the whole potential energy only $\frac{1}{2} \frac{CV^2}{n}$.

The arrangement is, therefore, not so good as a single jar fully charged by the same machine. It was fancied by Franklin, who invented this method of charging, that some advantage was gained by it in the time of charging, the notion being that the overflow was caught by the successive jars and that electricity was thereby saved. Charging by cascade was treated by Green. Some of the experiments of Riess bear on the matter (*vide* Mascart, §§ 190, 191), which, after all, is simple enough.

free and bound electricity. In the theory of accumulators, or condensers as they are often called, much stress has been laid on the difference between "free" and "bound" electricity. To illustrate the meaning of these terms, let us take a case where the calculations can be carried out in detail.

Suppose we have two concentric spherical shells, an inner, A, and an outer, B. Let the outer radius of A be a , and the inner and outer radii of B be b and c , so that the thickness of the latter is $c-b$. We shall suppose that we can, when we please, connect the inside sphere with the earth. It is clear that there can never be any electricity on the inner surface of A. Let the charges on the other surfaces in order be E, F, G. Let us suppose in the first instance that A is at potential V , and B at zero. Then we have to find E, F, G. Draw a surface in the substance of B; no lines of force cross it, therefore the whole amount of electricity within is zero. Hence $F = -E$. Also, considering the external space, which is inclosed between two surfaces of zero potential, we see that $G = 0$. Thus, since A is at potential V , we have $\frac{E}{a} - \frac{E}{b} = V$.

$$E = \frac{ab}{b-a} V = pV \quad \left(\text{where } p = \frac{ab}{b-a} \right). \quad (51).$$

In this case, then, there is no electrification on the outside of B, and an electric pendulum suspended there would give no indication.

Let us now connect A with the earth, so that its potential becomes zero; we have now to find the charges and potentials, our datum being that the whole charge on B is $-E$.

As before, we have $F' = -E'$, but G is no longer zero. We have, however, $F' + G' = -E$. Hence $G' - E' = -E$.

Also, since A is at zero potential, $\frac{E'}{a} - \frac{E'}{b} + \frac{G'}{c} = 0$,

$$\text{therefore } G' = \frac{-cE'}{p}; \quad -F' = E' = \frac{pE}{p+c}; \quad G' = \frac{-cE}{p+c}.$$

The potential of B is $\frac{G'}{c}$, or $\frac{-cpV}{p+c}$.

In this process, therefore, a quantity $E - E'$, or $\frac{cp}{p+c} V$, of electricity has flowed away to earth from A, and a quantity $\frac{-cp}{p+c} V$ has passed from the inner to the outer surface of B, while the potential has altered, on A from V to 0, and on B from 0 to $\frac{-cp}{p+c} V$.

Suppose now we connect B with the earth, thus reducing it to zero potential. Since the charge on A remains the same, and that on the inner coating of B is equal and opposite to it, it follows that now the charges on A, &c., are $\frac{pq}{c} V$, $\frac{-pq}{c} V$, 0, where q denotes $\frac{cp}{p+c}$; and the potentials of A and B are $\frac{q}{c} V$ and 0. After another pair of such operations the charges will be $\frac{pq}{c} \frac{q}{c} V$, &c., and the potential, $\frac{q^2}{c^2} V$; after a third, charges, $\frac{pq}{c} \frac{q^2}{c^2} V$, &c., and potential, $\frac{q^3}{c^3} V$. Hence the charges and potentials go on decreasing in geometrical progression. Amounts of electricity flow away from A equal to qV , $q^2 \frac{q}{c} V$, $q^3 \frac{q^2}{c^2} V$, &c., in the successive operations, and equal amounts of opposite signs are discharged from B. The sum of all these discharges is the whole original charge on A, for

$$qV \left(1 + \frac{q}{c} + \frac{q^2}{c^2} + \&c., \text{ ad. inf.} \right) = \frac{q}{1-\frac{q}{c}} V = pV.$$

Hence by an infinite number of alternate connections we shall finally discharge the jar completely. The electricity which flows out at each contact is called the "free electricity," and that which remains behind the "bound electricity." The quantity which we have denoted by p Capacity is clearly the capacity of a spherical Leyden jar; it increases indefinitely as the distance between the conducting surfaces decreases, and is very nearly proportional to the surface of the inside coating, when the distance is small compared with the radius of either surface.

It is very easy to extend our reasoning to any condenser.

If, in fact, q_{11}, q_{12}, q_{22} be the coefficients of self and mutual induction for the armatures, then this potential after operating n times as above is $\left(\frac{q_{11}^2}{q_{11} q_{22}} \right)^n V$, the charges, $q_{11} \left(\frac{q_{11}^2}{q_{11} q_{22}} \right)^n V$ and $q_{12} \left(\frac{q_{11}^2}{q_{11} q_{22}} \right)^n V$, and the amounts of electricity which leave 1 and 2 in the n th operation are $\pm q_{12} \left(\frac{q_{11} q_{22} - q_{12}^2}{q_{11} q_{22}} \right) \left(\frac{q_{11}^2}{q_{11} q_{22}} \right)^n V$ respectively.

We must not omit one more interesting case. If we have two infinite coaxial cylinders of radii a and b ($b > a$), then obviously the potential is symmetrical about the common axis, and Laplace's equation becomes

$$\frac{d^2 V}{dr^2} + \frac{1}{r} \frac{dV}{dr} = 0.$$

The integral of this is $V = C \log r + D$. Let the inner cylinder be at potential V_1 , the outer at potential V_2 , then

$$V = (V_1 - V_2) \frac{\log r}{\log a - \log b} + V_2 \frac{\log a - \log b}{\log a - \log b}. \quad (52)$$

Hence the surface density on the inner cylinder is given by

$$= -\frac{1}{4\pi} \frac{dV}{dr} = \frac{V_1 - V_2}{4\pi a \log \frac{b}{a}};$$

and the capacity per unit of length of same is

$$\frac{1}{2a \log \frac{b}{a}} \quad (53).$$

This result has important applications in the theory of telegraph cables, and to a form of graduated accumulator, invented by Sir William Thomson, and used by Messrs Gibson and Barclay in their experiments on the specific inductive capacity of paraffin (see Maxwell, vol. i. § 127).

ON THE INSULATING MEDIUM.

It has been assumed hitherto that the medium interposed between the conductors in the electric field is in all cases air—the most prevalent of all dielectric media; or, where any other medium actually occurred, as in the case of the Leyden jar, it has been assumed that the result is the same as if the glass were replaced by air. Experimenters soon recognized, however, that the capacity of a Leyden jar depends very much on the quality of the glass of which it is made. But the nature of this action was very little understood, until Faraday showed by a number of striking experiments that the dielectric has a specific function in all phenomena of induction.

Faraday's experiments.

Faraday used in his experiments two identical pieces of apparatus, which were virtually two spherical Leyden jars. The outer coating EF (fig. 16) was divided into two hemispheres, which could be fitted together air-tight. The lower hemisphere F was fitted to a perforated stem, provided with a stop-cock G, so that it could be screwed to an air-pump while the apparatus was being exhausted, and afterwards screwed into a foot H. The upper hemisphere was pierced by a tube, into which was cemented a shellac plug B. C is a metal wire passing down through B, which supports the hollow metal sphere D, forming the inside armature, and carries the metal ball A, by means of which D can be charged and discharged. To give an idea of the size of the apparatus, it may be mentioned that the diameters of the inner and outer spheres were 2.33 in. and 3.57 in. respectively. Two jars were made on the above pattern, as nearly alike as possible. The equality of their capacities was tested as follows. Both were filled with air at the same temperature and pressure. Apparatus I. was then charged, by bringing A in communication with the knob of a Leyden jar, while the coating EF was connected to earth. I. and II. were then placed at a moderate distance from each other, as symmetrically as possible with respect to the observer and other external objects, the outer armatures in both cases being in conducting communication with the earth. The ball of I. was touched by a small proof sphere, the repulsion of which on the movable ball of a Coulomb balance was measured; after a short interval this measurement was repeated. The balls of I. and II. were then brought into communication, and the charge divided between the internal armatures. The ball of II. was immediately tested as before, and then the ball of I. again. Finally I. and II. were discharged and tested for permanent "stem effect." The result of one such series of measurements was

I.	...	254,250	...	124	1...
II.	0,	...	122	...	2.

Neglecting the slight dissipation of the charge, and taking account only of the "stem effect" in I., we see that the charges on I. and II. after division are represented by 122 and 124, each of which is not far from the half of the whole disposable charge in I., viz., 124.5; so that the capacities of the two jars must be equal. This will perhaps be clearer if we consider what would happen were the capacities unequal. Let the capacities be C and C', the potential of I. before division V, and the common potential after U, the charge on I. Q, and on I. and II. q and q' after division. Then Q = CV, q = CU, q' = C'U, and q + q' = Q. The indication of the torsion balance is proportional to the charge of

the proof sphere, that is (owing to the symmetry of the arrangements), to the potential of the knob with which it was in contact; or at all events this is true if we consider only readings taken from the knob of the same jar, and that is all we shall ultimately want. But (C + C') U = CV; hence

$$\frac{C}{C'} = \frac{V - U}{U}$$

Hence the ratio of the capacities is equal to the ratio of the excess of the first over the last reading to the last reading, both being taken from the knob of I. Thus, taking the uncorrected values in the above experiment, the ratio of the capacities would be (250 - 124) ÷ 122, i.e. 1.02. By various experiments of this kind, Faraday convinced himself of the equality of his two jars. To test the sensibility of his method, he reduced the distance between the lower hemispheres and the ball in II. from .62 in. to .435 in., by introducing a metal lining. The capacity of II. was then found to be 1.09 (the mean of two observations). He next compared the capacities of the jars when the lower half of the space between the armatures of one of them was filled with shellac. The ratio of the capacities was found to be 1.5 (mean of several experiments), the shellac jar having the greater capacity.

It appears, therefore, that, other things being equal, the Specific capacity of an accumulator is greater when the insulating inductive medium, or, as it is called, the "dielectric," is shellac, than when it is air. The ratio of the capacity in the former case to that in the latter¹ is called the *Specific Inductive Capacity* of shellac. This we shall in general denote by K. According to this definition, air is taken as the standard, and its specific inductive capacity is unity. Properly speaking, we ought to state the temperature and pressure of the air; we may assume 0° C. as our temperature, and the average atmospheric pressure (760 mm.) as our standard barometric pressure.

It is easy to obtain an approximate value of K from the above result for the shellac apparatus. Remembering that the shellac occupies only one hemisphere, and assuming that the lines of force are not disturbed at the junction of the air and shellac, we have, if ρ denote the ratio of the capacities,

$$\frac{1 + K}{1 + 1} = \rho, \text{ and } K = 2\rho - 1.$$

This gives for shellac K = 2.0, the real value being probably greater. Similar experiments gave for glass and sulphur K = 1.76 and 2.24 respectively.

Thus the specific inductive capacities of shellac, glass, and sulphur are considerably larger than that of air. Faraday was unable to find any difference in this respect between the different gases, or in the same gas at different temperatures and pressures, although he made careful experiments in search of such differences.

It would lead us too far to discuss in detail the precautions taken by Faraday to remove uncertainty from his experimental demonstration of the existence of a specific dielectric action. The reader will find a minute description in Faraday's own surpassingly lucid manner in the eleventh series of the *Experimental Researches*.

His discovery of the action of the medium led Faraday to invent his well-known theory of the dielectric. According to him, the fundamental process in all electrical action is a polarization of the ultimate particles of matter; this polarization consists in the separation of the positive and negative electricities *within* the molecules, exactly as the two magnetic fluids are supposed to separate in the theory of magnetic induction. In this view a dielectric is supposed to consist of a number of perfectly conducting particles, immersed in a medium or menstruum, which is either a non-conductor or a very imperfect conductor. When electrical action starts, the two electricities separate in the molecules; but, in the first instance at least, there is no interchange of electricity between different molecules.

¹ It must be noticed that the assumption is tacitly made that the air is to be replaced by shellac *everywhere*, or at least wherever there are lines of force.

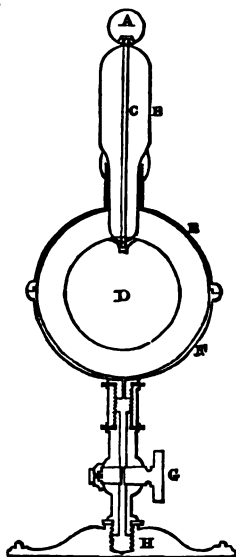


Fig. 16.

Faraday assumed that the electrical action is propagated from molecule to molecule by actions whose sphere of immediate activity is very small. He denied the existence of "action at a distance," and regarded his results about induction in curved lines as at variance with it. Thomson¹ showed, however, that Faraday's results were perfectly consistent with the theory of action at a distance, provided the polarization of the dielectric be taken into account, and that the mathematical treatment of the subject is identical with Poisson's theory of induced magnetism. The theory of action at a distance as applied to this subject will be found under MAGNETISM. Helmholtz, whose memoirs we have already mentioned, takes this view of the matter. We do not propose to follow Faraday's theory any further at present; its main features are involved in Maxwell's theory, to which we shall afterwards allude.

W. Siemens² examined and confirmed the conclusions of Faraday. He used voltaic electricity in comparing the capacities of condensers. By means of a kind of self-acting commutator³ (*Selbstthätige Wippe*), the armatures of the condenser were connected alternately with a battery of Daniell's cells and with each other; so that the condenser was charged and discharged about 60 times per second.

Figure 17 gives a scheme of the arrangement. F and G are two insulated metal screws, with which the vibrating tongue E of the Wippe comes alternately into contact; CD and AB are the armatures of the condenser, H the battery, and K the galvanometer. Theory indicates, and experiment confirms, that the deflection will be the same whether the galvanometer is put in the charge or in the discharge circuit. The former arrangement is that indicated in the figure.

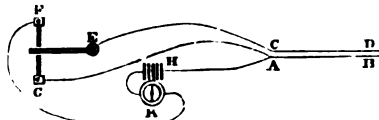


Fig. 17.

The amount of electricity which flows through the galvanometer each time the condenser is charged, is proportional to the product of the capacity C of the condenser and the electromotive force E of the battery. E is proportional to the number of cells in the battery. If, therefore, the speed of the Wippe be constant, the galvanometer deflection, or its sine or tangent as the case may be, will be proportional to EC. By varying E and C independently, we can verify the laws that regulate the charge of condensers. If we keep E the same, and the speed the same, we can compare the capacities of two condensers, or of the same condenser with two different dielectrics, and thus find the specific inductive capacities of various substances with respect to air. Siemens found that C is independent of E, and concluded that the effect of solid dielectrics on the capacity of a condenser is not to be explained by a penetration of the electricity into the dielectrics. We shall give some of his values of the specific inductive capacity farther on.

Gauguin⁴ studied the effect of the insulator on the capacity of condensers. He used in his researches the discharging electroscope (see art. ELECTROMETER), an instrument which does not at first sight look likely to lead to very accurate results, but which seems to have worked satisfactorily in his hands. Many of Gauguin's results concerning the gradual increase of the charge are very interesting; their bearing on theory is difficult to estimate, however, owing to the mixture of effects due to surface and body conduction. His results concerning the "limit-

ing" value of the specific inductive capacity are at variance with those of subsequent experimenters who have worked with more delicate instruments.

In their experiments on the specific inductive capacity of paraffin, Gibson and Barclay⁵ employed a method due to Sir William Thomson, in which an instrument called the Platymeter is used in conjunction with the quadrant electrometer. They found for the specific inductive capacity of paraffin 1.97, and showed that this value alters very little, if at all, with the temperature.

The most extensive measurements of this kind that have been made of late are those of Boltzmann⁶ and Schiller.⁷ Boltzmann used a sliding condenser, whose plates could be placed at measured distances apart. Plates of different insulating materials were introduced between the parallel plates of the condenser, so as to be parallel with them and at different distances from one of them.

According to the mathematical theory, the capacity of the condenser is independent of the position of the plate, and varies inversely as $m - n + \frac{n}{K}$, where m is the distance between the plates of the condenser, and n the thickness of the plate of insulating material whose specific inductive capacity is K. In other words, the plate may be supposed replaced by a plate of air of thickness $\frac{n}{K}$. If therefore λ denote in absolute measure the reciprocal of the capacity of the condenser, then

$$\lambda = G \left(m - n + \frac{n}{K} \right),$$

where G is a constant. The capacity of the condenser was measured by charging it with a battery of 6 to 18 Daniell's cells, and then dividing its charge with the electrometer. One pole of the battery and one armature of the condenser are connected to earth. The other pole of the battery is first connected with the electrode A of the electrometer, whose other electrode B is connected to earth. Let the reading thus obtained be E, then E is proportional to the potential of the battery pole. The condenser is next charged by connecting its insulated armature with the battery; the battery connection is then removed, and the electrode A of the electrometer, which has meanwhile been connected with the earth, is now connected with the condenser. If C be the capacity of the condenser, C' that of the electrometer (in certain cases artificially increased), we have, if F be the common potential of the condenser and connected parts of the electrometer, $(C + C')F = CE$, and

$$C = \frac{FC'}{E - F}, \quad \text{or} \quad \lambda = \frac{E - F}{F} \cdot \frac{1}{C'}.$$

But F is proportional to the second reading of the electrometer, hence λ is known in terms of C'. As only relative measures are wanted, C' is not required. Boltzmann made a variety of experiments, all of which confirmed the theory, and showed the applicability of the above formula.

If we make three measurements, first with the plates at distance m_1 ; secondly, at distance m_2 , with only air between in each case; and thirdly, at distance m_3 , with an insulating plate of thickness n between, we have, if $\lambda_1, \lambda_2, \lambda_3$ be the corresponding values of λ ,

$$G = \frac{\lambda_2 - \lambda_1}{m_2 - m_1}, \quad \text{and} \quad \frac{1}{K} = \left(\frac{\lambda_3 - \lambda_1}{G} - m_3 + m_1 + n \right) \div n.$$

The advantage of this procedure is that only differences of m_1, m_2, m_3 come in, and no absolute length has to be measured. Measurements were also made with condensers, in which there was no air between the armatures and the insulating plates; in them the armatures were formed by means of mercury. To give an idea of the agreement of the results by different methods, we give K for paraffin as determined on plates of different thickness; with the ordinary condenser, $K = 2.28, 2.34, 2.31$ for plates I., II., and III.; and $K = 2.31, 2.33$ for plates I. and II. used with mercury armatures.

Boltzmann convinced himself that, in the case of ebonite, Effect of paraffin, sulphur, and rosin, the time during which the condenser was charged was without sensible influence. He found that the result was the same whether the charge

¹ *Camb. and Dub. Math. Journ.*, 1845, or *Reprint of Papers*, p. 15.

² *Pogg. Ann.*, cli., 1857.

³ For a description of this instrument, see Wiedemann's *Galvanismus*, Bd. I. § 451.

⁴ *Ann. de Chim. et de Phys.*, 4 ser. t. ii. (1862).

⁵ *Phil. Trans.*, 1871.

⁶ *Pogg. Ann.*, cli., 1874, or *Sitzb. der Wiener Akad.*, lxxvii.

⁷ *Pogg. Ann.*, clii.

⁸ It is supposed that the plates are near enough to allow us to neglect the effect of the rims.

was instantaneous or lasted for a considerable time. The case was different with the imperfect insulators, glass, stearine, and gutta percha, for which he has given no results. To test still farther the influence of the time, Boltzmann measured the attraction between a sulphur and a metal sphere—first, when the latter was charged continuously positive or negative, and, secondly, when it was charged positive for $\frac{1}{300}$ th of a second, negative for the next $\frac{1}{300}$ th, and so on; he found the attraction to be the same in both cases, provided the charges without respect to sign were equal. This experiment establishes beyond a doubt the existence, in the case of sulphur, of a specific dielectric action, which is fully developed in less than $\frac{1}{300}$ th of a second. From experiments of this kind values of K were deduced, which agreed fairly well with those obtained by other methods. A very important result which he obtained was, that for a certain crystalline sphere of sulphur the values of K were different in the directions of the axes, being 4.773, 3.970, and 3.811 respectively. The result realizes an expectation of Faraday.¹

Schiller
Method
of elec-
trical
oscilla-
tions.

Schiller employed two methods—the method of Siemens, which we have already described, in which the duration of charge was from $\frac{1}{300}$ th to $\frac{1}{300}$ th of a second, and the method of electrical oscillations devised by Helmholtz. In the latter method K is given by the equation $K = (T^2 - T_0^2) \div (T'^2 - T_0'^2)$, where T_0 , T , T' , are the periods of oscillation of a certain coil, firstly, by itself, secondly, when connected with an air-condenser, and thirdly, with the same condenser when the air is replaced by the insulator to be tested (see below, p. 82). In this method the duration of charge varied from $\frac{1}{30000}$ th to $\frac{1}{3000}$ th of a second.

The following table gives some of the results of Boltzmann and Schiller:—

	Boltzmann.	Schiller.	
Ebonite	3.15	2.76	2.21
Paraffin (clear)	2.32	1.92	1.68
Do. (milky) ... }		2.47	1.81
Sulphur	3.84
Rosin	2.55
Indiarubber (pure)	2.34	2.12
Do. (vulcanized)	2.94	2.69
White mirror glass..	...	6.34	5.88

The first column of Schiller's results was obtained by Siemens's method, the second by the method of oscillations. It will be seen that the shortness of the time of charge has affected the value of K in the last column, reducing it considerably in all cases. Boltzmann's results are on the whole the largest obtained by any physicist; he attributes this to the care with which he constructed his plates. Gibson and Barclay found 1.97 for paraffin, and Siemens 2.9 for sulphur.

Among the more recent researches on the theory of dielectrics may be mentioned those of Rood,² whose results for crystals are interesting, and Wüllner,³ who has studied the course of induction when the charge is maintained for a considerable time.

There are very few fluids which are sufficiently good insulators to allow an easy determination of their specific inductive capacity. Measurements have, however, been made by Silow.⁴ He used (1) Siemens's method, and (2) a method in which he observed the deflection of a quadrant electrometer corresponding to the same potential, first, when the quadrants were filled with air, and secondly, when they were filled with the fluid to be examined; the ratio of the latter deflection to the former is the specific inductive capacity of the liquid.

The instrument actually used was a glass vessel, inside which were pasted pieces of tinfoil corresponding to the quadrants of

Thomson's electrometer. The shape of the needle was also slightly different. A fine silver wire replaced the bifilar suspension, and the deflections were read off by means of a scale and telescope. The needle and one pair of quadrants were connected with the earth, and the other pair of quadrants charged to a constant potential by connection with a battery. The results were for oil of turpentine by method (1), 1.468; by (2), 1.473; for a certain specimen of petroleum, by (1), 1.439; for another specimen, by (2), 1.428; for benzol, by (1), 1.483.

In the researches in which Siemens's method was used, the speed of the commutator was varied considerably, but no effect was thereby produced on the value of K , which is therefore, within certain limits at least, independent of the duration of the charge.

Perhaps the most important of all the recent additions to our knowledge in this department is due to Boltzmann,⁵ Boltzmann who has succeeded in detecting and measuring the decrease of the specific inductive capacity of gases when rarefied.

The principle of his method is as follows. Suppose we have an ordinary air-condenser inside a receiver, which we can exhaust at will. Let one of the armatures A of the condenser be connected with a battery of a large number n of cells (Boltzmann used about 300 Daniell's), while the other armature B is connected with the earth. If we now insulate B , and if the condenser does not leak, then on connecting B with the electrometer no deflection will be indicated. If, however, we increase the number of cells by one, the potential of A will increase from np to $(n+1)p$, while that of B will rise from 0 to an amount which is proportional to p . Let the corresponding electrometer reading be β . Suppose now that we altered the specific inductive capacity of the gas from K_1 to K_2 , both armatures being insulated, A originally at potential np , and B at potential zero; the potential of A will, by the mathematical theory, become $K_1^{-1}np$, while that of B remains zero. If now we reconnect A with the battery of n cells, the potential of A becomes again np . If we then connect B with the electrometer we shall get a deflection α proportional to

$$np \left(1 - \frac{K_1}{K_2} \right); \text{ hence we have } \frac{\alpha}{\beta} = n \left(1 - \frac{K_1}{K_2} \right).$$

Let us now assume, what experiment shows to be the case, that the increase of K is very nearly proportional to the pressure, then, b_1 and b_2 denoting the manometric reading in millimetres corresponding to K_1 and K_2 , we may write

$$K_1 = c \left(1 + \frac{\lambda b_1}{760} \right), \quad K_2 = c \left(1 + \frac{\lambda b_2}{760} \right).$$

Here λ is a constant, the meaning of which is very simple, if we assume our law of proportionality to hold up to absolute vacuum; in fact, $1 + \lambda$ is in that case the specific inductive capacity⁶ of the gas at 760 mm. pressure, at the temperature t of observation, and $1 + \lambda(1 + \alpha t)$ is the corresponding coefficient at 0°C . The formula written above becomes therefore

$$\lambda = \frac{\alpha \cdot 760}{8n(b_1 - b_2)}.$$

In this way Boltzmann arrived at the following values for \sqrt{K} at 760 mm. pressure, and temperature 0°C .:—for air, 1.000295; carbonic acid, 1.000473; hydrogen, 1.000132; carbonic oxide, 1.000345; nitrous oxide, 1.000497; olefiant gas, 1.000656; marsh gas, 1.000472. These results are of great importance in connection with the electromagnetic theory of light.

Residual Discharge.

When an accumulator, whose dielectric is glass or shellac, is charged up to a moderately high potential, and one armature insulated, a gradual fall of the potential occurs. This fall is tolerably rapid at first, but it gets slower and slower till at last it reaches a certain limit, after which it remains sensibly constant for a considerable time. This fall is not entirely due to loss by conduction or convection of the ordinary kind, for we find that if an accumulator that has been charged to potential V , and has been allowed to stand till the potential has fallen considerably, be again charged up to potential V , then

⁵ *Pogg. Ann.*, lv., 1875.

⁶ K is now taken to be $=1$ for absolute vacuum.

¹ *Exp. Res.*, 1689.

² *Pogg. Ann.*, clviii., 1876.

³ *Pogg. Ann.*, N.F. i., 1877.

⁴ *Pogg. Ann.*, clvi., 1875; clvii., 1876.

Silow.
Results
for
fluids.

Pheno-
mena of
latent
charge.

the rate of loss is much less than before, being now very nearly constant, and not far from the limit above mentioned. It would appear, therefore, that this constant limit, which on favourable days is very small, represents the loss due to convection and conduction in the usual way, and that the larger varying loss is due to some other cause. When an accumulator, let us say a Leyden jar, has been repeatedly charged up to potential V , until the rate of dissipation has become constant, we shall say that it is saturated. If we discharge a saturated jar, by connecting the knob for a fraction of a second with a good earth communication, and then insulate the knob, the outer coating being supposed throughout in connection with the earth, we find that the instant after the discharge the potential of the knob is zero; after a little, however, it begins to rise, and by and by it reaches a value which is a considerable fraction of V , and has the same sign. This phenomenon justifies the assumption we made as to the peculiar nature of the variable loss of potential experienced by a freshly charged jar. The charge which reappears in this way subsequent to the instantaneous discharge is called the residual charge.¹ If at any time during the appearance of the residual charge the jar be discharged, the potential of the knob becomes for a short time zero, but begins to rise again; and this may be repeated many times before all trace of charge disappears. Faraday made a variety of experiments on the subject, and established that whenever a charge of positive electricity disappeared or became latent in this way, an equal negative charge disappeared in a similar way. He concluded that the cause of the phenomenon was an actual penetration of the two electricities (*Exp. Res.*, 1245) by conduction into the dielectric. This is not the view which is favoured by the best authorities of the present day; it is indeed (see Maxwell, *Elect. and Mag.*, vol. i. § 325) at variance with the received theories of conduction, and alike untenable, as far as we know, whether we adopt the theories of Weber, of Maxwell, or of Helmholtz. Faraday established that time was a necessary condition for the development of the phenomenon; and he was thus enabled to eliminate its influence in the experiments on the specific inductive capacity of sulphur, glass, and shellac. The phenomenon is most marked in the last of these; and in spermaceti, which relatively to these is a tolerably good conductor, the phenomenon is very marked, and develops very rapidly.

Kohlrausch² studied the residual discharge in an ordinary Leyden jar, in a jar whose outside and inside coatings were at one time quicksilver and at another acidulated water, and in a Franklin's pane, one side of which was coated with tinfoil in the usual way, while the other was silvered like a piece of looking-glass. He showed, by taking measurements with an electrometer and a galvanometer, that the ratio of the free or disposable charge to the potential is constant. By the disposable charge is meant the charge which is instantaneously discharged when the knob of the jar is connected with the earth. This ratio is the capacity of the jar, and it appears that it is independent of the "residual" or "latent" charge. He showed that the "latent" charge is not formed by a temporary recession of the electricity to the uncovered glass about the neck and upper part of the jar; and that it does not to any great extent depend on the material used to fasten the armature to the glass, or on the air or other foreign matter between them. On the other hand, his results led him to suspect that the "latent" charge depended on the thickness of the glass, being greater for thick plates than for thin. This

conclusion has been questioned, however.³ He separated by a graphical method the loss by latent charge from the loss by conduction, &c., and found that the amount of charge which becomes latent, or, which amounts to the same thing, the loss of potential owing to the forming of latent charge in a given time, is proportional to the initial potential so long as we operate with the same jar.

Kohlrausch recognized the insufficiency of Faraday's His explanation of the residual charge, and sought to account theory for it by extending Faraday's own theory of the polarization of the dielectric. The residual charge is due according to him to a residual polarization of the molecules of the dielectric, which sets in after the instantaneous polarization is complete, and which requires time for its development. This polarization may consist in a separation of electricity in the molecules of the dielectric, or in a setting towards a common direction of the axes of a number of previously polarized molecules, analogous to that which Weber assumes in his theory of induced magnetism. It is easy to see that such a theory will to a great extent account for the gradual reduction of the potential of a freshly charged jar, and the gradual reappearance of the residual charge.

If the charge, and consequently the potential, of the jar were kept constant at Q_0 , the residual charge tends to a limit pQ_0 (p const.) Kohlrausch assumes that the difference $r_t - pQ_0$ between the residual charge actually formed and the limit decreases at a rate which is at each instant proportional to this difference, and furthermore, to a function of the time, which he assumes to be a simple power. In any actual case, where the jar is charged and then insulated, the charge varies, owing to conduction, &c., and to the formation of residual charge, so that the limit of r is continually varying, and we must write Q_t for Q_0 , Q_t denoting the charge at time t . The equation for residual charge is then

$$\frac{d}{dt}(r_t - pQ_t) = -bt^m(pQ_t - r_t).$$

From this he deduces the formula

$$r_t = p\left(Q_t - Q_0 e^{-\frac{b}{m+1}t^{m+1}}\right),$$

which he finds to represent his results very closely. m has very nearly the same value (-0.5744 , or $-\frac{1}{2}$ nearly) in all his experiments, p had the values 0.4289 , 0.5794 , 0.2562 ; and b 0.0897 , 0.0223 , 0.0446 in his three cases.

Kohlrausch called attention to the close analogy between the residual discharge and the "elastic recovery" (*elastische Nachwirkung*) of strained bodies, which had been investigated by Weber⁴ in the case of a silk fibre, and which has of late excited much attention. The instantaneous strain which follows the application of a stress is analogous to the initial charge of the jar, and the gradually increasing strain which follows to the gradual formation of the latent or residual charge. The sudden return to a position near that of unconstrained equilibrium corresponds to the instantaneous discharge, and the slow creeping back to the original state of equilibrium to the slow appearance of the residual discharge. Another analogy may be found in the temporary and residual or subpermanent magnetism of soft iron or steel. If we wish to make the analogy still more complete, we have only to introduce the permanent polarity of tourmaline, the permanent set of certain solids when strained, and the permanent magnetism of hard steel. The phenomena of polarization furnish yet another analogy.

In justifying the introduction of a power of the time into his equation for the residual discharge, Kohlrausch makes the important remark that the time which a residual charge of given amount takes to reappear fully may be different according to the way that charge is produced. The charge reappears more quickly when it is produced in a short time by an initial charge of high potential, than when produced by a charge of lower potential acting

¹ When we think of the part of the charge that has disappeared, i.e., ceased to effect the potential of the knob, we may talk of the "latent charge." This part of the charge is sometimes said to be absorbed.

² *Pogg. Ann.*, xci., 1854.

³ Willner, *Pogg. Ann.*, N.F. i. pp. 272, 369.

⁴ *De fili bombycini vi elastica*, Göttingen, 1841.

longer. He suggests that the same thing may be true of elastic recovery. He does not allude to the fact (possibly he was unaware of it) that two residual charges of different sign may be superposed and reappear separately, although the possibility of this is to a certain extent involved in his remark. The analogous elastic phenomenon has recently been observed by F. Kohlrausch.

Maxwell¹ has shown that phenomena exactly like the residual discharge would be caused by conduction in a heterogeneous dielectric, each constituent of which by itself has not the power of producing any such phenomenon, so that the phenomenon in general might be due to "heterogeneity" simply.

Hopkinson has lately made experiments on the residual discharge of glass jars. He observed the superposition of residual charges of opposite signs, and he suggests theories analogous to those of Kohlrausch and Maxwell. He finds that his results cannot be represented by the sum of two simple exponential functions of the time, and concludes, therefore, that heterogeneity must be an important factor in the cause of the phenomenon.

The polarities of the different silicates of which the glass is composed rise or decay with the time at different rates, so that during insulation the difference of potential between the armatures E would be represented by a series $\sum A_n e^{-\lambda_n t}$. If, therefore, we charge a jar positively for a long time, and then negatively for a shorter time, the second charge will reverse the more rapidly changing polarities, while the sign of the more sluggish will not be changed; when, therefore, the jar is discharged and insulated, the first-mentioned polarities will decay more quickly at first and liberate a negative charge, and, finally, as the more sluggish also die away, a positive charge will be set free. Hopkinson also made the important observation that agitation of the glass by tapping accelerates the return of the residual discharge.

ON THE PASSAGE OF ELECTRICITY THROUGH BODIES.

We have hitherto supposed electricity to be either immovably associated with perfectly non-conducting matter, or collected on the bounding surfaces of conducting and non-conducting media in such a way that the force tending to cause it to move is balanced by an invincible resistance. We have now to consider what happens when there is a finite unbalanced resultant force at any point in a conducting medium. If a conducting sphere of radius a be charged with Q units of positive electricity, its potential will be $\frac{Q}{a}$. Connect this sphere by a long thin wire, whose capacity may be neglected, with another uncharged sphere of radius b , then we know that the potentials of the two spheres become equal; and since what we call electricity is subject to the law of continuity, the whole charge on the two spheres must be the same as before. Hence if U be the common potential, we must have $U = \frac{Q}{a+b}$. It appears, therefore, that the potential of a has fallen by $\frac{b}{a+b} \frac{Q}{a}$, and an amount $\frac{b}{a+b} Q$ of positive electricity has passed from a to b , and also a $\frac{b}{a+b}$ th part of the electric potential energy has disappeared. In accordance with our hypothesis that electricity obeys the law of continuity like an incompressible fluid, we explain this transference of electricity by saying that an electric current has flowed through the wire from the place of higher to the place of lower potential. We define the intensity or strength C of the current as the quantity of electricity which crosses any section of the wire in unit of time.

Owing to the law of continuity the current intensity is of course the same at every point of a linear conductor.

In the case which we have just given, the whole transference takes place in so short a time that we cannot study the phenomenon in detail. It is obvious that C will vary rapidly from a large initial value, when the difference between the potentials of the spheres is $\frac{Q}{a}$, to zero when

they are at equal potentials. It is possible, by replacing the wire by wetted string or other bad conductor, to prolong the duration of the phenomenon to any extent, so that C should vary very slowly; and we can imagine cases where C would remain constant for a long time. Machines for producing a continuous or "steady" current have been invented in considerable variety, the first of the kind having been the Pile of Volta. Of such machines we shall have more to say when we come to discuss Electromotive Force. We have seen, in the case of our spheres, that the passage of the electric current was accompanied by a loss of potential energy. The question thus arises, and the equalization of potential is complete? This leads us to look for transformations of energy depending on the electric current, or, in other words, to look for dynamical effects of various kinds due to it. Accordingly we find the passage of the electric current accompanied by magnetic phenomena, sparks, heating of the circuit, chemical decompositions, mechanical effects, &c. All these are observed in the discharge of the Leyden jar and other electrostatic reservoirs of potential energy. Exactly similar effects, some more, others less powerful, are observed accompanying the current of the voltaic battery and other machines which furnish a steady flow of electricity. In all such cases we have (1) a source of energy, (2) a flux of electricity, (3) an evolution of energy in different parts of the circuit. We reserve the consideration of (1) for the present, as being the most difficult, and devote our attention to (2) and (3).

Ohm's Law applied to Metallic Conductors.

We have already seen how to measure the strength of an electric current in a linear conductor. According to the definition we gave above, the unit current strength would be that for which a unit of electricity passes each section of the conductor in unit of time. If the unit of electricity is the electrostatic unit, this is called the electrostatic unit of current. We have supposed above that the current consists in the transfer of a certain amount of + electricity in a certain direction, which we shall call the positive direction of the current, and this for most purposes is convenient. We must remember, however, that no distinction can be drawn between the transference of + Q units of electricity in one direction and the transference of - Q units in the opposite direction; for we have no experimental evidence on which such a distinction can be founded.

We may measure the current by any one of its various effects. The method most commonly used, both for indicating and measuring currents, is to employ the magnetic effect. According to Oersted's discovery, a magnetic north pole placed in the neighbourhood of a straight current is acted on by a force such that, if the pole were to continually follow the direction of the force, it would describe a circle round the current as an axis, the direction of rotation being that of the rotation of a right-handed cork-screw which is traversing a cork in the positive direction of the current. If, therefore, we have currents of different strength in the same wire, the force exerted on a magnet which always occupies the same position relatively to the wire will be a measure of the current. The force exerted on the magnet may be found by balancing it against known forces, or by allowing the magnet to oscillate under it and finding the time of oscillation. It

¹ Electricity and Magnetism, §§ 327 eqq.

is easy, by applying the law of continuity to multiple circuits, to verify that the measure of current intensity thus got is proportional to the electrostatic measure.

Thus let AB (fig. 18) be a circuit splitting up into two exactly similar branches BCDG, BEFG, and uniting again at G. Then, since electricity behaves like an incompressible fluid, it is obvious that any current of intensity C in AB will split up into two currents each of intensity $\frac{1}{2}C$ in CD and EF. By placing a magnet in similar positions at the same distance with respect to AB, CD, and EF, it will be found that the magnetic action in the last two positions is just half that in the first.

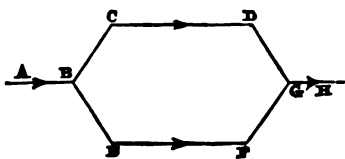


Fig. 18.

The appropriate unit in magnetic measurements of current intensity is that current which, when flowing in a circular arc of unit radius and unit length, exerts unit of force on a unit north pole placed at the centre of the arc, the unit north pole being such that it repels another equal north pole at unit distance with unit force. This is called the *electromagnetic unit* of current intensity. Unless the contrary is stated, all our formulæ are stated in terms of this unit.

To facilitate the detection and measurement of currents by magnetic means, an instrument called a galvanometer is used. It consists of a coil of wire, of rectangular, elliptical, or circular section, inside which is suspended a magnetic needle, so as to be in equilibrium parallel to the coil windings under the magnetic action of the earth, or of the earth and other fixed magnets. When a current passes through the coil a great extent of the circuit is in the immediate neighbourhood of the magnet, and the magnetic action is thus greatly accumulated. See article GALVANOMETER.

If we connect two points A and B of a homogeneous linear conductor, every point of which is at the same temperature, by two wires of the same metal to the electrodes of a quadrant electrometer, then, if a steady current C (measured in *electrostatic* units) be flowing from A to B, we shall find that the potential at A is higher than that at B by a certain quantity E , which we may call the *electromotive force* between A and B, and we may suppose E for the present to be measured in electrostatic units.

If we examine the value of the ratio $\frac{E}{C}$ for different positions of the points AB, we shall find that it varies directly as the length of linear conductor between A and B, provided the section of the conductor is everywhere the same. If we try wires of different section, but of the same length and the same material, we find that $\frac{E}{C}$ is

inversely proportional to the sectional area; in fact we may write

$$\frac{E}{C} = R = \frac{kl}{\omega} \quad (1),$$

where l denotes the length of the wire, ω its section, and k a constant depending on its material, temperature, and physical condition generally. This is Ohm's law.

In whatever unit measured, R is called the resistance of the conductor. The unit of resistance can always be conceived as established by means of a certain standard wire. The unit of electromotive force is then such that if applied at the end of the standard wire it would generate a unit current in the wire. The constant k is called the *specific resistance* of the material of which the wire is made; it is obviously the resistance of a wire of the material of unit length and unit section.

In the electrostatic system of unitation the unit of E is the work done by a unit particle of +electricity in passing to infinity from the surface of an isolated sphere of radius unity charged with an electrostatic unit of +electricity. The dimension of E is $[QL^{-1}]$, where $[Q]$ is the dimension of the electrostatic unit of quantity

(see p. 22), $[Q] = [L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}]$. Hence the dimension of E is $[L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}]$. The unit of C we have already discussed; its dimension is $[QT^{-1}] = [L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-2}]$. From these results, and equation (1), it follows that the dimension of R is $[L^{-1}T]$, i.e., that of the reciprocal of a velocity. We shall show hereafter that, if C be measured in electromagnetic units, its dimension is $[L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}]$; hence that of Q is $[L^{\frac{1}{2}}M^{\frac{1}{2}}]$, the unit of Q being the quantity of electricity conveyed across any section by the unit current. Also ECT = work done in time T in conveying C units of +electricity from potential $V + E$ to potential V , whence $[ECT]$ = dimension of energy = $[L^{\frac{1}{2}}MT^{-2}]$. Hence $[E] = [L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-2}]$. In this case then $[R] = [LT^{-1}]$; so that in electromagnetic measure R has the dimension of a velocity.

We can put the equation (1) into another form, which suggests Ohm's law generalized for any conductor. Consider two points P and Q on a linear conductor, at a distance dx from each other, x being measured in the direction of the current. Let the potentials at P and Q be V and $V + dV$, then $E = -dV$. If u denote the current per unit of area of the section, then $C = u\omega$, and since $l = dx$ we have $R = \frac{kdV}{u}$. Substituting these values in (1) we get

$$u = -\frac{1}{k} \frac{dV}{dx} = \frac{X}{k} \quad (2),$$

where X is the component electric force at P in the direction of the current. Since the electric current is of the nature of a flux, it is determined at any point of a conductor by the flux components u, v, w , representing the quantities of electricity which in unit of time cross three unit areas perpendicular to three rectangular axes drawn through P. If X, Y, Z be the components of the electric force at P, then the general statement of Ohm's law for a homogeneous isotropic conductor is

$$u = \frac{X}{k}, \quad v = \frac{Y}{k}, \quad w = \frac{Z}{k} \quad (3).$$

In such a conductor the resistance of a small linear portion of given dimensions, cut out of the substance any where or any how, will be the same. It is conceivable, however, that the resistance of such a small portion would be different if cut in different directions at any point, in which case the conductor would be *anisotropic*. The most general statement of Ohm's law would then be

$$\begin{cases} u = r_1X + p_1Y + q_1Z \\ v = q_2X + r_2Y + p_2Z \\ w = p_3X + q_3Y + r_3Z \end{cases} \quad (4),$$

Equations of conduction.

where $r_1, \&c., p_1, \&c., q_1, \&c.$, are constants for any one point. If they are the same for all points, the body is said to be homogeneous; if they vary from point to point, the body is said to be heterogeneous. If we may liken our conductor to an arrangement of linear conductors (see Maxwell, §§ 297, 324, vol. i.), then it may be shown that the skew system of (4) becomes symmetrical, inasmuch as $p_1 = q_2, p_2 = q_3, p_3 = q_1$. The great majority of the substances with which the electrician has to deal are, however, isotropic; and unless the experiments of Wiedemann on certain crystals point to anisotropic conduction, we do not know of any case which has been experimentally examined. The reader will find interesting developments of the subject in Maxwell, vol. i. § 297 *seqq.*

A very important remark to be made with regard to the equations (4) is that, being linear, the principle of superposition applies. Thus, if u, v, w be the current components due to electric forces X, Y, Z and u', v', w' similar components for X', Y', Z' , then the current for $X + X', Y + Y', Z + Z'$ is given by $u + u', v + v', w + w'$. It is obvious, moreover, that (4) are the most general equations that can be written down to connect current with electromotive force, subject to the condition that the currents due to superposed electric forces are to be found by the superposition of the currents due to the separate forces.

Besides the equations (4), u, v, w are subject like any other flux components to an equation of continuity. This equation, investigated in the usual manner, is

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} + \frac{d\rho}{dt} = 0 \quad (5),$$

where ρ is the electric volume density at the time t . At a surface of discontinuity (5) must be replaced by

$$(u - u')l + (v - v')m + (w - w')n - \frac{d\sigma}{dt} = 0 \quad (6),$$

where u, v, w , and u', v', w' are components of flux on the first and second sides of the surface, l, m, n the direction cosines of the normal

drawn from the first to the second side, and σ the electric surface density at time t .

If we consider the particular case of homogeneous isotropic media, and suppose further that $X = -\frac{dV}{dx}$, $Y = -\frac{dV}{dy}$, $Z = -\frac{dV}{dz}$, these equations reduce to

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} = k \frac{d\rho}{dt} \quad (7),$$

and

$$-\frac{1}{k_1} \frac{dV_1}{dv} + \frac{1}{k_2} \frac{dV_2}{dv} = \frac{d\sigma}{dt} \quad (8).$$

In the last equation V_1 and V_2 are the potentials on the two sides of a boundary between media of specific resistance k_1 and k_2 .

In the particular case of steady motion, the right-hand sides of (7) and (8) are zero. The analytical treatment of problems about steady currents is therefore precisely analogous to that of problems about electrostatical equilibrium, steady flow of heat, hydrodynamics, &c.: to every solution in one such physical subject corresponds a solution in each of the others. Many valuable details on this subject are to be found in Thomson's papers on electrostatics and magnetism.

Results of Ohm's law.

The consequences of Ohm's law have been followed out mathematically, and verified in a variety of cases. We shall notice a few which are interesting, either from the accuracy of the experimental results, or from the interest or practical importance of some method or principle involved.

Application to uniform linear conductors.

In the case of a steady current in a uniform linear conductor, say a wire, it is obvious that the potential must fall uniformly in the direction in which the current is flowing. Hence, if we suppose the wire stretched out straight, and erect at different points lines perpendicular to it, representing the potential at each point, the locus of the extremities of these lines will be a straight line.

This may be arrived at by integrating equation (5), which becomes in this case $\frac{d^2V}{dx^2} = 0$, x being measured along the wire supposed to be straight; we thus get for the potential V , at any point distant x from the origin, at which potential is V_0 ,

$$V = V_0 - \frac{Ck}{\omega} x \quad (9).$$

If V be taken as ordinate, this represents a straight line, the tangent of whose inclination to the x -axis is $-\frac{Ck}{\omega}$, or $-uk$.

Voltaic circuit.

We cannot apply Ohm's law at the junction of two different substances. The condition of continuity of course applies; in other words, if the flow has become steady, the current is the same at all points of the circuit, whether homogeneous or not. We shall see, when we come to discuss electromotive force, that there is a constant difference between the potentials at two points infinitely near each other, but on opposite sides of the boundary between two conductors of different material. If we knew this potential difference for each point of heterogeneous contact in the circuit, we could draw the complete potential curve for the circuit by applying Ohm's law to each conductor separately. The diagram (fig. 19) represents (on the contact theory, as held by Ohm, see Origin of Electromotive Force) the fall of potentials and the discontinuities in a voltaic circuit, consisting of zinc, water, and copper, in which the current flows from Cu to Zn across the junction of the metals. We assume for the present that Ohm's law applies to the liquid conductor.

Let us denote by V_Q , V_R , &c. the potentials at Q and R, &c., or what is the same thing, the ordinates BQ, BR, &c., in our diagram. Then applying Ohm's law to the homogeneous parts of the circuit, we have $V_V - V_Q = CR'$, $V_R - V_S = CS$, $V_T - V_U = CR''$, where R' , S , R'' , denote the

resistances of the zinc, the water, and the copper respectively. Now, denoting $V_V - V_U$, the potential difference, or as it is sometimes called, the "contact force" between Zn and Cu by E_{ZC} , and so on, let us add the above three equations; we thus get

$$E = E_{ZC} + E_{AZ} + E_{CA} = C(R' + R'' + S).$$

Here E is called the *whole electromotive force* of the circuit, being the sum of all the discontinuities of potential, taken with their proper signs, or, what is equivalent to the same thing, the whole amount of work which would be done by a unit of + electricity, in passing round the whole circuit once, supposing it to get over the discontinuities without gain or loss of work. Defining E in this way, we may extend Ohm's law to a heterogeneous circuit, the resistance R being now the sum of all the resistances of the different parts, or the whole resistance. In accordance with this definition, if we take two points, p and q (fig. 19) in the Cu and Zn respectively, the whole electromotive force will be $V_p - V_q + E_{ZC}$ and the current will be given by

$$V_p - V_q + E_{ZC} = RC \quad (10),$$

where R is the whole resistance of pq . $V_p - V_q$ is sometimes called the "external," and E_{ZC} the "internal" electromotive force. If p, q include more than one contact of heterogeneous metals, we have only to add on the left-hand side of (10) the corresponding internal electromotive force for each discontinuity.

If p and q be connected by wires of the same metal, say copper, to the electrodes of a Thomson's electrometer, then the electrometer will indicate a potential difference, $V_p - V_q + E_{ZC}$, and not $V_p - V_q$, as might at first sight be suspected.¹ No electricity can flow through the electrometer, hence the copper wire attached at p , and the pair of quadrants to which it leads (we may suppose the quadrants made of copper, but in reality it does not matter, see below, Origin of Electromotive Force), will be at potential V_p . But owing to the contact force between the Zn and Cu at q , the wire from q and the quadrant to which it leads will be at potential $V_q - E_{ZC}$. It appears, therefore, that the electrometer indication corresponds to the *whole* electromotive force between p and q , and is proportional to the whole resistance between p and q , no matter what metals the circuit may include.² This conclusion was verified by Kohlrausch. His method rested on the principle of Volta's condensing electroscope.

He used an accumulator consisting of a fixed plate B, and an *Verd* equal movable plate A, which could be lowered to a very small fixed distance from B, and raised to a considerable distance, so as Kohl to touch a fixed wire leading to a Dellmann's electrometer. The rause plate A was lowered and connected with p , while q and the fixed plate were connected with the ground; the connection with p was then removed, and A raised, its potential thereby greatly increasing owing to its greatly diminished capacity. This increased potential was measured by the electrometer, with which A was in connection through the fixed wire. In one of Kohlrausch's experiments, he found for the electromotive force between a fixed point of the metallic circuit and four points, such that the resistance between each adjacent pair was very nearly equal, the values 0.85, 1.81, 2.69, 3.70; the values calculated by Ohm's law were 0.93, 1.86, 2.80, 3.73. He also examined the fluid part of the circuit, and still found a good agreement between theory and experiment. (See Wiedemann, § 102.)

The laws of current distribution in a network of linear circuits were first studied by Kirchhoff. He laid down two general principles which are very convenient in practical calculations.

I. The algebraical sum of all the currents flowing from any node of the network is zero.

II. If we go round any circuit of the network, then no

¹ It is supposed that all the wires are at the same temperature.

² This more general statement follows at once from the above reasoning in conjunction with Volta's law (cf. below, Origin of Electromotive Force).

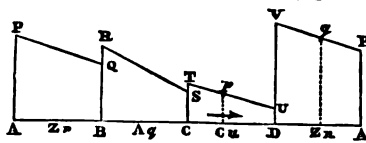


Fig. 19.

Netw of lin cond tors.

matter how many meshes it may include, or what conductors may branch off at different parts, we have

$$E = R_1 C_1 + R_2 C_2 + \dots + R_n C_n,$$

where E is the whole internal electromotive force, and $R_1, R_2, \dots, C_1, C_2, \dots$ are the resistances and current strengths in the different parts of the circuit.

The first of these principles is simply the law of continuity, and the second is got at once by applying equation (10).

We give here an investigation of the currents and potentials in a network of conductors. The method and notation are taken from Maxwell, vol. i. § 280. Let A_1, A_2, \dots, A_n be n points, connected by a network of $\frac{1}{2}n(n-1)$ conductors (that being the number of different pairs of conductors that can be selected from the n). Let C_{pq}, E_{pq}, K_{pq} denote the current strength, internal electromotive force, and conductivity, i.e., the reciprocal of the resistance, for the conductor $A_p A_q$. Let, moreover, the potential at A_p be P_p , and the current of electricity which enters the system there be Q_p . It is obvious from our definitions of the symbols that

$$K_{pq} = K_{qp}, \quad C_{pq} = -C_{qp}, \quad E_{pq} = -E_{qp},$$

and, by the condition of continuity, that

$$Q_1 + Q_2 + \dots + Q_n = 0.$$

At the point A_p we have

$$C_{p1} + C_{p2} + \dots + C_{pn} = Q_p \quad \dots (a).$$

Now

$$C_{pq} = K_{pq}(P_p - P_q + E_{pq}) \quad \dots (b).$$

Hence (a) becomes

$$K_{p1}(P_1 - P_p) + K_{p2}(P_2 - P_p) + \dots + K_{pn}(P_n - P_p) = K_{p1}E_{p1} + \dots + K_{pn}E_{pn} - Q_p \quad \dots (c).$$

The symbol K_{pp} does not occur in this equation, and has no meaning as yet. Let us define it to mean $-(K_{p1} + K_{p2} + \dots + K_{pn})$, where K_{pp} does not occur. Then we have

$$K_{p1} + K_{p2} + \dots + K_{pp} + \dots + K_{pn} = 0, \quad \dots (d)$$

and, multiplying by $P_p - P_r$,

$$K_{p1}(P_p - P_r) + \dots + K_{pp}(P_p - P_r) + \dots + K_{pn}(P_p - P_r) = 0.$$

Adding this last equation to (c) we get

$$K_{p1}(P_1 - P_r) + K_{p2}(P_2 - P_r) + \dots + K_{pn}(P_n - P_r) = K_{p1}E_{p1} + \dots + K_{pn}E_{pn} - Q_p \quad \dots (e).$$

In this equation the term whose coefficient is K_{pp} of course vanishes. By giving p all possible values except r , we get a set of $n-1$ equations to determine the $n-1$ quantities $P_1 - P_r, P_2 - P_r, \dots$, &c. Hence if M_{rr} denote the minor of K_{rr} in the determinant $\Delta = (K_{11}, K_{12}, \dots, K_{nn})$,¹ and if M_{rrp} denote the minor of K_{rp} in M_{rr} , we have

$$(P_p - P_r)M_{rr} = \{K_{11}E_{11} + K_{12}E_{12} + \dots + K_{1n}E_{1n} - Q_1\}M_{rrp} + \{K_{21}E_{21} + K_{22}E_{22} + \dots + K_{2n}E_{2n} - Q_2\}M_{rrq} + \dots \quad (f),$$

where of course E_{11} and E_{22} are zero, and M_{rrrr} does not occur. This expression is linear in the letters E and Q , and the principle of superposition holds, as we saw it ought to do in all applications of Ohm's law.

Consider the particular case in which all the Q s and E s vanish, except E_{im} and E_{im} ($= -E_{im}$), we then have the case of a linear circuit in which an electromotive force E_{im} is introduced into $A_i A_m$. We get from (f)

$$P_p - P_r = \frac{K_{im}E_{im}}{M_{rr}} (M_{rrip} - M_{rrmp}),$$

and

$$P_t - P_r = \frac{K_{im}E_{im}}{M_{rr}} (M_{rrit} - M_{rrmt}).$$

Hence

$$P_p - P_t = \frac{K_{im}E_{im}}{M_{rr}} (M_{rrip} - M_{rrit} - M_{rrmp} + M_{rrmt}),$$

and

$$C_{pq} = \frac{K_{pq}K_{im}E_{im}}{M_{rr}} (M_{rrip} - M_{rrit} - M_{rrmp} + M_{rrmt}) \quad \dots (g).$$

Similarly, if C_{im} be the current in $A_i A_m$ due to an electromotive force E_{pq} in $A_p A_q$, we get

$$C_{im} = \frac{K_{im}K_{pq}E_{pq}}{M_{rr}} (M_{rrpi} - M_{rrpm} - M_{rrqi} + M_{rrqm}) \quad \dots (h).$$

¹ This determinant has many properties of interest to the mathematical student; e.g., in our notation $M_{11} = M_{22} = \dots = M_{nn}$, $M_{rrpp} - M_{rrrp} = M_{pprr} - M_{pprp}$, &c. &c.

Now, since Δ is a symmetrical determinant, $M_{rrpi} = M_{rrip}$, &c., and the expressions within brackets in (g) and (h) are identical. Hence follows the important proposition:—

If an electromotive force equal to unity, acting in any conductor $A_i A_m$ of a linear system, cause a current C to flow in the conductor $A_p A_q$, then an electromotive force equal to unity, acting in $A_p A_q$, will cause an equal current C to flow in $A_i A_m$.

If we suppose all the conductors of the system except $A_i A_m$ and $A_p A_q$ removed, and $A_i A_p$ and $A_m A_q$ joined by two wires, in such a way that for electromotive force unity in $A_i A_m$ the current in $A_p A_q$ is C then the conductivity of the circuit which we have thus constructed would be

$$\frac{K_{im}K_{pq}}{M_{rr}} (M_{rrpi} - M_{rrpm} - M_{rrqi} + M_{rrqm});$$

this might be called the reduced conductivity of the system with respect to $A_p A_q$ and $A_i A_m$. When the expression within brackets vanishes, the conductors $A_p A_q$ and $A_i A_m$ are said to be conjugate. The reduced resistance in this case is infinite, and no electromotive force in $A_i A_m$, however great, will produce any current in $A_p A_q$, and reciprocally.

Similarly, we may prove that if unit current enter a linear system at A_i and leave it at A_m , the difference of potential thereby caused between A_p and A_q is the same as that caused between A_i and A_m , when unit current enters at A_p and leaves at A_q . (See Maxwell.)

The case of several wires forming a multiple arc very often occurs in practice.

Let AB, CD (fig. 20) be two parts of a circuit whose resistances are R and S , and let the circuit branch out between

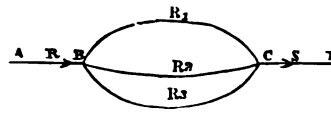


Fig. 20.

B and C into three branches of resistances R_1, R_2, R_3 .

We have $V_B - V_C = R_1 C_1 = R_2 C_2 = R_3 C_3$, and

$$C_1 = \frac{\frac{1}{R_1} C}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}, \quad C_2 = \&c.$$

Also

$$V_A - V_D = V_A - V_B + V_B - V_C + V_C - V_D = (R + \rho + S)C,$$

where

$$\frac{1}{\rho} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

Hence current in each branch is inversely proportional to the resistance, that is directly proportional to the conductivity; and the reduced conductivity of the multiple arc is equal to the sum of the conductivities of its branches. These statements are obviously true for any number of branches.

Some of the most important applications of the theory of linear circuits occur in the methods for comparing resistances. The earliest method for doing this consisted simply in putting the two conductors, whose resistance it was required to compare, into a circuit which remained otherwise invariable; if the current, as measured by a galvanometer, was the same, whichever conductor was in the gap, it was concluded that their resistances were equal. The difficulty in this method is that the electromotive force and internal resistance of the battery are supposed to remain constant, a condition which it is excessively hard to fulfil.

This difficulty can be avoided by using a differential galvanometer, or the arrangement of conductors called Wheatstone's bridge. The differential galvanometer differs from an ordinary one simply in having two wires wound side by side instead of a single wire. If we pass equal currents in opposite directions through the two wires, the action on the needle is zero, provided the instrument be perfectly constructed. If the currents are unequal, the indication will be proportional to the difference of the current strength.

If the coils are not perfectly symmetrical, but such that

Resistance measurement.

Differential galvanometer.

the deflection¹ due to a current c in one is mc , and in the other nc , where m and n are the "constants" of the two coils, then the deflection for currents c_1 and c_2 is $mc_1 - nc_2$.

Fig. 21 gives a scheme of the arrangement for measuring resistances with this instrument. V is the battery inserted in the common branch ED of the two circuits, which convey currents dividing off at D , and going in opposite directions round the coils of G . If we wish to measure the resistance of a wire, it is inserted at AB by means of binding screws or mercury cups, and the resistance of the other circuit is varied until there is no deflection; then AB is replaced by a known resistance, which is made up until there is zero deflection as before.

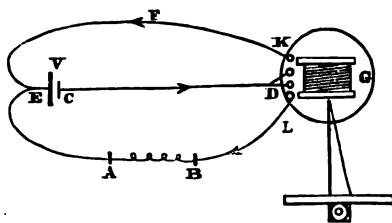


Fig. 21.

It is obvious that the only requisite here is that the resistances of EFK , EA , BL , and the galvanometer coils should remain constant. Variations in the electromotive force or internal resistance of the battery do not affect the result.

The method which we have thus sketched is the best way of using the differential galvanometer, and it does not matter even if the coils are not exactly symmetrical. Let the constants of the coils M and N be m and n , so that the deflection due to currents c_1 and c_2 in M and N is $mc_1 - nc_2$. Let the resistance from E to D in the single branch be B , and in the circuits EFK and $EABL$, which pass round M and N respectively, R and $S + U$, U being the resistance between A and B , which is such that the deflection is zero. Then

$$0 = mc_1 - nc_2 = \left\{ m(S + U) - nR \right\} \frac{E}{D} \quad \dots (a),$$

where E is the electromotive force of the battery, and

$$D = (R + S + U)B + R(S + U).$$

Suppose we substitute U' for U , and arrange U' so that we have again zero deflection. Then

$$0 = \left\{ m(S + U') - nR \right\} \frac{E}{D'} \quad \dots (b).$$

From a and b we get $U = U'$.

For farther details concerning this method, see Maxwell, vol. i. § 346, and Schwendler, *Phil. Mag.*, 1867.

Wheatstone's bridge.

The differential galvanometer method was much used by Becquerel and others, but it is now entirely superseded as a practical method in this country by the Wheatstone's bridge method. Suppose we have a circuit $ABDC$ of four conductors. Insert a galvanometer G between B and C , and a battery between A and D . Adjust say the resistance AB until the galvanometer in BC indicates no current. The bridge is then said to be balanced, and the potentials at B and C must be equal. But the whole fall of potential from A to D along ABD is the same as that along ACD ; hence if the fall from A to B is to be equal to that from A to C , we must have

$$\frac{R}{S} = \frac{T}{U},$$

where R, S, T, U are the resistances in AB, BD, CA, DC . This is the condition that BC and AD be conjugate. We might have deduced it as a particular case of the general theory given above. Hence if we know the resistances S, T, U , we

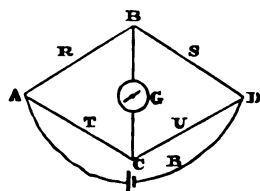


Fig. 22.

get in terms of these $R = \frac{ST}{U}$. S is often called the standard resistance, and T, U the arms of the bridge or balance. The sensibility of this arrangement may be found practically by increasing or decreasing R so as to derange the balance. The largest increase which we can introduce without producing an observable galvanometer deflection measures the sensibility of the bridge.

If we had a given set of four conductors, and a battery and galvanometer of given resistance, then it may be shown (see Maxwell, vol. i. § 348) that the best arrangement is that in which the battery or galvanometer connects the junction of the two greatest resistances with that of the two least, according as the former or the latter has the greater resistance. The practical problem might take another form. We might have given a resistance, and have at our disposal known resistances of any desired magnitude to form our bridge. We might also suppose further that we had given the total area of the plates of our battery, and the dimensions of the channel in which the galvanometer wire was to be wound. We may neglect the thickness of the silk coating, or assume that it is proportional to the thickness of the wire.

Then, B and G being the resistances of the battery and galvanometer, the electromotive force $E \propto \sqrt{B}$, and the number of turns in the galvanometer $\propto \sqrt{G}$.

Let us put $S = yR$, $T = zR$, and $U = yzR$. These resistances would balance; let us however put $(1 + x)R$ in the branch AB instead of R , the others being unchanged, and calculate the effect on the galvanometer in G , which we put proportional to the current in BC , and to the number of turns on galvanometer. Then, from equation (7) (or Maxwell, vol. i. 349), we find that the deflection δ varies as

$$\frac{yz\sqrt{BG}}{(1+y)(1+z)BG + y(1+z)^2BR + z(1+y)^2GR + yz(1+y)(1+z)R^2};$$

in order that δ may be a maximum, we must have

$$G\{(1+y)(1+z)B + z(1+y)^2R\} = y(1+z)^2BR + yz(1+y)(1+z)R^2 \quad (a),$$

$$B\{(1+y)(1+z)G + y(1+z)^2R\} = z(1+y)^2GR + yz(1+y)(1+z)R^2 \quad (b),$$

$$BG = zR^2 \quad (c),$$

$$BG = yR^2 \quad (d),$$

α and β give at once by addition and subtraction

$$\frac{R}{G} = \frac{z(1+y)^2}{y(1+z)^2} \text{ and } BG = yzR^2,$$

$$\text{or } B = z \frac{1+y}{1+z} R \quad (e),$$

$$G = y \frac{1+z}{1+y} R \quad (f).$$

Combining the four equations (c), (d), (e), (f), we get

$$y = z - 1 \text{ and } B = G = R = S = T = U.$$

It appears, therefore, that when all the resistances on the bridge are at our disposal, we ought to make them all equal to the resistance to be measured, or come as near this as we can; e.g., if we had a very small resistance to measure, we should make the arms of the bridge small, and take a small-resistance in preference to a high-resistance galvanometer.

In order to carry out measurements of resistance with ease we must possess a series of graduated resistances, with which we can compare any unknown resistance, and of which we can make the arms of our balance, &c. Again, if the measurements of one electrician are to be of any use to another, there must be a common standard. It would be most convenient to have only one standard for all nations, and this standard might be either arbitrary, like the standard of length, or absolute in some sense such as we have defined above. Arbitrary standards have at different times been proposed by Jacobi and others. The mercury standard of Siemens, to which we alluded in the historical sketch, has obtained great prevalence on the Continent. The British Association unit or ohm is an absolute unit,

¹ The deflections are supposed small.

inasmuch as it professes to represent in electromagnetic measure a velocity of 10^9 centimetres per second, or, taking the original definition of a metre, an earth quadrant per second. It happens, by a curious accident, that the mercury unit and the ohm are very nearly equal, the latter being expressed in terms of the former (according to Dehms and Hermann Siemens; see Wiedemann, Bd. ii. 2, § 1074) by the number 1.0493.

One of the earliest instruments for furnishing a graduated resistance was the rheostat, brought into use by Wheatstone, but also invented independently by Jacobs at St Petersburg about 1840.

It consisted of two cylinders of equal diameter, one of wood and one of brass. A wire, whose extremities were in connection with the metallic axes of the cylinders, was wound in opposite directions round the cylinders. The axes of the cylinders were connected with two binding screws by means of sliding contacts. The part of the wire which does not lie on the metal cylinder is the only part that produces resistance between the binding screws; and, by winding and unwinding, we can increase or diminish the resistance continuously to a known extent, means being provided for measuring the angular rotation of the metal cylinder.

We shall not stop to consider the defects of this instrument, which is now never used for delicate work. Its place is taken by resistance boxes, containing coils of wire whose resistances are different multiples of the unit of resistance (in this country always the ohm). The reader will find a full account of the methods by which the standards are reproduced in the collected reports of the Committee on Electrical Standards. The usual material for the wire of resistance coils is German silver. Most of the copies of the ohm issued by the British Association were made of an alloy of two parts of silver to one of platinum. The great advantage of alloys is that the variation of resistance with temperature is small for them; in the PtAg alloy, for instance, it is less than a tenth of the value for an average pure metal. To secure insulation the wires are carefully coated with silk, and after winding the coil is immersed in melted paraffin. To get rid of electromagnetic and inductive effects, the wire on resistance-coils is doubled on itself before being wound, so that, when a current passes through the coil, there are always two equal and opposite currents at each point. The terminals are formed by stout pieces of copper rod, whose resistance is either included in the coil, or is so small that it may be neglected. The connections for small resistances are managed by means of mercury cups, with pieces of amalgamated copper at the bottom, on which the copper electrodes are made to press.

For ordinary purposes the coils are arranged in a box (fig. 23), the terminals being stout pieces of brass fixed on the ebonite lid;

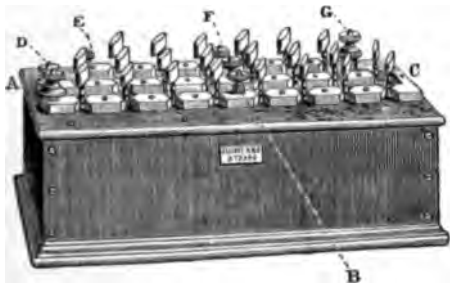


Fig. 23.

conical brass plugs inserted between these pieces serve to throw the coils in and out of circuit. The box represented in fig. 23 is specially arranged for use in Wheatstone's bridge. In E, F, G we have a series of coils, 1000, 100, 10, 10, 100, 1000; these are used for the arms of the bridge. In A, C, D there are sixteen coils, 1, 2, 2, 5, 10, 20, 20, 50, &c., which give us any resistance of a whole number of ohms from 1 up to 10,000. In actual use the resistance to be measured is inserted between A and G, D and E are connected by a stout piece of copper, the galvanometer is inserted between F and

A, and the battery between E and G. The resistances of the arms of the bridge are taken equal, and as near the resistance to be measured as possible. In this way the resistance of any conductor may be very quickly found to an ohm. If it is desired to go farther, we may proceed thus. Suppose that we have found that a resistance lies between 5 and 6, put in the arm FE 100, and in FG 10, let the resistance in DCA, when there is a balance, be 57, then the resistance of the conductor is $\frac{1}{57} \times 57$, or 5.7. Similarly we might go to a second place of decimals by putting 1000 in FE and 10 in FG. There is a limit, however, to this process, because the increase in the resistance of the arm decreases the "sensitivity" of the bridge. Another method is to balance as nearly as possible, and then interpolate by taking the deflection of the galvanometer. Suppose, for instance, in the above case, that, with 5 ohms in DCA, the deflection was 21 in one direction, and, with 6 ohms, 9 in the other direction, then, taking the deflection proportional to the deviation from balance (see formula for δ above), we have resistance = $5 + \frac{21}{30} \cdot 1 = 5.7$.

We might also construct small graduated resistances; Conductivity boxes, rheocord, &c. and this would enable us to use smaller arms in the bridge, and thus increase the "sensitivity" when used to measure small resistances. Owing to the multiplication of connections, there is a limit to the ordinary resistance box arrangement. The difficulty may be evaded to a certain extent by using conductivity boxes, according to Sir W. Thomson's suggestion, where the resistances are arranged abreast, so that a small alteration of the resistance is brought about by adding on a *very great* resistance to the multiple arc. The rheostat principle has been used by Poggendorff in his rheocord for producing small resistances. He stretches two platinum wires side by side; on these is strung a hollow box filled with mercury, whose longitudinal motion is read off on a scale. If this arrangement be thrown into any circuit by means of two binding screws connected with adjacent terminals of the wires, the parts of the two wires up to the bridge give a small resistance, which may be adjusted at pleasure.

In the quicksilver agometer of Müller (Wiedemann, i. § 160), the resistance is formed by a column of mercury of variable length. We may remark here that difficulties equally arise in constructing very large resistances. To get such within reasonable compass the wire must be exceedingly thin and the insulation very good. Messrs Warden and Muirhead have wound coils of fine wire, giving a resistance of 100,000, and have constructed in compact form resistance boxes up to 1,000,000, or a megohm, and beyond. They have also given practical form to a suggestion of Phillips to utilize the resistance of carbon, by drawing fine pencil lines on ebonite or glass; they mix plumbago with the pulp in the ordinary process of paper manufacture, and thus produce a species of carbon paper. A strip of this about 21 in. long and .5 in. broad gives a resistance of about 50,000. This seems a valuable invention; but we are not aware how far it has stood the test of practical use.

Selenium and tellurium have been proposed as material for high resistances, but owing to the variability of their resistance under the action of light, &c., they are unfit for the purpose.

The best method for comparing resistances with great Kirch-

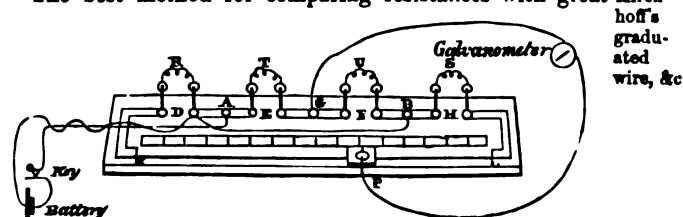


Fig. 24.

accuracy is the modification of Wheatstone's bridge introduced by Kirchhoff (fig. 24).

KL is a platinum-iridium wire, DK and HL are stout copper terminals to which it is soldered, DAE, EGF, FBH are stout copper pieces with binding screws and terminals for mercury cups, by means of which resistances R, T, U, S can be inserted at D, E, F, H. A, B, and G are binding screws for the battery wires and one end of the galvanometer wire. The other end of the galvanometer wire is screwed to a spring contact piece fixed to a sliding block at P; when the button of this block is depressed, contact is made with KL, at a spot which is definite to an eighth or tenth of a millimetre. Platinum iridium is chosen for KL, because it is hard and tough, not liable to be scratched or abraded by the contact piece, does not oxidize or amalgamate with mercury, and changes very slightly in resistance when the temperature alters. The wire must be calibrated to find what correction, if any, must be applied for variation of resistance per unit of length at different parts; for methods of doing this see Matthiessen and Hockin; *Brit. Assoc. Reports on Electrical Standards*, p. 117; or Foster, *Journ. of Society of Telegraphic Engineers*, 1874.

Foster's method.

Kirchhoff's arrangement may be used in the ordinary way after we have made special experiments to determine the resistance of the connections, &c. Professor Foster (*l.c.*) has given a very useful method, by which the difference of two resistances can be got independently of the resistances of the connections. Suppose we wish to find the difference between R and S, which we suppose so near each other that, with the arms T and U approximately equal, there will be a balance when P is somewhere on KL. Let the reading for the position of the block be x , taken from left to right. Interchange R and S, balance again, and let the new reading be x' (we suppose the difference between R and S so small that P is still on KL); then, if μ be the resistance of unit length of KL, $R - S = \mu(x' - x)$.

For, if α represent the resistance of the connections in DK, β the same for the other end of the wire, and if T and U include the resistance of the invariable connections, then we have

$$\frac{R + \alpha + \mu x}{S + \beta + \mu(l - x)} = \frac{T}{U},$$

where l = length of KL. Hence

$$\frac{R + \alpha + \mu x}{R + S + \alpha + \beta + \mu l} = \frac{T}{T + U}.$$

Similarly

$$\frac{S + \alpha + \mu x'}{R + S + \alpha + \beta + \mu l} = \frac{T}{T + U},$$

therefore

$$R - S = \mu(x' - x).$$

Methods of Matthiessen and Hockin and of Sir W. Thomson.

If we have to find the resistance of a thick cylindrical body, what is really wanted is the ratio of the current strength to the difference of potential between the two ends, when the current flows parallel to the axis at every point. The last condition is not generally fulfilled. It is obviously not so in the case where the cylinder is joined up with a thin wire. In cases where we wish to compare the specific resistance of two metals which we possess in cylindrical pieces, we get over the difficulty by observing the potential at a point at some distance from the end of the piece, where the flux is parallel to the axis at all points of the section.

Matthiessen and Hockin used the following method for this purpose (fig. 25). The two pieces XZ, YZ are soldered together and connected in circuit with two resistance coils A and C, and a graduated wire PR as before. S, S' are two sharp edges, at a measured distance apart, fixed in a piece of ebonite or hard dry wood, and connected with mercury cups. T, T' is a similar arrangement for YZ. The galvanometer is inserted between S and Q, and the position of Q is found for balance; then the terminal is shifted to S', and if necessary the resistances A and C altered, so as to keep their sum constant, until balance is again found. The same is done for T and T'. Then, XS denoting the resistance between X and S, and A, C_1 the values of A and C in the first case, and so on, we have

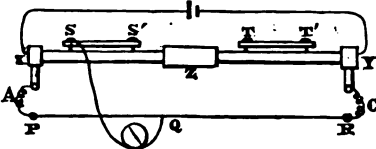


Fig. 25.

$$\frac{XS}{XY} = \frac{A_1 + PQ_1}{R}, \quad \frac{XS'}{XY} = \frac{A_2 + PQ_2}{R},$$

where

$$R = A_1 + C_1 + PR = A_2 + C_2 + PR$$

Hence

$$\frac{SS'}{XY} = \frac{A_2 - A_1 + Q_2 Q_1}{R}$$

Similarly

$$\frac{TT'}{XY} = \frac{A_2 - A_1 + Q_2 Q_1}{R}$$

Therefore

$$\frac{SS'}{TT'} = \frac{A_2 - A_1 + Q_2 Q_1}{A_2 - A_1 + Q_2 Q_1}.$$

This gives us the ratio of the resistances between SS' and TT'. The method does not depend for its success on the goodness of the contacts at SS', &c. Another ingenious arrangement for effecting a similar purpose is due to Thomson, and will be found described in Maxwell, vol. i. § 351.

In measuring very large resistances, such as the insulation resistance of a telegraph cable, it is convenient to use the quadrant electrometer. One end of the cable is connected with one electrode of a condenser, the other end of the cable is insulated, and the other electrode of the condenser put to earth. The condenser is charged, and the difference of potential between its electrodes measured by means of the electrometer. If E_1, E_2 be the value of the difference at the beginning and end of an interval of t seconds, and if S be the capacity of the condenser in electromagnetic measure, then the resistance of the cable is

$$\frac{t}{S(\log_e E_1 - \log_e E_2)}$$

in electromagnetic measure. If the condenser itself leaks, we must determine its resistance by insulating the electrodes and operating as before. Then, regarding the circuit in the first experiment as a multiple arc, composed of the insulation of cable and the dielectric of condenser, the true conductivity of the cable envelope is the difference of the conductivities obtained in the two cases. Several other methods might be used to compare metallic resistance but they are of small importance compared with those we have now been describing.

The reader who desires information concerning the application of Ohm's law to conductors other than linear will find the sources sufficiently indicated in Wiedemann's *Galvanismus*; some of them have been alluded to in the Historical Sketch.

Application of Ohm's Law to Electrolytes.

In our discussion of Ohm's law, we have hitherto had in view principally the metallic part of the voltaic circuit. We now turn our attention more particularly to the fluid parts. It is of no importance in the present connection whether the fluid forms part of the "battery" or "electromotor," or whether it is inserted outside the battery; the only difference in these two cases is, as we shall hereafter see, that in the former case energy is being absorbed by the current, and in the latter it is being evolved. In many respects the properties of the metallic and fluid parts of the circuit are alike: the electromagnetic action is the same for both; heat is also developed in the body of the conductor, whether metallic or fluid, according to the same law. But there is one peculiarity about a large class of fluids which has no analogue in purely metallic conduction, viz., that in them the passage of a steady current of electricity is invariably accompanied by chemical decomposition, definite in kind and quantity. To such fluid substances Faraday gave the name of electrolytes.

For example, suppose we fill a small beaker with a solution of zinc chloride ($ZnCl_2$), and suspend in the liquid two strips of platinum foil (called *electrodes*), at a moderate distance apart. Let a current enter at one of these strips, which we shall call the anode, and leave at the other, which we shall call the cathode. It will be

found that the solution continues to decompose so long as the current passes, zinc appearing at the cathode, and chlorine at the anode. The metallic zinc precipitates, and the chlorine combines with the platinum of the anode to form platonic chloride.

It is obviously essential in an electrolyte that it should be a compound in some sense or other. It is not, however, true that all compound bodies are electrolytes. Fluidity is also a necessary condition, whether attained by heating to the melting-point, or by dissolving in water or other solvent. Faraday established as a law, to which there appear to be few, if indeed any, exceptions,—that all substances which in the solid state are very bad conductors, but conduct on being heated to the melting-point, are electrolytes, i.e., are decomposed by the passage of the electric current. Faraday thought that periodide of mercury, fluoride of lead, and some other bodies were exceptions to this law; but later researches seem to have established that this is not so. (Cf. *Experimental Researches*, 414, 439, 1340, &c., and Wiedemann's *Galvanismus*, i. § 191, &c.) The conductivity of electrolytes in solution also increases rather quickly with increase of temperature, while the conductivity of metallic conductors, on the other hand, diminishes, but more slowly, as the temperature rises.

In considering the passage of the current through electrolytes, it is convenient to distinguish two cases. First, let there be a steady, or at least permanent current, and a continuous evolution of the products of electrolytic decomposition (these are called the "ions," anion and cation at the anode and cathode respectively). *The amount of ion that appears at an electrode in a second is equal to the strength of the current (supposed constant during a second) multiplied by a constant called the electrochemical equivalent of the ion.*

The electrochemical equivalent is proportional to the chemical equivalent, account being taken of the "valency" of the ion. (See art. ELECTROLYSIS.)

For instance, if C be the strength of the current in the illustrative case above, then the amount of zinc deposited at the cathode in time t will be zCt , and the amount of chlorine liberated at the anode cCt , where z and c the electrochemical equivalents of zinc and chlorine, and $z : c :: \frac{65}{2} : 35.5$, zinc being divalent. If a cell containing lead chloride ($PbCl_2$) were also inserted into the circuit, the same amount of chlorine would be liberated at the anode, and the amount of lead precipitated at the cathode would be pCt , where

$p : z : c :: \frac{207}{2} : \frac{65}{2} : 35.5$, i.e. $103.5 : 32.5 : 35.5$

As the electrochemical decomposition ("electrolysis") goes on, the surface of the electrodes is altered. In some cases the ion is merely deposited on the electrode, in other cases it combines more or less intimately therewith; but in general there is an alteration of the nature of the contact, and a consequent alteration of the electromotive force at the surface of the electrode. Experiment shows that this electromotive force, in a great many cases, tends to oppose the passage of the current. So that if we insert an electrolyte into any circuit, the current starts with a certain value, and falls more or less quickly, until it reaches a limit at which it remains steady. The opposing electromotive force of "polarization," as it is called, has then reached its maximum, and the deposition of the ions goes on without further alteration of the contact surfaces. It is obvious that this limit may be reached under a variety of different circumstances (*vide infra*, p. 86). There is also another phenomenon, the possibility of which we must not overlook, viz., an alteration of resistance, owing to the presence of the ions at the electrodes. This resistance, due to the ions, has been called the "transition resistance." The enfeebling of the current by the electromotive force of polarization might, as far as the observed result is concerned, be due entirely to an increase of resistance, or to a transition resistance, and such was the explanation given by the earlier physicists. It is easy, however, to show that there is an

actual electromotive force of polarization; for, if we disengage our electrolytic cell from the battery, and connect its electrodes with a galvanometer, a current is indicated, which passes through the cell in the opposite direction to the original current. This could not be due to any transition resistance, but must arise from an opposing electromotive force generated by the passage of the battery current. This point can be illustrated by a hydrodynamical analogy. If we attempt to force water through a narrow capillary tube, or through a wide vertical tube against gravity, there is an opposing force in both cases. But, when we remove the pressure, the water has a tendency to return in the latter case, but none in the former. The former case represents a transition resistance, the latter an electromotive force of polarization.¹

Without denying the existence of a transition resistance, we see that an electromotive force of polarization actually exists. In some cases, e.g., amalgamated zinc in zinc sulphate, it is very small; in other cases, e.g., platinum electrodes in dilute sulphuric acid, it may considerably exceed the electromotive force of a Daniell's element.

We have, up to this point, been treating the case where a permanent current finally flows through the electrolyte; but there are cases where the existence of such a current would violate the principle of the conservation of energy.

Suppose that a single Daniell's cell is the electromotor, then (see below, p. 90) if a current C is sent for a time t , an amount of energy dCt is absorbed in the cell, d being constant. Suppose, farther, that the excess of the intrinsic energy of the ions, in the state in which they are being delivered in the electrolytic cell, over that which they possess when in combination is w , then if a current C pass for a time t , an amount of energy wCt will be evolved. But if $w > d$, this cannot go on for any time however short, no matter how feeble the current may be, otherwise more energy would be evolved in the cell than is absorbed in the battery.

If we insert an electrolytic cell containing dilute sulphuric acid along with a galvanometer into a circuit in which there is a single cell of Daniell, we observe the galvanometer needle swing out vigorously, and then settle down to a small and gradually decreasing deflection. The current ultimately becomes zero;² but the time it takes to do so may be considerable, and varies with the nature of the electrodes. If we remove the battery after the current has stopped, and connect the polarized cell with the galvanometer, we observe an initial swing very nearly equal to the former but in the opposite direction, and a corresponding deflection, which after a time disappears entirely. Although, as a rule, a sensible time elapses before the polarization reaches its maximum, yet it is important to remark that it may rise to a very considerable fraction of the maximum in a very short time indeed. Edlund³ found that in a certain case the electromotive force of polarization reached 0.57 of a Daniell in about $\frac{1}{10}$ of a second. Bernstein has recently arrived at results of a similar kind. He found, for instance, that platinum plates, polarized to 1.85 of a Daniell, fell, when the resistance of the circuit was 7.46 Siemens units, to 1.57 in .00111 sec.⁴ This rapidity of the rise and fall of the polarization is of very great importance, and has, we think, been overlooked by some experimenters.

In cases where the polarization does not reach its maximum, no liberation of gas or other ion is observed, such as is seen with a permanent current, and it might of course be denied that chemical decomposition takes place at all. We shall, however, assume that Faraday's law holds for this case also, and assert that the current in the first instance actually passes through the liquid and produces chemical decomposition, according to the same law as a permanent current, and that this goes on until the accumu-

¹ Maxwell, *Electricity*, vol. I. § 266.

² For an exception to this statement see below, p. 87.

³ *Pogg. Ann.*, lxxxv., 1852.

⁴ *Pogg. Ann.*, clv., 1875.

lation of the ions has generated an opposing electromotive force, equal to that of the battery, when of course the current must stop. We cannot justify this position very easily by direct experiment; yet there are many facts to support it, and so long as it is tenable it seems to afford the most philosophical view of the matter.

Having explained the phenomena of polarization so far as is necessary for our immediate purpose, we now proceed to inquire how far experience justifies the application of Ohm's law to electrolytes, or, which is much the same thing, to examine how far the methods of different physicists for measuring electrolytic resistance have led to concordant results.

Mea-
sure-
ment of
electro-
lytic
resist-
ance.
Hors-
ford.

One of the earliest methods, in which polarization was eliminated, was that of Horsford.¹ He filled a rectangular trough with the electrolyte, and inserted in the trough two electrodes very nearly fitting the cross section. These electrodes could be set at different measured distances apart. They were coated on the further side with non-conducting substance, so that the current could flow between the opposed sides only. In this way he secured that the stream lines in the neighbourhood of the electrodes should depend as little as possible on the distance between them. This trough was inserted in the battery circuit along with a tangent galvanometer; then the distance between the plates was decreased, and a metallic resistance R inserted in the circuit, so as to bring the current to the same strength as before. The current being the same in both cases, it is assumed that the polarization in both is the same, in which case the resistance of a length of the electrolyte equal to the difference of the distances between the electrodes in the two cases is equal to R . Knowing the section of the trough, we might calculate from R the specific resistance of the electrolyte. If the values arrived at be the same when deduced from different lengths of the electrolyte, and for different strengths of current, it may be concluded that Ohm's law applies. The application of this method requires the passage of a permanent current, in consequence of which the ions appear at the electrodes, and the solution in the neighbourhood becomes altered; so that it is difficult to make certain that the polarization is exactly the same in the two cases, and that no resistance of transition is generated. Matters may be mended a little by passing the current for the same time in both cases; but this is scarcely a satisfactory remedy. Still valuable results were obtained with this method by Horsford and Wiedemann; the latter, in applying it to silver and copper solutions used electrodes of silver and copper respectively, whereby the polarization to be eliminated was very much reduced.

Beetz.

Taking advantage of the discovery of Matteucci and Du Bois Reymond,² that carefully amalgamated zinc electrodes in a neutral³ solution of zinc sulphate are not polarizable, Beetz⁴ determined, by means of Wheatstone's bridge, the resistance of various solutions of this electrolyte.

The liquid was inclosed in a cylindrical tube, 29.7 cm. long, with a mean section of 1.4051 sq. cm. Amalgamated zinc plates were applied to the ends of the tube, and fastened on by india-rubber collars. The ends were then inserted tightly into openings in the sides of two bottles which were filled with the solution (the same as that contained in the tube). The thick electrodes leading to the discs, and the backs of the zinc discs themselves, were lacquered, to insulate them from the liquid in the bottles. The whole apparatus was immersed in a trough of water, which could be heated to any desired temperature.

In the course of his experiments Beetz demonstrated the absence of polarization when amalgamated zinc electrodes are used, and eliminated the transition resistance by boiling the electrodes in zinc sulphate, and transferring them to the ends of the tube without exposure to the air.

Beetz farther proposed to find the specific conductivity of other electrolytes in terms of that of zinc sulphate, by experimenting on

closed circuits consisting *entirely* of the electrolyte to be examined. He tried damping experiments for this purpose, but the effects to be observed turned out too small for accurate observation.

Paalzow⁵ inclosed the electrolyte to be examined in a Paalzow siphon, the two ends of which dipped into vessels of porous clay also filled with the electrolyte. The clay vessels were immersed in beakers filled with zinc sulphate, at the bottoms of which were placed large amalgamated zinc discs, which formed the electrodes. The only polarization or transition resistance to be feared is that at the boundary of the two liquids, and this is very small. What little remained was eliminated, as in Horsford's method, by taking differences.

The resistance of the whole arrangement was measured by means of Wheatstone's bridge, and then the siphon was replaced by a shorter one filled with the same liquid. If R_1, R_2 be the resistances found in the two cases, $R_1 - R_2$ is obviously the resistance of a length of the electrolyte equal to the difference between the lengths of the siphons. If R_1', R_2' be similar values obtained when the electrolyte is replaced by mercury, then the specific resistance of the electrolyte is $\frac{R_1 - R_2}{R_1' - R_2'}$, that of mercury being taken as unity.

The most important of all the recent researches on the Kohl application of Ohm's law to electrolytes are those of F. Kohlrausch and Nippoldt. In order to avoid the effects of polarization, they used the alternating currents of an electromagnetic machine. These currents varied very nearly as the sine of the angle of rotation, and could be sent in rapid succession through the electrolyte. The whole quantity of electricity that passes in the first part of any alternation is exactly equal and opposite to that which passes in the second; hence equal quantities of the two ions (say H and O) will be separated at each electrode. If the H₂ and O combine to form water, it is obvious that, on the whole, there will be no resultant electromotive force of polarization either way; and if they coexist side by side without combining, there will still be no resultant electromotive force, provided the electrodes be exactly similar. There are two advantages in this method. There is no evolution of gas or other ion, and consequently no alteration of the solution and electrode, such as goes on with a constant current. We have, besides, another great advantage, which is denied⁶ us with constant currents,—viz., that by increasing the size of the electrode, we can diminish the effects of polarization.

The whole amount of electricity which passes in each induction current is the same, and consequently the whole amount of ion deposited on the electrode is the same; hence, if we increase the surface of the electrode, the density of the deposit is decreased in an inverse ratio. Now, the researches of Kohlrausch and Nippoldt have shown⁷ that, within certain limits, the electromotive force is proportional to the surface density of the deposit. Hence, by sufficiently increasing the surface of the electrodes, the polarization may be made as small as we please.

In the earlier experiments platinum electrodes, having a surface of 1.08 cm. were used, and it was found that each induction current of the magneto-electric machine deposited on each square millimetre of the positive electrode only $\frac{1}{13,500,000}$ c.cm. of oxygen.

It was therefore expected that the polarization would be insensible, and that the electrolyte would behave like a metallic resistance. The magneto-electric machine and the electrolyte were connected up with an electro-dynamometer, and it was found that the deflection of the suspended coil of the electro-dynamometer was scarcely sensible when the machine made 10 revolutions per second, although it was 15 scale divisions when the electrolyte was replaced by 70 Siemens units. On the other hand, when the velocity reached 77 revolutions per second, the deflection was much greater with the electrolyte than with 70 Siemens units. It was found, however, that when the surface of the electrodes was increased to 29 cm. a metallic resistance could be found, which gave the same deflection (within errors of observation) as the electrolyte for speeds varying from 4.8 to 76.9 revolutions per second.

⁵ Pogg. Ann., cxxxvi., 1869.

⁶ The advantage gained even with constant currents by increasing the size of the electrodes is, however, appreciable (see below, p. 88).

⁷ Pogg. Ann., 1873, and "Jubilbd.," 1874.

¹ Pogg. Ann., 1847.

² Monatsber. der Berl. Akad., 1859.

³ Patry, Pogg. Ann., cxxxvi., 1869. ⁴ Pogg. Ann., cxvii., 1862.

The above results seem to compel us to one or other of two conclusions,—either that Ohm's law does not apply to rapidly alternating currents, where the maximum of polarization is not reached, or else that the electromotive force of exceedingly small deposits of the ions must be very considerable. The fact that, under certain conditions, the electrolyte is apparently a better, and under others, apparently a worse conductor than a certain metal wire, seems at first sight rather to point to the former conclusion. On the other hand, the result with the 29 cm. electrodes, is a direct verification of Ohm's law. Kohlrausch, therefore, adopted the latter conclusion, and justified his doing so by special researches on the electromotive force of small gas deposits. He showed that, with the currents he used, the electromotive force is proportional to the surface density of the deposit, and estimated that the products of decomposition of $\frac{1}{10}$ mg. of water per square metre would generate an electromotive force equal to that of a Daniell's cell. It is of the greatest importance to remark that the polarization effects, from which this result is deduced, must have arisen and disappeared in some cases in much less than $\frac{1}{10}$ of a second. The anomalous behaviour of the electrolyte with small electrodes is explained by Kohlrausch by taking into account the self-induction of the circuit.

A little consideration will show that the electromotive force due to this cause always opposes the electromotive force of polarization, when the current strength is a simple harmonic function of the time. Let i denote the current strength, reckoned positive in a given direction, then, according to Kohlrausch's law, the electromotive force of polarization at time t is $-p \int i dt$, where p is the

electromotive force generated by the passage of a unit of electricity; its value depends on the electrolyte and on the electrode being, *ceteris paribus*, nearly inversely proportional to the surface of the latter. Let n be the number of revolutions of the machine per second, and let $2\tau = \frac{1}{n}$; then we may represent the electromotive force of the

machine at time t by $\frac{k}{\tau} \sin \frac{\pi}{\tau} t$, and the electromotive force

of self-induction by $-q \frac{di}{dt}$, where k and q are constants, the latter being the coefficient of self-induction of the circuit (see Electromagnetism, p. 76). If w be the whole resistance of the circuit, we may write

$$wi = \frac{k}{\tau} \sin \frac{\pi}{\tau} t - q \frac{di}{dt} - p \int i dt,$$

or

$$q \frac{d^2 i}{dt^2} + w \frac{di}{dt} + pi = \frac{k\pi}{\tau} \cos \frac{\pi}{\tau} t.$$

Neglecting disturbances that die away very soon after starting the machine, we get for the value of i ,

$$i = \frac{\frac{k}{\tau} \sin \frac{\pi}{\tau} t}{\sqrt{w^2 + \left(\frac{p\tau}{\pi} - q\frac{\tau}{\pi}\right)^2}},$$

where the origin of time has been thrown back by

$$\frac{\tau}{\pi} \tan^{-1} \frac{1}{w} \left(\frac{p\tau}{\pi} - q\frac{\tau}{\pi} \right).$$

The deflection α of the dynamometer is proportional to $\frac{1}{\tau} \int i^2 dt$, and may be written

$$\alpha = \frac{A n^3}{w^2 + \left(\frac{p}{2\pi n} - 2\pi n q \right)^2}.$$

Kohlrausch found that this formula completely accounted for all the peculiarities in the behaviour of the electrolyte (for the numerical verifications see the papers quoted). We see that the deflection is increased or diminished by the insertion of the electrolyte, according as n is greater or less than $\frac{1}{\pi} \sqrt{\frac{p}{8q}}$, and, if $n = \frac{1}{\pi} \sqrt{\frac{p}{8q}}$, the insertion of the electrolyte makes no difference. Again, if $n = \frac{1}{2\pi} \sqrt{\frac{p}{q}}$

the deflection will be the same as if there were no extra current and no polarization. So that, for any given electromagnetic machine, working at any given speed, a certain electrolytic arrangement can be found, which will exactly eliminate the effect of self-induction, and thereby render the efficiency of the machine a maximum. It is obvious too that, with a given electrolytic cell, the deflection reaches a maximum when

$$n = \frac{p}{2\pi \sqrt{pq - \frac{1}{2} w^2}};$$

this maximum was actually observed by Kohlrausch (*l.c.*).

Having due regard to these circumstances, Kohlrausch and Nippoldt found that Ohm's law was applicable to Ohm's their alternating currents, for electromotive forces varying from over $\frac{1}{2}$ to under $\frac{1}{4}$ of a Grove's cell. By using the constant current of an iron-copper thermo-electric pair, they found Ohm's law applicable to zinc sulphate with amalgamated zinc electrodes, when the electromotive force was reduced to $\frac{1}{100000}$ of a Grove's cell.

It is important to remark that the fact that the electrolyte behaves like a metallic conductor through a considerable range of velocities of the sine inductor, is not a conclusive proof that the last trace of polarization has been eliminated.

In fact, let x be the resistance of the electrolyte, W that of the rest of the circuit, and w the metallic resistance that gives the same electro-dynamometer deflection for n revolutions of the inductor per second, then the above formula gives

$$x - w = \frac{2}{2W + x + w} \left(pq - \frac{p^2}{8\pi^2 n^2} \right) = \frac{1}{W + w} \left(pq - \frac{p^2}{8\pi^2 n^2} \right),$$

since we suppose x very nearly $= w$. If now p be reduced to a very small value, it may happen, especially for tolerably high speeds,

that $\frac{p^2}{8\pi^2 n^2}$ is very small compared with pq , in which case $x - w$ will be independent of n through a considerable range of speed, and the electrolyte will be replaceable by a wire whose resistance is less than the real resistance of the electrolyte by a small constant quantity.

The earlier results of Kohlrausch and Nippoldt for sulphuric acid, in which they used 29 cm. electrodes, were affected with an error due to this cause, amounting to about 4 per cent. In the later experiments of Kohlrausch and Grottrian,¹ this error was finally eliminated by "platinizing" the platinum electrodes. Kohlrausch had found that, with "platinized" electrodes of only 1 sq. cm. surface, the polarization of the currents of his sine-inductor was insensible; he therefore concluded that, with 25 sq. cm. platinized electrodes, the residual polarization would be finally eliminated. To make quite certain, he instituted three tests, which were carried out on the method used in all the later experiments on this subject.²

The Wheatstone's bridge arrangement was adopted. Fig. 26 gives a scheme of the arrangement. The fluid and a rheostat occupy two arms of the bridge, the remaining two contain each 100 Siemens units; A is the fixed and B the suspended coil of the electro-dynamometer, and S the sine-inductor.

In this way, (1) the resistance of a receiver with 25 cm.

platinized electrodes was found, when filled first with H_2SO_4 of maximum conductivity, and secondly, with NaCl, the driving weight of the inductor being varied, so as to give speeds of 10 to 100 revolutions. The results, reduced

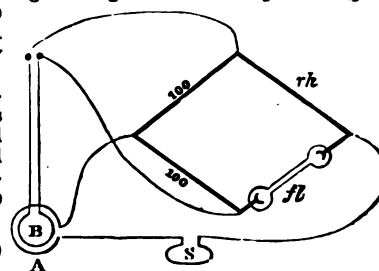


Fig. 26.

¹ Pogg. Ann., cliv., 1875.

² Kohlrausch and Grottrian, Pogg. Ann., cliv., 1875; Kohlrausch, *Ibid.*, cliv., 1876.

to a common temperature, were, for the H_2SO_4 , 141.73, 141.64, 141.52, 141.53, 141.55, and, for the NaCl , —, 366.27, 366.23, 366.25, 366.21 Siemens units, with the driving weights 5, 7.5, 10, 15, 20 kgr. respectively. (2) The resistance of a solution of zinc sulphate was found, first, in Beetz's manner with constant current and amalgamated zinc electrodes; secondly, using alternating currents and the same electrodes as before; thirdly, with alternating currents and the platinized electrodes; the three results reduced to a common temperature gave 537.49, 537.41, 537.20. The greatest divergence from the mean might have been caused by an error of $\frac{1}{30}$ degree in the temperature measurement. The agreement may therefore be pronounced complete. We think that it must be conceded that the experimental methods just described have solved in a satisfactory manner the problems involved in the determination of electrolytic resistance. We have dwelt on them so long partly because nearly all the information on the subject we possess has been obtained by their means, and partly because they present points of great theoretical interest.

Another method has been employed by Ewing and Macgregor.² The electrolyte was inclosed in a narrow tube with wide ends, in which were set platinum electrodes. This arrangement was inserted in a Wheatstone's bridge, and its resistance measured in the usual way. The precautions against polarization consisted in operating with currents of very short duration, sent through the bridge by means of a "rocker" worked by hand; the resistances in the arms of the bridge were also made large, in order to reduce the rate of polarization as much as possible; another essential feature of the method is the use of a "dead beat" galvanometer with a mirror of very small moment of inertia. The paper of Ewing and Macgregor has formed the subject of a somewhat bitter criticism by Beetz,³ to which Macgregor has replied.⁴

Battery
resistance.

Battery Resistance.—If the electromotive force and internal resistance of a battery in action were the same, whatever the external resistance, there would be no difficulty in finding the internal resistance by Ohm's method. We have simply to give two different values to the external resistance, and measure the current in the two cases. The electromotive force does not appear in the ratio of the two current measures; hence, knowing this ratio, we can find the internal resistance. Or we may use an electrometer, and measure the difference of potentials between the two poles of the battery, first, when the external resistance is infinite, secondly, when the external resistance is R . Then, if r be the internal resistance, the ratio of the first electrometer reading to the second is $\frac{R+r}{R}$, by Ohm's law; hence r can be found.

Difficulties
in
measuring.

Unfortunately, however, the electromotive force of a battery is *not* independent of the external resistance. In general, when a battery is circuited through a small resistance, its electromotive force is much smaller than when the external resistance is very great. This arises from the polarization set up by the passage through the battery of its own current, and possibly in some degree from other causes as well. There is also reason to believe that the internal resistance of the battery is a function of the current. This being so, it is clear that a theoretically satisfactory determination of battery resistance cannot be arrived at by such methods as we have described. Since, however, the increase of the electromotive force is very slow after the external resistance has reached a certain value, and since the alteration of the internal resistance takes some time, we can get in many cases measurements sufficiently accurate for practical purposes. A variety of methods have been devised with this object, and applied mostly to the so-called constant batteries. It must be remembered, however, that there is something indefinite in the term in-

ternal resistance, unless the circumstances be given under which it is found. In the method of Von Waltenhofen, the battery is "compensated" by another battery so arranged that no current passes through it; and then this arrangement is slightly altered, so that a very small current passes through the battery. This amounts to finding the internal resistance for very small currents. The method of Beetz also involves the principle of compensation; two batteries are used, but the one whose resistance is to be found is compensator and not compensated. The circuit of the compensator is joined for an instant, and then the compensated battery is thrown in. The assumption in the method is that the electromotive force is the same in the first instant whether the battery is closed through a resistance R or a resistance R' . The results seem to justify the assumption, and to establish the practical value of the method; but there are clearly limits to its application which it would not be very easy to define. Beetz himself shows that the electromotive force of a battery is greater when it is compensated than when it is compensating. A similar objection may be urged against the method of Siemens, which again gives good results when properly used. We refer the reader interested in this matter to the sources of information already quoted (see Historical Sketch), and content ourselves with an account of Mance's method, which, although subject to the same objection as all the others, is very convenient for rough purposes, and is much employed in this country.

Let A, B, C, D be four resistances arranged in circuit, B being the battery whose resistance is required. Insert a galvanometer between $\hat{A}\hat{B}$ and $\hat{C}\hat{D}$, and a circuit which can be closed and opened by means

of a key between $\hat{A}\hat{D}$ and $\hat{B}\hat{C}$. We thus have an ordinary Wheatstone's bridge, with a key in place of a battery, and a battery in place of the ordinary resistance to be measured. Owing to the presence of the battery, there will be a current through the galvanometer, which will deflect the needle; this deflection is compensated by means of a magnet, and the needle brought back to zero. Then the resistances A, C, D are arranged so that the galvanometer is not affected when the key circuit is opened or closed; when this is so the key and galvanometer circuits are conjugate, and we have $AC - BD = 0$, from which we can find B , since A, C, D are known. In practice, however, it is impossible in the great majority of cases to fulfil the direction printed in italics. Suppose for a moment we had arranged the resistances so that $AC - BD$ is very nearly but not quite zero, and suppose we close the key circuit, which had been formerly open, then, since this is not conjugate to the battery circuit, the external resistance opposing the battery is reduced; hence its electromotive force falls, the current through the galvanometer is altered, and the deflection of the needle alters. At the same time there is a current owing to the fact that $AC - BD$ is not exactly zero. These two effects may either conspire or oppose each other. No data, so far as we know, have been obtained which would enable us to tell how quickly this fall in the electromotive force of any given battery comes on. In practice we see a sudden jerk of the galvanometer, and then a slow swing. The former is due to the deviation of the bridge from balance, and the latter to the alteration of the electromotive force. It is easy to decide which is which, for the direction of the former can be changed by making $AC - BD$ positive or negative, while the direction of the latter is not affected in this way. This disturbing effect is very great with one-fluid batteries; it would, for instance, be a hopeless undertaking to measure in this way the resistance of a cell of Smee while sending a large current. The effect is not so great with a Daniell's cell, and can be reduced *ad libitum*, by introducing metallic resistance into the battery circuit. The effect having been thus reduced within reasonable limits, we operate thus:—Arrange the bridge until the deflection owing to deviation from balance is opposite to that due to the change in the electromotive force; then, by gradual adjustment, work down the initial jerk to nothing, so that the needle appears to start off on its slow swing without any perceptible struggle. When this state of matters is reached, there is a balance, and $B = \frac{AC}{D}$. Then subtracting from B

the resistance put into the battery circuit, we get the resistance of the battery. Of course this does not solve the problem of finding the resistance of any battery sending any current; but we believe that as much can be done in this way as in any other. Various modifications of Mance's method have lately been proposed, but their practical advantages over the original method have scarcely as yet been established.

¹ No observation made for NaCl in the first case.

² *Trans. R.S.E.*, 1873. ³ *Pogg. Ann.*, cliv. ⁴ *Proc. R.S.E.*, 1875.

On Resistance in General.

We have drawn no distinction between statical and dynamical electricity in our application of Ohm's law, and no such essential distinction has ever been proved to exist. In proportion as a body is a good conductor for galvanic electricity, it is a bad insulator for statical electricity. In general, however, bodies which are good enough insulators to retain a charge of statical electricity are so bad conductors that it is with difficulty that we can compare their conductivities by means of the voltaic current. On the other hand, it is difficult by means of statical electricity to compare satisfactorily the conductivities of very good conductors. Determinations of the last-mentioned kind have, however, been made by Riess (*vide infra*,—Heating Effects), and the results agree with those obtained by other methods. The insulating power of a substance depends practically to a great extent on the nature of its surface. The dissipation of statical electricity by insulating supports is due, in most cases, almost entirely to the conducting power of a thin surface layer of moisture condensed from the atmosphere, or of some product of chemical decomposition caused by exposure to the air, or of dust or other foreign matter accidentally deposited. As far as high specific resistance is concerned, paraffin, shellac, ebonite, and glass at ordinary temperatures would all be about equally good insulators; but in practice they stand in the order in which we have named them. Paraffin and shellac surpass the other two in their power of preserving for a long time a clean dry surface; ebonite is very good for a time, but ultimately its surface becomes covered with a layer of sulphuric acid, arising from the decomposition of the material; glass, again, is very hygroscopic, although white flint glass, when kept dry by artificial means, is said to be one of the best insulators known.

ala. Highest in the order of conductivity stand the metals and their alloys. In this class of bodies the passage of the electric current is unattended by chemical decomposition, and the conductivity decreases as the temperature increases. Along with the metals may be ranked a few other bodies, which have anomalous conductivity, but are not decomposed; such as graphite, red phosphorus, chloride and oxide of lead under the melting-point, various sulphides and selenides, tellurium, and selenium. In the great majority of the bodies included in this supplementary class the conductivity increases with the temperature; the last two present several anomalies, to which we shall refer farther on.

ctro- A second class of bodies is formed by those which are decomposed by the electric current. The specific conductivity of these is much lower than that of the metals, and it increases when the temperature is raised. To this class belong, when in solution or in the melted state, most simple binary compounds composed of equal equivalents of two elements, and compounds derived from these by "double decomposition" (see, however, art. ELECTROLYSIS); also some sulphides which have an anomalous conductivity, and glass and some bodies like it, which in the melted state, and in the soft state preceding fusion conduct as electrolytes.

non- Non-conductors, on the other hand, are:—All gases and vapours, whether at ordinary pressures or in what is called a vacuum, diamond, sulphur, amorphous phosphorus, amorphous selenium, fluid chlorine, bromine, solid and melted iodine, bichloride and biniodide of tin, sulphuric anhydride, solid silicic acid, oxide of iron, oxide of tin; most compounds that are not binary, that is, do not consist of an equal number of equivalents of two components, e.g., many organic compounds—etheric oils, resins, wood fibre, caoutchouc; also "binary compounds" in the solid state. To these may be added pure water, pure hydro-

chloric acid, &c., which are very bad conductors, if not absolute non-conductors.

Before leaving this part of our subject, it will be interesting to throw together a few of the general principles that have been arrived at, and to give a few numerical results, which will convey to the reader an idea of the position of the different classes of bodies in the scale of conducting power. For farther details we refer to Wiedemann's *Galvanismus*.

Metals.—(1.) It was remarked by Forbes that the order of conductivity is the same for electricity as for heat. The measurements of Wiedemann and Franz have established that the ratio of the conductivities for heat and for electricity is very nearly constant, temperature not only for pure metals, but also for alloys. (See Wiedemann's *Galvanismus*, bd. i. § 194.)

(2.) The conductivity of the pure metals decreases as the temperature rises from 0° to 100° C., the rate of decrease becoming smaller towards the upper limit. Matthiessen expresses the conductivity by the formula $k = k_0(1 - \alpha\theta + \beta\theta^2)$, where k_0 denotes the conductivity at 0° C., θ the temperature, and α and β constants. He found that α and β had nearly the same value for all pure metals in the solid state, with the exception of thallium and iron, and gives as the mean values for pure metals $\alpha = 0.00376470$, $\beta = 0.0000083402$. The values for iron are $\alpha = 0.0051182$, $\beta = 0.000012915$; for mercury, $\alpha = 0.007443$, $\beta = 0.000008263$. Although there can be no doubt about the general agreement in the formulæ for the different pure metals, yet the actual formulæ arrived at is purely empirical and must be used only between 0° and 100° C. If we carried its application beyond, it would give a minimum conductivity for pure metals about 300° C. The direct experiments of Müller and Siemens give no indication of such a minimum. The latter represents the results of his experiments (extending in some cases as far as 1000° C.) by means of the formula $r = \alpha\sqrt{T} + \beta T - \gamma$, where r is the specific resistance, T the absolute temperature, α , β , γ constants. Relying on a formula of this kind for platinum, Siemens has constructed a pyrometer for determining the temperature of furnaces by means of resistance measurements.

(3.) As we have seen, the specific resistance of pure metals goes on increasing continuously as the temperature rises. At the melting-point there is a sudden rise in the resistance, and after that the resistance goes on increasing with a smaller temperature coefficient than before. This is in accordance with the fact, that both the specific conductivity and temperature coefficient of mercury are smaller than those of the other metals in the solid state. Bismuth and antimony are exceptions to this rule, in that there is a sudden decrease of resistance at the melting-point. According to the results of L. de la Rive, the resistance of metals in general is about doubled in passing the melting-point. We should therefore expect the specific conductivity of frozen mercury to be about 3.31, that of silver being 100.

Alloys.—(1.) Matthiessen found that the metals could be divided into two classes, according to the conducting properties of their alloys:

a. Lead, Tin, Cadmium, and Zinc.

b. Most of the other metals—Bismuth, Antimony, Platinum, Palladium, Iron, Aluminium, Sodium, Gold, Copper, Silver.

Let v, v' be the volumes, s, s' the specific gravities, k, k' the conductivities of the two components of any alloy; and let $\bar{s} = \frac{vs + v's'}{v + v'}$, and $\bar{k} = \frac{vk + v'k'}{v + v'}$, be called the mean specific gravity, and mean

conductivity of the alloy. Then alloys of any one metal of class a, with any other of the same class, have very nearly the mean specific gravity and conductivity calculated by the above formulæ.

Alloys of a metal a with a metal b have specific gravity and conductivity always less than the mean. If a metal a is alloyed with a considerable percentage of b, the conductivity is not much altered, but if a metal b be alloyed with even a very small quantity of a, the conductivity is greatly reduced.

Alloys of the metals b among themselves have in general a conductivity much inferior to that of either component. The conductivity remains constant through a considerable range of percentage, but rises very quickly as the percentage of either metal approaches 100. This property is very marked in an alloy of gold and silver. Matthiessen recommended an alloy of two parts by weight of gold to one of silver for the reproduction of the standard of resistance. The resistance of such an alloy would be very slightly affected by small variations in its composition.

Mercury, and melted metals generally, are not subject to the foregoing laws. A very small percentage of another even worse conducting metal raises the conductivity of mercury, but the

addition of larger quantities of the foreign metal lowers the conductivity.

(2.) The formulæ for the temperature variation for alloys of the metals α among themselves agree very closely with the mean formulæ calculated from the volume percentages.

If P denotes the fraction of itself by which the conductivity at 0° exceeds that at 100° for an average pure metal ($P=0.29307$), and P the same fraction, observed in the case of any alloy for which the observed and mean or calculated conductivities at 0° and 100° are k_0 , k_{100} , and \bar{k}_0 , \bar{k}_{100} —then, according to Matthiessen, the following relation holds for alloys of metals α among themselves, and metals β among themselves:—

$$P : \bar{P} :: k_{100} : \bar{k}_{100},$$

or, which is the same thing, R_0 , &c., denoting resistances,

$$R_{100} - R_0 = \bar{R}_{100} - \bar{R}_0.$$

For alloys of α with β , the observed value of P is in general greater than that calculated by this formulæ.

Effect of
physical
condi-
tion.

Other Physical Conditions affecting the Resistance of Solid Bodies.

—Besides temperature, a variety of other circumstances affect the specific resistance of metals. As a general rule, metals are worse conductors in the hard than in the soft state. Tempering steel increases its resistance considerably, but subsequent heating and gradual cooling reduces the resistance again. The resistance of a wire stretched by a weight is increased more than can be accounted for by the mere decrease of the section.¹ Winding on a bobbin has the same effect. The finer a metal is drawn into wire, the greater is its specific resistance in the case of iron, the smaller in the case of copper. Magnetization has also in certain cases been found to affect the resistance. These effects were studied by Sir William Thomson; the results of his researches are given in his Bakerian Lecture, *Phil. Trans.*, 1856. The experiments are very instructive, and many of them well worth repeating now that we have more delicate apparatus. The most curious case of alteration of resistance is that of tellurium and selenium. We have already mentioned that selenium in the amorphous state is a non-conductor. After continued heating it passes into the crystalline state and conducts. Sale found² that the conductivity of this crystalline form of selenium is greatly affected by light, and that, too, differently by light of different colours. Prof. W. G. Adams³ has lately made a series of experiments on the subject, and concludes that there is an action of light, which varies as the square root of the illuminating

power, and is distinct from any heating effect. He found the resistance of selenium in one case diminished by a fifth when it was exposed to the light of a certain paraffin lamp; the change in tellurium under similar circumstances was $\frac{1}{15}$ th. He found that the passage of a strong current through selenium sets up a kind of polarization, which opposes a current in the same direction as that which produced it, and aids a current in the opposite direction. This led him to suspect that the action of light might of itself start a current in the selenium, and he found that under certain circumstances this is the case.

Fluids.—The verification by the experiments of Kohlrausch and Nippoldt of Ohm's law for electrolytes, through a wide range of electromotive force, has greatly increased the interest of all data relating to the resistance of this class of conductors. We have no difficulty in working with electrolytes whose composition and physical state is perfectly definite, a thing next to impossible in the case of solids. Hence the resistance of an electrolyte has, far beyond the resistance of a solid metal, a value as datum for physical speculations concerning the ultimate properties of matter, which underlie Ohm's law. We refer the reader to Wiedemann's *Galvanismus* for an account of the earlier results in this department of Pouillet, Hankel, Becquerel, Horsford, Wiedemann, Becker, Lenz, and Saweljew. We recommend to his notice particularly the careful experiments of Beetz on zinc sulphate (his temperature determinations are the most extensive of the kind), also the researches of Paalzow, who examined the conductivity of various mixtures of two solutions, the conductivities of which had been separately determined. He finds that if R and R' be the resistances of the components, the resistance of the

mixture is not $\frac{RR'}{R+R'}$; so that the current is not divided between the liquids as if they were metals in multiple arc; nor is it the mean of R and R' , but it lies nearer the smaller of the two. A similar result was arrived at by Ewing and Macgregor.⁴

Kohlrausch and Grotrian⁵ have made the most recent as well as the most extensive investigations; and we shall best describe the present state of scientific knowledge on this subject by giving an analysis of their results and conclusions. Their experiments deal with the chlorides of the metals of the alkalies and alkaline earths. Kohlrausch has also examined a number of the commoner acids. For convenience we have transcribed the diagram given by Kohlrausch, which embodies certain of the results obtained by himself and Grotrian. Fig 1 of the diagram gives the conductivities (k_{18}) at

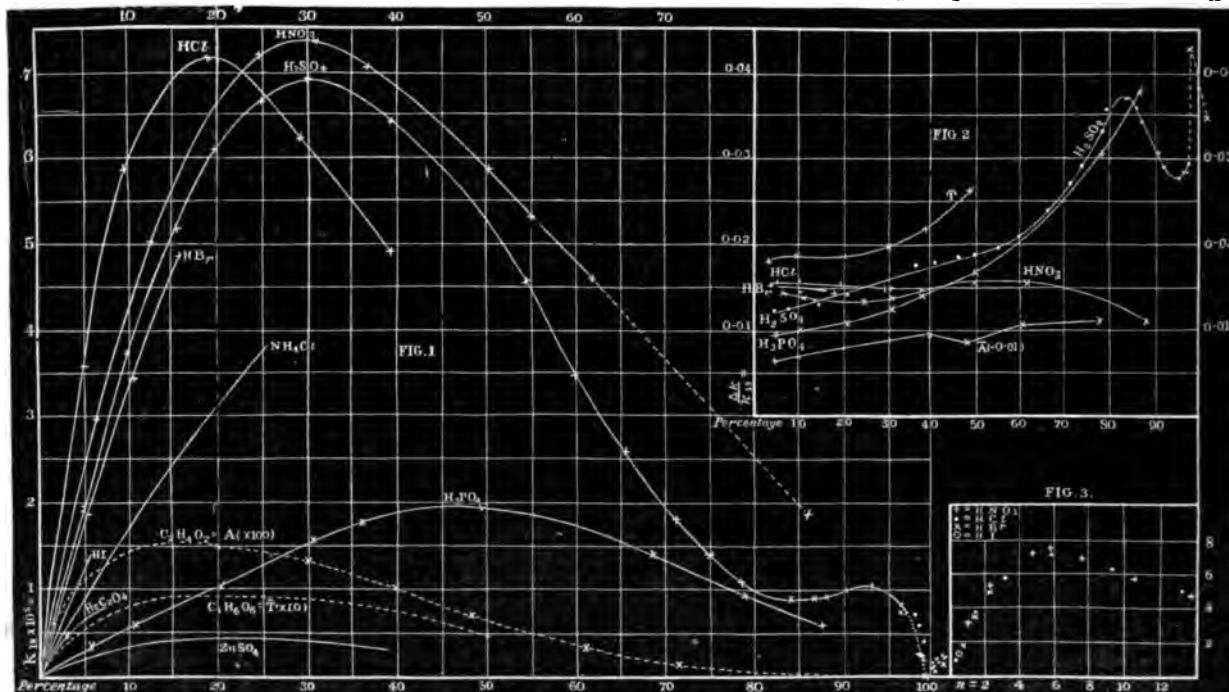


Diagram illustrating Electrical Conductivity.

18°C. ; the ordinates represent $k_{18} \times 10^5$, except for acetic and tartaric acid, where they represent $k_{18} \times 10^6$ and $k_{18} \times 10^6$ respectively, the abscissæ represent percentages by weight in the solution of HCl , H_2SO_4 , NH_4Cl , &c. In fig. 2 the values of the temperature

¹ For recent experiments on this subject see *Proc. R. S.*, Dec. 1876, and June 1877. Authorities for some of the other facts stated will be found in Wiedemann, i. § 207.

² *Proc. R. S.*, 1873.

³ *Proc. R. S.*, vol. xxiii. xxiv. xxv.

coefficient ($\frac{\Delta k}{k_{18}}$) for 18°C. are given by the ordinates, the abscissæ being percentages as before. For convenience of drawing the coefficient of acetic acid is decreased by 0.01.

The curves which appear in the diagram include all the distinct varieties; and it will be seen that in all cases the conductivity varies

⁴ *Trans. R. S.E.*, 1873.

⁵ *Pogg. Ann.*, cliv., 1875, and cliv., 1876.

⁶ Mercury is the standard.

continuously with the concentration, an approach to zero for infinitely weak solutions being indicated in all cases. The chlorides may be divided into two classes. (1.) CaCl_2 and MgCl_2 reach maximum conductivities 1968×10^{-8} , 1810×10^{-8} , at 18°C . for percentages 24 and 19.8 respectively, in each case short of saturation. LiCl probably does the same, and NaCl appears to reach a maximum between 23.9 p.c. and its saturation percentage 26.5. (2.) KCl , NH_4Cl , SrCl_2 , and BaCl_2 increase in conducting power up to the point of saturation.

Taking the best conducting solutions, the order of conductivities is NH_4Cl , KCl , NaCl , LiCl , CaCl_2 , SrCl_2 , BaCl_2 , MgCl_2 , the alkaline chlorides heading the list. A 25 p.c. solution of NH_4Cl is in fact half as good a conductor as the best acid solution known.

It was found that, if the conductivity for small percentages be represented by $k = \kappa p - \kappa' p^2$, so that κ may be called the *specific conductivity in watery solutions*, then κ varies inversely as the specific gravity, that is, directly as the "specific volume."

The temperature coefficients for the chlorides are very nearly independent of the temperature. There is a slight increase for higher temperatures, which is most marked in the case of highly concentrated and viscous solutions of CaCl_2 , MgCl_2 .

For weak solutions the coefficients are all very nearly equal; at 18°C . the extreme value for 5 p.c. solutions lies between $\frac{1}{4}$ (for LiCl) and $\frac{1}{5}$ (for NH_4Cl). There is a tendency, as seen by the curves, to a value $\frac{1}{4}$, or .022 for very weak solutions. It will be noticed (see table below) that this coefficient is much larger than .0039, which is about the corresponding number for a pure metal.

When the percentage is increased from five upwards, the temperature coefficient for 18°C . decreases at first for all the chlorides; it reaches a minimum for NaCl , CaCl_2 , MgCl_2 , which belong to class (1); but there is no minimum for KCl , NH_4Cl , BaCl_2 , which belong to class (2), and have no maximum conductivity.

The acids investigated were nitric, hydrochloric, sulphuric, phosphoric, oxalic, tartaric, and acetic. In every case, except that of oxalic acid, a maximum conductivity was obtained. The order in which we have named the acids is that of the conductivity of the best conducting solutions at 18°C .; for the first three we have respectively $k_1 \cdot 10^8 = 7330$, 7174, 6914, the corresponding percentages being 29.7, 18.3, 30.4, so that the maxima are very nearly equal, and the maximum percentages not far apart. The curve for sulphuric acid is exceedingly remarkable. Between 0 and 100 p.c. of H_2SO_4 , it shows two maxima. The first minimum occurs at the percentage corresponding to the hydrate $\text{H}_2\text{SO}_4 + \text{H}_2\text{O}$. The conductivity corresponding to H_2SO_4 is also a minimum; for when SO_3 is added, causing supersaturation, the conductivity again increases, there must therefore be at least one more maximum, since melted SO_3 is a non-conductor. There is no peculiarity in the curve corresponding to the hydrate $2\text{H}_2\text{O} + \text{H}_2\text{SO}_4$, which is distinguished from $\text{H}_2\text{O} + \text{H}_2\text{SO}_4$ in not being crystallizable. A striking similarity in the case of sulphuric and acetic acid is remarked between the curves of resistance and of solidification temperature; wherever the latter is high, the former is so also; there is a maximum in both cases for $\text{H}_2\text{O} + \text{H}_2\text{SO}_4$ and for H_2SO_4 , and a minimum in both cases near 92.5 p.c.; the other minima do not agree so well.

A remarkable relation is given, which appears to connect the resistance of the monobasic acids HCl , HBr , HI , and HNO_3 . If any percentage be multiplied by the specific gravity of the solution, and divided by the molecular weight of the acid, the result is the number of molecules (n) in unit of volume of the solution. On forming a table of resistances with n for argument, it was found that for solutions with the same n , whether of HCl , HBr , HI , or HNO_3 , the conductivity is the same. This appears very clearly from the dotted curve in fig. 3 of the diagram, calculated from the different acids, the regularity of the curve, and in parts the coincidences, are very marked. This result may be stated thus:—*In solutions containing an equal number of molecules, whether of HNO_3 , HCl , HBr , or HI , the components of electrolysis under equal electromotive forces pass in opposite directions with equal relative velocities.*¹

The temperature coefficients for the four monobasic acids are nearly equal, and nearly independent of the concentration. The same increase of temperature coefficient with increase of concentration as was noticed in the case of viscous chloride solutions appears also in the viscous acid solutions of phosphoric, tartaric, and sulphuric acid. It is also found that where the conductivity is a minimum, the temperature coefficient is correspondingly great; so that, with increasing temperature the maxima and minima tend to get smoothed out. It appears also that the proximity of the maxima for H_2SO_4 , HNO_3 , HCl , becomes more marked as the temperature rises.

The existence of the maxima in most cases, and of the minima in the sulphuric acid curve, led Kohlrausch to suggest the principle that no stable chemical compound in a pure state is a conductor, and that mixture of at least two such compounds is necessary for conduction. He mentions many instances of this principle, e.g., water, sulphurous acid, carbonic acid, acetic acid, melted boric

acid, chromic acid, anhydrous SO_3 , &c. In a recent paper² he gives some very interesting results concerning the conductivity of pure water and other bad conductors. The lowest conductivity he got for water was 71 ($\text{Hg} = 10^{13}$). This was after careful purification and repeated distillation in glass, and finally in platinum vessels. After standing under a glass bell jar for 4.3, 20, 78, and 1080 hours, the water rose in conductivity from 78 to 133, 350, 850, and 3000 respectively. He calculates that, if pure water were a non-conductor, the presence of 0.1 mgr. per litre of HCl would be sufficient to account for the observed conductivity. He also found conductivities for SnCl_4 (< 200), alcohol (commercial distilled) 30, acetic acid (glacial melted) 4, ether (< .8. Among recent researches of interest may be mentioned Braun's attempt³ to measure the conductivity of melted salts, and Grotrian's⁴ on the relation between the viscosity and the electric conductivity of electrolytes. For the speculations of Kohlrausch, Hankel, Beetz, Wiedemann, and Quincke on the ultimate nature of electrolytic resistance, see the papers of the first-mentioned, or Wiedemann's *Galvanismus*, Bd. i. § 434 sqq.

Gases.—We are not aware that any experiments have hitherto established that any gas or vapour at ordinary temperature and pressure is a conductor. Boltzmann⁵ has arrived at the negative result that air at ordinary temperature and pressure must have a specific resistance at least 10^{18} times that of copper. Sir William Thomson has, we believe, arrived at a similar result for steam; and recent experiments by Prof. Maxwell⁶ on air, steam, mercury, and sodium vapour (at high temperatures) have led him to a similar negative conclusion. It was found, however, that the heated air from a Bunsen's burner conducts remarkably well.⁷ The so-called unipolar conductivity of flames presents many anomalies, which have been examined by various experimenters. For the literature see F. Braun, *Pogg. Ann.*, 1875.

It would appear, therefore, that the loss of electricity from insulated conductors at moderate potentials, observed by Coulomb and Riess, cannot be due to conduction or convection by the air, but must arise almost wholly from the insulating supports. Warburg, who has experimented much on this subject, appears to be of the same opinion (*vide* Boltzmann, *l.c.* p. 415). Varley has lately investigated the passage of the current of a large number of Daniell's cells through a Geissler's (hydrogen?) tube. He found that it required 323 cells to start the current, but that once it was started it could be maintained by 308 cells; the current which flowed was proportional to the excess of the number of cells over 304. Thus, for $317 - 304 + 13$ the current was proportional to $25\frac{1}{2}$, for $330 - 304 + 26$ it was proportional to 51. Accordingly, if E_0 be a constant, and R another constant (the resistance of the gas?) we get for the electromotive force E , required to send a current I , $E = E_0 + RI$. E_0 is analogous to the electromotive force of polarization. For further details about the resistance of dielectrics we refer the reader to Maxwell's *Electricity and Magnetism*, vol. i. § 366 sqq.

The following table will give an idea of the conducting power of General different bodies; r denotes the specific resistance in C.G.S. units (to table reduce to ohms divide by 10^9); α is the percentage of itself that r increases in the case of metals and decreases in the case of electrolytes per deg. C.; t is the temperature at which r is given.

	t	r	α
Silver (annealed).....	20°	1521	.37
Copper (annealed).....	20	1615	.38
„ (hard drawn).....	20	1652	—
Platinum (annealed).....	20	9158	—
Iron (annealed).....	20	9827	—
Lead (pressed).....	20	19850	.38
Mercury (liquid).....	20	21170	.04
German silver.....	20	96190	.07
H_2SO_4 (max. soln.).....	18	1.39×10^9	1.5
NH_4Cl (sat.).....	18	2.55×10^9	1.5
ZnSO_4 (max. soln.).....	10	26.60×10^9	2.3
H_2SO_4 (pure).....	18	120.20×10^9	4.2
H_2O (pure).....	18	135×10^{13}	—
Glass.....	200	227×10^{14}	—
„.....	400	735×10^{11}	—
Gutta percha.....	24	353×10^{21}	—
„.....	0	7×10^{24}	—

² The residual conductivity he would attribute to residual impurities, or, as in the case of H_2SO_4 and melted salts, to dissociation, whereby the solution becomes in reality a mixture of different compounds. — *Pogg. Ann.*, clviii. 1876. ³ *Pogg. Ann.*, cliv. 1875.

⁴ *Pogg. Ann.*, clviii. 1876; clx. 1877.

⁵ *Pogg. Ann.*, clv. 1875.

⁶ Unpublished results.

⁷ Herwig (*Pogg. Ann.*, 1874) has recently concluded from some experiments that Hg vapour does conduct in a certain anomalous way. His experiments were complicated by the conductivity of the glass tubes containing the heated vapour; steps were taken, however, to eliminate this. Considerable doubt hangs over the whole subject.

¹ A similar law might be stated for the chlorides, but it holds only for very weak solutions.

On the Passage of Electricity through Insulators.

Disruptive discharge.

Hitherto we have divided bodies into *conductors*, through which electricity passes under the influence of any electromotive force, however small, and *non-conductors* or *insulators*, through which electricity will not pass, no matter how great the urging force. In practice, however, when the value of the electromotive force reaches a certain limit, electricity *does* pass through a non-conductor. A discharge of electricity taking place suddenly in this way through a non-conductor is called a "*disruptive discharge*." The power of a non-conductor to resist up to a certain limit the passage of electricity through it has been called its *dielectric strength*. The dielectric strength of any medium is greater the greater the electromotive force it will stand, when placed say between two parallel metal plates arranged in a given way, before it is broken through by the disruptive discharge. We shall by and by attach a definite quantitative signification to the term, but the general notion will be sufficient for the present.

Although it may be found when both phenomena have been more fully analysed, that conductive and disruptive discharge are really two different aspects of one and the same phenomenon, yet for the experimenter they are two distinct things, which must not be confounded.

This would be the place to set forth the quantitative relations which regulate the electromotive forces required to produce disruptive discharge, the quantity of electricity that passes under given circumstances, and the dielectric strength of different media; in fact, to lay down for disruptive discharge a law corresponding to the law of Ohm for metallic and electrolytic conduction. The present state of electrical science, however, does not permit us to do this in a satisfactory manner. Experiment has not as yet led to a single dominant principle, like Ohm's law, which will account for all the phenomena of disruptive discharge. The best theory of the subject is Faraday's, which will be gone into under "*disruptive discharge in gases*." Observation and experiment, on the other hand, have been occupied for the most part with the various transformations of energy which accompany the disruptive discharge. We prefer, therefore, to discuss the whole matter under the single head "*disruptive discharge*."

TRANSFORMATIONS OF ENERGY ACCOMPANYING
THE ELECTRIC CURRENT.

Under this head we propose to discuss (to use a word of Rankine's) the energetics of electricity. It may be objected that this heading might have been put over a good deal of what has gone before, and we shall, for convenience, treat certain matters under it which, in a strictly logical division, would have found a place elsewhere. If we had formed a definite conception of what we call electricity—had, for instance, assumed that it is a material fluid, having inertia like other fluids, then no doubt the energetics of the subject could have been much extended. As it is, we think that advantage is to be gained by associating in our minds the experimental laws which we are now to arrange under the above heading.

We shall consider (1) the heat developed in metallic and electrolytic conduction, and at the junctions in heterogeneous circuits; (2) the mechanical, sound, heat, and particularly light effects accompanying disruptive discharge; (3) the energy of magnetized iron and steel, and of electric currents in the neighbourhood of the electric current (electromagnetism); (4) the energy of the electrotonic state, or electrokinetic energy (magneto-electric induction). In this list ought to be included the potential energy of chemical separation, which would come under the head of electrolysis. At present, however, electrolysis is quite as much a chemi-

cal as an electrical subject, and it has been found convenient to treat it in a separate article (see ELECTROLYSIS). Some points in connection with it have already been touched upon, and a few more will come up in (5), which treats of sources of electromotive force, and deals with the question, whence comes the energy which is evolved in the voltaic circuit? a question the answer to which is for the most part experimental and practical—the only one, in fact, that the state of electrical science permits us to give.

Heating Effects.

It is easy to show, by a variety of simple experiments, that a current of electricity heats a conductor through which it passes. In the case of moderately strong currents the heat developed is perceptible to the touch; the wire may, in the case of very strong currents, be raised to a white heat; it may melt, and even be volatilized. In the case of very weak currents, the heating effect may be demonstrated by passing the current through the spiral of a delicate Breguet's thermometer. We find, when we examine the experimental data on the subject, that heating effects may be conveniently divided into two distinct classes. In the first of these the fundamental law is that the development of heat in any part of a linear circuit varies as the resistance of that part multiplied by the *square* of the current. In the second class the development of heat varies as the *first power* of the current. The heating effects of the first class are obviously independent of the direction of the current, and are irreversible; and the more we examine them the more they appear to correspond to the loss of energy by the frictional generation of heat in ordinary machines. In the language of the dynamical theory of heat, the part of the energy of the electric current which disappears in this way is said to be dissipated. The effects of the second class change their sign when the direction of the current is changed; so that, if anywhere there was evolution of heat when the current flows in one direction, then, when the current is reversed, there will be absorption of heat to an equal extent. We shall find that we have great reason to believe that such effects are strictly reversible.¹ In order to get a satisfactory foundation for the simple theoretical views which we have thus indicated, it is essential to be able to separate the two classes of effects. Now, this is possible to a very great extent even in practice. The effects of the first class increase much more rapidly with the strength of the current than those of the second, so that, by sufficiently increasing the current, we can make the effects of the second class as small a fraction of the whole heating effect as we please; while, on the other hand, by sufficiently decreasing the current, the preponderance of the second class may be increased to any desired extent. We shall in what follows suppose the two classes of effects separated in this way.

Discharge of Statical Electricity.—One of the earliest attempts to study the heating effects of the electric discharge was made by Kinnersley. He constructed an thermoelectrometer, which consisted of a closed glass vessel, in which were fixed two metal balls communicating with electrodes outside the vessel. The bottom of the vessel was filled with a little coloured fluid, which communicated with a tube having a vertical arm rising outside the vessel. When a spark passed between the balls, the heat developed caused the air to expand and force the liquid into the vertical tube, the rise of level in which indicated the degree of expansion, and, by inference, the amount of heat developed in the spark.

Sir Wm. Snow Harris² revived this instrument of Kinnersley's, and improved it by stretching a fine wire between

¹ That is, in the thermodynamic sense.

² *Phil. Trans.*, 1827.

Development
of heat in
circuit

Heating
effects
of discharge

the terminals inside the vessel, so that the heat measured was now that evolved in a metallic conductor.

With this improved instrument he made a number of valuable experiments on the heating of wires by the discharge of a Leyden battery, whose charge was measured by a Lane's electrometer. Assuming that the heat developed varies inversely as the conductivity of the wire (which is not the case), he arranged the metals in a series which agrees with that given later by Riess, although the numbers given do not properly represent the conductivities owing to the erroneous assumption on which they are deduced. Harris observed that the specific conductivity of alloys is often less than that of either metal, and that a very small admixture of another metal considerably reduces the conductivity of pure copper. He also arrived at the result that the amount of heat developed in a wire varies as the quantity of electricity which passes in the discharge, but seems to have concluded that the amount of battery surface used had no effect.¹

Riess made two very important improvements on the thermoelectrometer by substituting spirals for the straight wire of Harris, and by inclining the tube containing the liquid so as to be nearly horizontal. The sensibility of the instrument was thus greatly increased. Riess took up the whole question of the heating of wires, and investigated it thoroughly.

The actual instrument which he used is represented in figure 27 (taken from his *Reibungselectricität*). It consists of a glass tube of

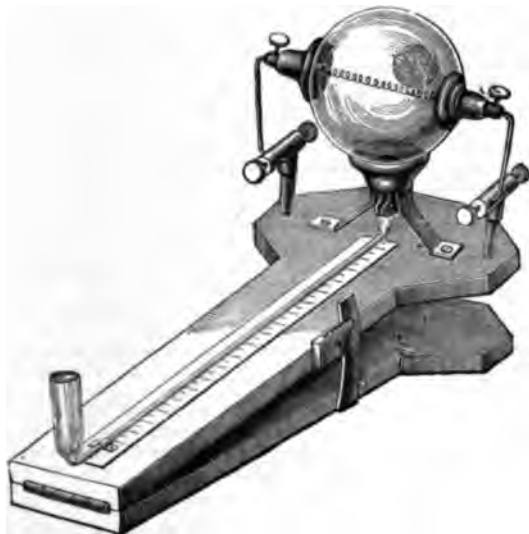


Fig. 27.

narrow bore, 16 to 17 inches long, to which is blown a glass globe 3 to 4 inches in diameter. This tube is partially filled with some coloured fluid which confines the air in the globe, a wide reservoir at the other end of the tube allows the fluid to accumulate without sensible change of level. The stand of the instrument consists of two pieces hinged together, so that the tube can be placed at a small inclination to the horizon. The rest of the instrument will be understood from the figure. Details concerning the manipulation will be found in the *Reibungselectricität*, Bd. i. § 410. When the fine wire is heated by a current of electricity, the heat developed is divided between the wire and the air; the expansion very quickly reaches a maximum, and the level of the liquid in the fine tube becomes stationary for a moment. If m be the number of scale divisions between its original and final positions, we have (see Riess, *l.c.*, or Mascart, t. i. § 325)

$$T = mA \left(1 + \frac{B}{CW} \right), \text{ and } H = mA(CW + B) \quad \dots \quad (1),$$

where T denotes the amount by which the temperature of the wire would have risen had no heat left it, and H the whole amount of heat developed by the current. C and W are the mean specific heat and weight of the wire, and A and B constants, which depend on the make of the instrument, and on the initial temperature and pressure of the air.

A very convenient form of thermoelectrometer, called the *thermomètre inscripteur*, has been used by Mascart (*l.c.*).

¹ *Phil. Trans.*, 1834.

The alterations of pressure are registered automatically on a revolving drum, after the manner of the pulse-registering instrument of Marey. One advantage of this instrument is that it gives a representation of the course of the temperature in the apparatus.

In most of his experiments Riess used batteries of Leyden jars. The jars were all as nearly as possible alike, and the inner armatures were in general connected together. The quantity of electricity given to the battery was measured by means of a Lane's jar, the balls of which were placed at a distance of about a line apart. The battery was then discharged through the thermo-electrometer along with any external circuit connected with it.

It is of great importance in such experiments as we are now describing to examine what happens at the place where the circuit is closed. This closure is effected by bringing two metallic balls into contact. But before contact is reached, a spark passes in which sound, light, and heat are given forth,—in a word, energy evolved. When the resistance of the circuit is small, this spark passes at a considerable distance, and is very intense, no matter how quickly the conductors are brought together. The energy consumed in this case is a considerable fraction of the whole energy given out by the discharge. If, however, the resistance of the circuit through which the discharge takes place be considerable, the electricity takes longer to accumulate sufficiently to raise the electromotive force between the balls to the discharging limit. We may, therefore, by operating quickly, get the balls very nearly in contact before the spark passes. In this case the spark is much less intense, and the fraction of the whole energy which appears in it is very small. Riess made some very valuable experiments on this point. He arranged an air-break in the circuit of the thermoelectrometer, which he could widen or narrow at pleasure, and discharged his batteries through this circuit in the usual way. He found that as the gap is widened the amount of heating in the thermometer is at first increased, but after a certain length of break is attained it decreases again. It must be remembered that we have now two air-breaks in our circuit of discharges, the discharging break and the inserted break. One effect of the inserted break is to diminish the intensity of the spark at the discharging break, and cause a decrease of the energy which appears there. On the other hand it makes the discharge of the battery incomplete, so that part of the potential energy is not exhausted. It is very likely to the opposition of these two effects that the peculiarity observed by Riess is due. Mascart has observed a similar phenomenon in disruptive discharge through oil of turpentine. At all events Riess showed that, when the inserted break was not longer than $\frac{1}{10}$ ths of a line, the heating in the thermometer was the same as when there was no break at all. Hence, if we make the resistance of our circuit so great that the spark at the discharger is not longer than $\frac{1}{10}$ ths of a line, the energy consumed there may be neglected.

The resistance of the connections belonging to the battery and the thermometer were always very small compared with that of the thermometer wire, and the wire, if any, inserted outside the thermometer; so that, if the resistances of these be R and S , the resistance of the whole circuit may be taken to be $R + S$. The law to which the experiments of Riess led can be expressed by means of the formula

$$H = \frac{S}{R + S} Q \quad \dots \quad (2),$$

where Q is the amount of electric potential energy which has disappeared, and H the amount of heat (measured by its dynamical equivalent) developed in the wire of the thermometer, whose resistance is S .

In the case of the complete discharge of a battery of n

General considerations.

jars, each of capacity C , if q be the whole charge, we get immediately, from (48) of Mathematical Theory (p. 34),

$$Q = \frac{q^2}{2Cn}, \text{ and} \\ H = \frac{S}{R+S} \cdot \frac{q^2}{2Cn} \dots \dots \dots (3).$$

Hence, if we keep the thermometer and inserted wires the same, the thermometer indications will be proportional to $\frac{q^2}{n}$, or, in words—the heat evolved in the whole or in any given part of the circuit is proportional to the square of the battery charge directly, and to the number of jars (i.e., to the battery surface) inversely.

If the thermometer wire remain the same, while the length, section, and material of the inserted wire is varied, then, r being the specific resistance, l the length, and ρ the diameter of that wire, $R = \frac{4rl}{\pi\rho^2}$. Then, according to (3), the heat developed in the thermometer is given by

$$H = \frac{A}{1 + B\frac{rl}{\rho^2}} \dots \dots \dots (4),$$

where A and B are constants.

If, again, we use two wires of the same material of lengths l and l' and diameters ρ and ρ' , and make two observations with these for inserted and thermometer wires respectively and *vice versa*, then, if H_1 and H_2 be the heat evolved in the two cases,

$$\frac{H_1}{H_2} = \frac{l\rho'^2}{l'\rho^2} \dots \dots \dots (5),$$

since $R + S$ is the same in the two cases.

When the discharge is not complete, we have only to substitute for Q in (3) the appropriate expression for the exhaustion of the electric potential energy. Similarly we may find the heating effect caused by the discharge of a battery of jars arranged in series and charged by cascade in Franklin's manner (p. 35). If we discharge through a multiple arc, we may assume that the discharge divides itself between the branches in the ratio of the conductivities, so that the conductivity of the whole arc is the sum of the conductivities of its parallel branches. On these principles it is easy to calculate the heat generated in the whole circuit or in any branch of the arc.

All the cases we have alluded to were treated experimentally by Riess, and satisfactory agreement with formula (2) established in every case.

Comparison of conductivities.

By means of formula (4) he compared the specific conductivities of a variety of metals. A and B were determined, and a standard wire of platinum of given length kept in the thermometer; the wires to be compared with it were inserted in the outside circuit, and the heating in the thermometer observed. From the result the specific conductivity (in terms of platinum) of the wires could be calculated, their dimensions being known. The results agree very well with those got by other means.¹

Heating effects of constant current.

Heating by Constant Current.—The heating effect of the current furnished by a voltaic battery was recognized as a distinct and often very remarkable phenomenon for a considerable time before any definite quantitative law was established regarding it. Davy² experimented on wires of the same dimensions but of different materials, and found that the metals could be arranged in the following order:—silver, copper, lead, gold, zinc, tin, platinum, palladium, iron,—those standing nearer the beginning of the list being less heated by a given current than those nearer the end.

Joule³ was the first, however, to establish a definite law connecting the amount of heat evolved per second with the current strength and the resistance of the wire. He wound the wire in which the heat generated was to be measured round a glass tube which was immersed in a calorimeter. The resistance of the water is so great that we may assume without sensible error that the whole of the current passes through the wire. The temperature of the water was determined by means of a mercury thermometer immersed in the calorimeter. The amount of heat developed in the wire per second could then be found by the usual calorimetric methods. The strength of the current was measured by means of a galvanometer inserted in the battery circuit along with the wire. By experiments of this kind Joule established that *the amount of heat generated in a given time varies directly as the product of the resistance of the wire into the square of the strength of the current*. So that, if we choose our units properly, we may write

$$H = RI^2t \dots \dots \dots (6),$$

where R is the resistance of the wire, I the strength of the current, and H the quantity of heat generated in time t .

The experiments of Joule were repeated with increased precautions against error by Becquerel,⁴ Lenz,⁵ and Bötti. Becquerel allowed the wire to disengage heat till the calorimeter reached such a temperature that the loss of heat by radiation and convection, &c., was just equal to the gain from the wire, so that the temperature became stationary. The current was then stopped, and the loss of heat per second found by observing the fall of temperature in the calorimeter. Bötti used an ice calorimeter. Lenz⁶ made a series of very careful experiments with a calorimeter, in which the liquid used was alcohol, which is a much worse conductor than water. He first cooled his apparatus a few degrees below the temperature of the surrounding air, and then allowed the current to generate heat in the wire till the temperature of the whole calorimeter (which was kept uniform by agitation) had risen to an equal number of degrees above the temperature of the air. The current was then stopped, and the time t which it had flowed noted. According to Joule's law, RI^2 ought to be constant, and it was found to be so very nearly. A very convenient instrument for demonstrating and measuring the heat generated by the electric current in a wire is the galvanometer of Poggendorff, which consists simply of an alcohol thermometer with a large bulb, into which is let a spiral of fine wire. The heat generated is deduced from the expansion of the alcohol, which is measured by means of a scale fastened to the stem of the thermometer. The value of the graduations is found by comparison with an ordinary thermometer. The thermoelectrometer of Riess might also be used in a similar way.

Heating in Electrolytes.—Joule's law applies also to electrolytes. The phenomenon, however, is not so simple as it generally is in the case of metallic conductors. Disturbances arise, owing to the heat evolved and absorbed in the secondary actions that take place at the electrode; and superadded to this we have in all probability an absorption or evolution of heat corresponding to the Peltier effect between different metals, of which we shall have to speak directly. Joule eliminated these disturbing influences by using a solution of copper sulphate with copper electrodes. In this case copper is dissolved from one electrode and deposited on the other, so that if we except the slight difference in the states of aggregation of the dissolved and deposited copper, the secondary processes are exactly equivalent, and must compensate each other. Joule⁷ found that in a certain solution of CuSO_4 , 5.50 units of heat were generated in a certain time, while in a wire of equal resistance 5.88 units were generated by an equal current in the same time. In a similar manner E. Becquerel⁸ found that a current, which would produce a cubic centimeter per minute of explosive gas, generated in certain solutions of CuSO_4 and ZnSO_4 0.213 and 0.365 units of

¹ See Wiedemann's *Galvanismus*, Bd. i. § 194.

² *Phil. Trans.*, 1821

³ *Phil. Mag.*, 1841.

⁴ *Ann. de Chim. et de Phys.*, 1848.

⁵ *Pogg. Ann.*, lxi., 1844.

⁶ Wiedemann's *Galvanismus*, Bd. i. § 670.

⁷ *Phil. Mag.*, 1841.

⁸ *Ann. de Chim. et de Phys.*, 1843.

heat; while the same current would have generated in wires of equal resistance 0.26 and 0.32 units respectively.

Reversible Heating Effects.—Peltier¹ was the first to discover an effect of this nature. He found that, when an electric current passes over a junction of antimony with bismuth, the order of the metals being that in which we have named them, there is an *evolution* of heat at the junction; and, when the current passes in the opposite direction, there is an *absorption* of heat, so that the temperature of the junction falls. Here, therefore, there is an effect which cannot vary as the square of the current strength, but must be some function of the current strength, whose *principal* term at least is some odd power.

The Peltier effect, as it is now called after its discoverer, may be demonstrated by inserting a soldered junction of antimony and bismuth into a Riess's thermoelectrometer. When the current

goes BiSb, the fluid will rise in the stem, indicating absorption of

heat; when it goes SbBi, the fluid will fall, indicating evolution of heat. Or we may use Peltier's cross, which consists of two pieces, one of bismuth BB', and the other of antimony AA', soldered together in the form of a cross (fig. 28). A and B are connected by a wire through a galvanometer G. A' and B' are connected with a battery C through a commutator D, by means of which the current can be sent either from A' to B' or from B' to A' through the junction. The thermoelectric current indicated by the galvanometer shows that the junction is heated in the first instance and cooled in the second.

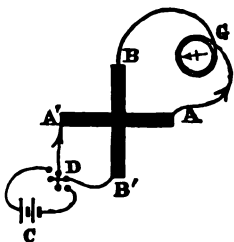


Fig. 28.

By leading the current of a Grove's cell for five minutes through a BiSb junction, Lenz² succeeded in freezing a small quantity of water which had been placed in a hole in the junction, and previously reduced to 0° C. The temperature of the ice formed fell to -4.5° C.

The Peltier effect is different for different pairs of metals. Peltier and Becquerel³ found that the metals could be arranged in the following order:—

Bi, Ga,⁴ Pt, Pb, Sn, Cu, Au, Zn, Fe, Sb.

If the current pass across a junction of any two of these metals, cold or heat is generated according as the current passes the metals in the direction of the arrow or in the opposite direction; and the Peltier effect between the metals is greater the farther apart they are in the series. We shall see later on that this is none other than the thermoelectric series.

Von Quintus Icilius⁵ showed that the Peltier effect is directly proportional to the strength of the current. He passed a voltaic current through a tangent galvanometer (serving to measure it) and a thermopile of 32 BiSb couples. The current was allowed to pass for a fixed time, then the battery was removed and the thermoelectric current of the pile measured by means of a delicate mirror galvanometer. The current of the battery heats the pile in part *uniformly* according to Joule's law: this causes no unequal heating of the junction, and therefore no thermoelectric current; and in part *unequally*, so that one set of junctions are cooler and the other warmer than the mass of the metal: this causes a thermoelectric current, which, since the temperature differences are small (see below, p. 97), may be taken to be proportional to the temperature difference, that is, to the double of the Peltier effect at each set of junctions.

It is interesting to note the analogy here with the polarization of an electrolytic cell. We turn a battery on to

the thermopile, and polarize it, as it were. Then, when we remove the battery and close the pile, we get a return current, which might be called the polarization current of the thermopile.

In general the Peltier effect is, as we have seen, mixed up with Joule's effect, and makes itself felt by producing a disturbance at the junction. Thus Children⁶ found that, when a strong current passed through two mercury cups joined by a thin platinum wire, so that the wire became red hot, the temperature of the mercury in the cups next the + pole of the battery rose to 121° F., while in the cup next the - pole the temperature was only 112° F. Frankenheim⁷ studied the two effects together. He made a Peltier's cross of the pair of metals to be examined, passed a current I through the cross first in one direction and then in the other, and determined by means of a delicate galvanometer the thermoelectric current generated in each case, which is very nearly proportional to the heat produced. If α and β be the heat from Joule and Peltier effects respectively, and i and i' the observed thermoelectric currents, then $i = C(\alpha + \beta)$, $i' = C(\alpha - \beta)$; whence $\alpha = (i + i') \div 2C$, and $\beta = (i - i') \div 2C$. In this way he found that α was proportional to I^2 , and β to I . Thus the whole heat developed may be expressed by $\alpha I^2 \pm \beta I$. We get in this way a verification of the results both of Joule and of Von Quintus Icilius.

Further experiments have been made on this subject by Thomson, Edlund and Le Roux; and Sir W. Thomson was led by a remarkable train of reasoning to discover another reversible heating effect. We prefer to leave these matters for the present, to return to them when we consider thermoelectric sources of electromotive force.

The Peltier effect between metals and liquids and other reversible effects will also come up again under the Origin of Electromotive Force.

Theoretical Deduction of the Formulae.—The above formulae for the heat developed in wires by statical and dynamical electricity may be deduced from a common formula, which can be deduced from Ohm's law.

Let P, Q be two points of a linear circuit, and let E be the difference between the potentials at P and Q, then, if there be no other electromotive force in the portion PQ, the work done by a unit of +electricity in passing from P to Q is E. Hence, if I be the strength of the current, so that $I dt$ units of electricity pass from P to Q in time dt , then the amount dW of work done by the current in time dt is $E I dt$. But, by Ohm's law, $E = RI$, hence

$$dW = RI^2 dt \quad (7).$$

Since the whole of this work is spent in heat, we may for W write H , which denotes the heat generated in PQ. If the current be constant, we get immediately $H = RI^2 t$, which is Joule's law (6). If the current be variable, $H = \int RI^2 dt$, from which we may very easily deduce the formula for the discharge of a battery of Leyden jars. For, applying Ohm's law to the whole circuit whose resistance is $R + S$, we have, if U denote the potential of the inside coatings at time t , $I = \frac{U}{R + S}$. Also the capacity of each of the n

jars being C , we have for the charge $q = nCU$, and $I = -\frac{dq}{dt} = nC \frac{dU}{dt}$. Hence

$$H = R \int I^2 dt = -\frac{nCR}{R + S} \int U \frac{dU}{dt} dt = -\frac{nCR}{R + S} \frac{V^2}{2} = -\frac{R}{R + S} \cdot \frac{q^2}{2nC} \quad (8),$$

where q and V have the same meanings as in (3). (8) agrees with (3), except that we have reckoned the heat developed in a portion of the circuit whose resistance is R instead of S , as in (3). It appears, therefore, that the theoretical formula (7), when properly interpreted, covers both cases.

If there were a junction of heterogeneous metals in the part PQ of the circuit, at which the potential suddenly fell by an amount Π , then work equal to $\Pi I dt$ would be done by the current in passing over the junction, and we should have to write

$$dW = RI^2 dt + \Pi I dt \quad (9).$$

Had there been a rise of potential at the junction, we should have written $-\Pi$ instead of $+\Pi$. If all the work done at the junction is transformed into heat, $W = H$ as before, and for a constant current,

$$H = RI^2 t + \Pi I t \quad (10).$$

¹ Ann. de Chim. et de Phys., 1834.

² See Wiedemann's Galvanismus, Bd. i. § 689.

³ Ann. de Chim. et de Phys., 1847.

⁴ Ga = German Silver. ⁵ Pogg. Ann., lxxxix. 1853.

⁶ Phil. Trans., 1815.

⁷ Pogg. Ann., xvi. 1854.

⁸ Measured, of course, in dynamical equivalents.

The first term is Joule's, the second Peltier's effect. Here the coefficient of the Peltier effect appears as an electromotive force. We shall return to this again.

Glowing,
melting,
&c., of
wires.

Glowing, Melting, Volatilization, &c.—If a wire lost none of the heat generated in it, then, for the same current, the rise in its temperature during a given time would vary as its specific resistance directly, and as the product of its specific heat and density into the fourth power of its diameter inversely. Thus, T, r, c, ρ, d denoting these quantities in the order named above, $T \propto \frac{r}{c\rho d^4}$.

If we have a given battery of electromotive force E , and a circuit connected with it of resistance R , and we insert a wire of length l specified in other respects as above, the current will be $\frac{E}{R+S}$, where $S = \frac{4lr}{\pi d^2}$. If the diameter of the wire be given, then $S \propto l$; and $T \propto \frac{S}{(R+S)^2}$, which is a maximum when $R = S$, that is, when the length of the wire is such that its resistance is equal to that of the rest of the circuit.

Owing to our ignorance of the exact law of cooling, and of the manner in which the resistance and specific heat of most metals change at very high temperatures, it is very difficult to predict beforehand to what temperature a given current will raise a given wire. It is, as may be supposed, still more difficult to predict the effect of a given discharge from a Leyden battery. According to Riess, the phenomenon of glow in this case is complicated by concomitant effects of specific nature.¹

If we assume Newton's law of cooling, i.e., that the heat given out is proportional to the surface of the wire and to the elevation T of its temperature over that of the surrounding medium, then, I denoting the strength of the constant current which heats the wire, we have, when a constant temperature has been attained, $I^2 = \text{const.} \times Td^2$, for wires of same length and material but different diameters. If we compare the apparent brightness of the wires, by causing them to illuminate a screen at a constant distance off, and assume that the light given out is proportional to Td , then, if two wires of diameters d_1 and d_2 have the same apparent brightness, $T_1 d_1 = T_2 d_2$, and $I_1^2 \div d_1^2 = I_2^2 \div d_2^2$. In other words, the strength of current requisite to bring a wire of given length and material to a given brightness of glow varies directly as its diameter. A law of this nature is, of course, merely a rough approximation; Müller and Zöllner, however, have made experiments which agree with it within certain limits. The method of Zöllner is interesting (see Wiedemann's *Galvanismus*).

The temperature of a glowing wire is very sensitive to external circumstances, such as air currents, &c. These effects may be very strikingly shown by balancing the wire in a Wheatstone's bridge against a resistance of thick wire, a strong current being sent through the bridge.

The behaviour of the wire in different gases is very remarkable. If a wire which is glowing in air be suddenly immersed in a jar of hydrogen or coal gas, the brightness will be very much reduced, in fact, in most cases the glow will entirely disappear.² This is owing to the greater cooling power of hydrogen, of which evidence is furnished by the experiments of Dulong and Petit.³ The cooling power of different gases was shown by Grove. He arranged a platinum wire in a glass tube, which could be filled with different gases. The current of the same battery was sent through the wire and through a voltmeter. When the tube was filled with hydrogen or olefiant gas, the amount of gas evolved in the voltmeter per minute was 7.7 and 7.0 cubic inches respectively. The numbers for the other gases experimented on varied from 6.6 to 6.1. They stood in the following order:—CO, CO₂, O, air (2 atmos.), N, air (1 atmos.), air (rarefied), Cl. Experiments of a similar nature were made on liquids. Clausius carried out a calculation of the cooling effect of different gases, and found that the experimental results could be satisfactorily accounted for.⁴

When the strength of the current is sufficiently increased, the wire ultimately fuses, or even volatilizes. The phenomenon is in general complicated. In air, for instance, the

wire burns, and the oxidization once started may take a greater share in raising the temperature than the current does, so that the destruction of the wire may take place under certain circumstances with a current, which, under other conditions, would scarcely make it glow. When discharges from a Leyden battery are used it is very difficult, if not altogether impossible, to get melting unaccompanied with mechanical disaggregation of the wire. The reader who wishes for further information concerning these matters, will find the sources sufficiently indicated in Wiedemann, Riess, and Mascart.

This department of electricity is very fruitful in popular lecture-room experiments. We shall quote one or two of these, and refer the reader to popular treatises for more of the same kind.

On a sheet of thin card-board is pricked a design, generally what is understood to be a portrait of Franklin, two pieces of tinfoil are pasted on the ends of the card by way of electrodes, and between these a piece of gold leaf is laid. On the other side of the card is placed a piece of white paper or silk. The whole is then tightly screwed up between two boards. When an electric discharge is sent through the gold leaf it volatilizes, sending the disintegrated particles through the holes in the card-board. In this way an impression of the portrait is obtained.

If a current be caused to heat a pretty long thin platinum wire to dull redness, and a portion of the wire be cooled by applying a piece of ice to it, the remainder of the wire will glow much more brightly than before; whereas, if a portion be heated by a spirit-lamp, the reverse effect takes place. The reason is that the current is strengthened in the one case by the decrease of the resistance in the cooled part, and weakened in the other by the increase of resistance where the wire is heated.

When two curved metal surfaces rest upon each other, a current passing from the one to the other encounters considerable resistance at the small area of contact. The heat developed in consequence of this causes the parts in the neighbourhood to expand very quickly when the contact is made. This very often gives rise to rapid vibratory movements in the conductors. The Trevelyan rocker⁵ can be worked in this way (see art. HEAT), bells rung, &c. The best known experiment of the kind is Gore's railway. This consists of two concentric copper hoops, whose edges are worked very truly into the same plane. A light copper ball is placed on the rails thus formed, a current from two or three Groves is sent from one hoop to the other, and the ball set in motion. If the ball be very true, and the railway be well levelled, the energy supplied by the swelling at the continually changing point of contact is sufficient to keep up the motion, and the ball runs round and round, emitting a crackling sound as it goes.⁶

The Voltaic Arc.—When two electrodes of volatile or readily disintegrable material forming the poles of a powerful battery (say 30 or 40 Grove's cells) are brought into contact and then separated, the current continues to pass across the interval, provided it is not too great. The conducting medium appears to be a continuous supply of heated matter, suspended in glowing gas or vapour. This phenomenon seems to be more akin to the subject we are now discussing than to the disruptive discharge of which we shall speak by-and-by. The light thus generated with a large battery, especially when electrodes of graphitic carbon are used, is brilliant in the extreme. It was thus that Davy first obtained the phenomenon.⁷ With a battery of 2000 cells he obtained a luminous arc 4 inches in length, and when the carbons were placed in an exhausted receiver the arc could be lengthened to 7 inches.

The fact that the electrodes must be brought in contact in order to start the light is quite in accordance with what we know of the extremely small striking distance of even very powerful batteries. When the contact is made, the place where the electrodes touch, owing to its small section, is intensely heated; the matter begins to volatilize, and then the current is kept up by the quickly increasing cloud of metallic

¹ *Reibungselectricität*, Bd. ii. §§ 557 sqq.

² Grove, *Phil. Mag.*, 1845, or *Wied. Galv.*, Bd. i. 679.

³ Poggendorff, *Pogg. Ann.*, lxxi., 1847.

⁴ *Wied. Galv.* (4. c.), or *Pogg. Ann.*, lxxvii., 1852.

⁵ *Wied. Galv.*, Bd. i. § 726.

⁶ This motion has been attributed to electromagnetic action. Such an explanation is quite inadmissible.

⁷ *Phil. Trans.*, 1821. According to Quetelet, Curtet observed the light between carbon points in 1802. *Wied. Galv.*, Bd. i. § 783.

experiments; Wiedemann, on the other hand, gives elaborate accounts of the more modern results of De la Rive, Plücker, Hittorf, and others.

Theoretical considerations.

When induction is exerted across a dielectric, we may consider the action at any point of it in one or other of two ways. We may regard the resultant electromotive force arising from the action at a distance of all the free electricity in the field as tending to separate the two electricities in the molecules of the dielectric. In this view, we might measure the dielectric strength of the medium by the value of the electromotive force, when the electricity is on the point of passing from one molecule to the next. We might, on the other hand, consider, with Faraday and Maxwell, that the dielectric is the seat of a peculiar kind of stress, consisting of a tension p along the lines of force, and an equal pressure perpendicular to them, p being equal to $\frac{K}{8\pi} R^2$ (Maxwell, vol. i. § 104). We shall adopt the latter alternative, and when we speak of tension henceforward it means $\frac{K}{8\pi} R^2$. In this view the dielectric strength may be defined as that tension under which the dielectric just begins to give way. The reader who prefers the other way of looking at the matter will find no difficulty in translating any statement from the one language into the other.

We have started by considering any point of the dielectric, and it is obvious that the dielectric (supposed homogeneous) will first give way at that point which first reaches the limiting tension w ; just as an elastic solid begins to give way where the stress first reaches the breaking limit. It may be proved, however, that R^2 cannot have a maximum value at any point where there is no free electricity, which shows us at once that the point at which the limiting tension is first reached must always be on *some* electrified surface, in general therefore on the surface of one of the conductors of the system.¹ Disruptive discharge, thus begun at the surface of a conductor, spreads out into the dielectric. Its farther course is influenced by a variety of circumstances very hard to define in the great majority of cases.

An attempt will be made by-and-by to give an idea of the varieties of luminous discharge that arise in this way; meantime we concentrate our attention on a feature common to all disruptive discharges, viz., the definite limiting tension at which under given circumstances they begin.

Dielectric strength.

Striking distance.

Dielectric Strength of Gases.—The earlier measurements bearing on this subject were conducted under circumstances which render a comparison of the results with the theory, as at present developed, very difficult. Harris found that the striking distance between two balls connected with the armatures of a condenser was directly proportional to the charge of the condenser as measured by a Lane's jar. Riess used a Leyden battery, and varied the number of jars and the charge of the battery. The balls of his spark micrometer were of diameters 5·7 and 4·4 lines respectively, while the distance between them varied from 0·5 to 2·5 lines. Under these circumstances, he found the striking distance to be proportional to the charge of the battery directly, and to the number of jars inversely. The results of Harris and Riess might be summed up in the statement that the striking distance between two balls connected with the armatures of a condenser varies as the electromotive force or difference of potential between the armatures. This result is purely empirical, and must not be extended beyond the experimental limits within which it

was found. Even Riess's experiments themselves show that the striking distance increases more rapidly than the difference of the potentials.

The experiments of Knochenhauer² led to a similar result. Gauguier³ made experiments of the same kind through a wider range of striking distances, and found, in conformity with the result of Riess, that, with balls of 10 or 15 mm. diameter, the striking distance is proportional to the potential difference between the balls, when the distance between them lies between 2 and 5 millimetres. Beyond these limits the ratio of potential difference to striking distance falls off; whereas, for smaller distances, it increases very rapidly. He also found that the deviation from the law of Harris and Riess is more marked when unequal spheres (3 mm. and 10 mm.) are used, and still more when a ball (3 mm. diam. used as + electrode) and a disc (35 mm. diam.) were used as electrodes. Experiments leading to similar conclusions are cited by Mascart,⁴ who finds that, for spheres of diameter 3 to 5 centimetres, the striking distance for given potential difference is sensibly the same; whereas for plates, both the striking distance and the law of the whole phenomenon is different. The same experimenter examined the striking distances between two equal balls (3 cm. diam.) from 1 mm. up to 150 mm. Taking the potential difference for one millimetre as unity, he found for 10, 20, 40, 80, 150 mm. the potential differences 8·3, 11·8, 15·9, 20·5, 23·3. The deviation from proportionality is obvious; the potential differences in fact tend to become constant. Wiedemann and Rühlmann, in their experiments on the passage of electricity through gases (see below, p. 61), made some experiments on the influence of the form and distance of the electrodes. They used two brass balls of 13·8 and 2·65 mm. diameter respectively, and sent between them the discharges of a Holtz machine. The distance (δ) between the nearest points varied from 3 to 22·3 mm. They found that the quantity of electricity (y) required to produce discharge, could be represented by the formulæ $y = A - \frac{B}{\delta}$ and $y = C + D\delta^2$, according as the larger sphere formed the positive or negative electrode. The constants A, B, C, D depend on the pressure, which varied in these experiments between 25 and 60 mm. of mercury.

In most of the experiments that have just been described the effect of the form of the electrodes and the surrounding conductors could not be estimated theoretically. Experiments in which the theoretical conditions are simple have been made by Sir Wm. Thomson.⁵ The spark was taken between two parallel plates of considerable area; one of these was plane, and the other very slightly curved, to cause the spark to pass always at a definite place. The electrical distribution on the opposing surfaces can be found (see above, Math. Theory of Electrical Equilibrium), as if the plates were plane and of infinite extent. This distance between the plates was measured by a micrometer, the contact reading being determined by observing when the electricity ceased to pass between the plates in the form of a spark. The potentials were measured in absolute electrostatic (C.G.S.) units, by means of Thomson's absolute electrometer (see art. ELECTROMETER). The limiting tension or dielectric strength is given in each case in grammes per centimetre, the formula for calculating it being

$$p = \frac{V^2}{8\pi \times 981 \cdot 4 d^2},$$

in which V represents the potential difference or electromotive force between the plates, and d the distance in centimetres. If we take the older view of Poisson's time that the action of the electricity on the surface of a conductor is simply a fluid pressure, then p represents that pressure.

If we could consider the air between the plates as a homogeneous dielectric, then, for air at a given pressure (and temperature?) and given state of dryness, p , which measures its dielectric strength, would have a constant value independent of the distance between the plates, and V would be proportional to d . A glance at Sir Wm. Thomson's⁶ tables shows that this is not the case. For a

¹ The dielectric is supposed to be homogeneous. Prof. Maxwell has pointed out that exceptions might occur in the case of a weak dielectric interposed between two strong ones, e.g., a current of hot air passing through cold.

² Mascart, t. i. § 463, or *Pogg. Ann.*, lviii.

³ Mascart (*l.c.*).

⁴ t. i. § 478.

⁵ *Proc. R.S.*, 1860, or *Reprint*, p. 247.

⁶ *Reprint*, pp. 252, 258.

distance of .00254 cm., $p = 11.290$, whereas for a distance .1524, $p = .535$. It appears, therefore, that the dielectric strength of a thin stratum of air is much greater than that of a thick one. It is very difficult to understand why this should be so. "Is it possible that the air very near to the surface of dense bodies is condensed, so as to become a better insulator; or does the potential of an electrified conductor differ from that of the air in contact with it, by a quantity having a maximum value just before discharge, so that the observed difference of potential of the conductors is in every case greater than the difference of potentials on the two sides of the stratum of air by a constant quantity equivalent to the addition of about .005 of an inch to the thickness of the stratum?"¹ It is remarkable that the limiting tension should be so small, somewhere about half a gramme per sq. cm., as compared with the atmospheric pressure, which is about 1032 gm. per sq. cm.

A series of absolute measurements of the potential required to produce a spark between equal spheres at different distances has been made by Mascart. The method employed was very ingenious.²

Effect of Pressure, Temperature, &c., on the Dielectric Strength of Gases.—The dielectric strength of a given gas depends on its pressure, or at all events on its density. Harris, who experimented on this subject, inclosed two balls in a receiver which could be exhausted to any required degree, and connected them with the armatures of a battery of jars. He found that the charge which had to be given to the battery in order to produce a spark between the balls was proportional to the density of the air in the receiver, while it seemed to be independent of its temperature. This amounts to asserting that the difference of potentials required to produce a spark between the balls is proportional to the density of the gas and independent of its temperature. Since we keep the distance between the balls the same throughout, this statement is equivalent to saying that the dielectric strength of a gas varies directly as its density, and does not depend on the temperature. Masson, using the method which Faraday had employed in comparing the dielectric strength of gases (*vide infra*) arrived at the same conclusion as Harris. Knochenhauer, however, experimenting with pressures ranging from 3 to 27.4 inches of mercury, found that for a given interval the difference of potentials required to produce disruptive discharge was proportional to the pressure increased by a small constant quantity.

Faraday, in the 12th and 13th series of his *Experimental Researches*, examines this subject; and the reader who desires to have a clear idea of what the issues involved really are will do well to begin by carefully studying Faraday's results, and still more his views on this matter. Faraday directs his attention to the specific behaviour of different gases.

The gas to be examined was introduced into a receiver in which were arranged two balls s and l , of diameters 0.93 in. and 2.02 in. respectively, at a constant distance 0.62 in. apart. Two balls, S and L , of diameters 0.96 in. and 1.95 in., were placed on suitable insulating supports outside the receiver. S and s were connected with an electric machine, and l and L to earth. The distance u between S and L could be varied at will; if it was greater than a certain value β , the sparks always passed between s and l in the receiver; if it was less than a certain value α , they always passed between S and L in the outer air. It might have been expected that α and β would be equal, or at least very nearly so, i.e. that there would be one definite value of u , for which the spark would hesitate between the alternative intervals. This is not so, however. Nor again is the value of u the same when s and l are negative as when they are positive. The following table will illustrate these points, as well as the relations of the different gases:—

Gas.	s and l positive.			s and l negative.		
	α	β	Mean.	α	β	Mean.
Air.....	0.60	0.79	0.69	0.59	0.68	0.63
Oxygen.....	0.41	0.60	0.50	0.50	0.52	0.51
Nitrogen.....	0.55	0.68	0.61	0.59	0.70	0.64
Hydrogen.....	0.30	0.44	0.37	0.25	0.30	0.27
Carbonic acid.....	0.56	0.72	0.64	0.58	0.60	0.59
Olefiant gas.....	0.64	0.86	0.75	0.69	0.77	0.73
Coal gas.....	0.37	0.61	0.49	0.47	0.58	0.52
Hydrochloric acid.....	0.89	1.32	1.10	0.67	0.75	0.72

It will be seen that the different gases present considerable variety, and cannot be classified in any way so as to connect the dielectric strength with any other physical property. The numbers given cannot be regarded as measuring the dielectric strength, owing to the disturbing influences which cause the inequality of α and β . This inequality is not by any means small; e.g., for air the uncertainty amounts to about 32 per cent. These experiments show very clearly that the sign of electrification of the surface at which the discharge begins has a great effect on the limiting tension. The discharge passes much more readily from a small ball to a large one when the former is negative than when it is positive. Faraday made a variety of experiments to elucidate this point, and he was driven to the conclusion "that, when two equal small conducting surfaces equally placed in air are electrified, the one positively the other negatively, that which is negative can discharge to the air at a tension a little lower than that required for the positive surface, and that, when discharge does take place, much more passes at each time from the positive than from the negative surface."

Positive and negative limiting tension.

The inequality of α and β may be due to various causes, among which may be mentioned the charging of the glass of the receiver, dust, &c., in the air, heating of the air, and the presence of finely divided metal dispersed by preceding sparks. The last of these causes would account to a considerable extent for the fact that the sparks show a tendency to persist in a path once opened, and that the interval $\beta - \alpha$ is less for the negative spark, which starts at a smaller limiting tension, and may therefore be supposed to produce less mechanical effect.

Wiedemann and Rühlmann have recently taken up this subject in a research which has already been alluded to.³

Wiedemann and Rühlmann.

The gas and the spark terminals were inclosed in a cylindrical metal receiver with rounded ends. A small window allowed the light from the spark to fall on a rotating mirror fixed on the axis of a Holtz machine, which furnished the electricity. The images of the successive sparks were observed by means of a heliometer. One-half of the divided object-glass was moved until one of the images of one discharge coincided with one of the images of the next; then a similar coincidence was brought about by displacing the half-lens in the opposite direction. The difference (y) of the two readings on the micrometer of the heliometer measures the rotation of the disc of the Holtz machine between the two sparks. Preliminary experiments showed that the amount of electricity furnished by the machine while the disc moves through a given angle is independent of the angular velocity of the disc. It varies from day to day, however, according to the quantity of moisture in the air and the arrangement of the machine; but, on the principle just laid down, correction can easily be made by taking the reading each day of a galvanometer through which the current of the machine is sent. It follows, therefore, that y is proportional to the quantity of electricity which passes at each discharge through the gas, and by means of a galvanometer observations on different days can be compared.

It was found that at the lowest pressures worked with (.5 to .25 mm. of mercury) the discharge of the Holtz machine was still discontinuous; and that in all the experiments the tension at the electrodes was such that the discharge was independent of the nature of the metal,—in

¹ Maxwell, *Electricity and Magnetism*, vol. i. § 57.

² *Electricité*, t. i. § 481.

³ *Abh. d. k. Sächs. Gesellsch.*, 1871, or Wiedemann, *Galv.* ii. 2, § 933, &c.

other words, that the disintegration of the electrode played no essential part in the discharge.

The quantity of electricity required to effect a discharge, other things being equal, increases with increasing pressure. This increase is at first rapid, then slower, and at high pressures it is nearly proportional to the increase of pressure. It was found that y could be expressed with sufficient accuracy in terms of the pressure p by the empirical formula, $y = A + Bp - Cp^2$, in which the constants A , B , C depend on the size and insulation of the electrodes, their distance apart, and so on.

They arrange the gases in the following order of dielectric strength:—hydrogen, oxygen, carbonic acid, air, nitrogen. It is not a little remarkable that this is the order given by Faraday in the second column (the best) of the results we quoted above.

They find, in agreement with Faraday, that a greater quantity of electricity is required to bring two unequal spheres to the discharging point when the small one is positive than when it is negative. When two equal spheres are used, the value of y is least when both are insulated, greater when the positive sphere is uninsulated, and very much greater when the negative one is uninsulated.

All this is in accordance with theory, provided we assume with Faraday that the limiting tension is greater at positive than at negative surfaces. For example, suppose the surface densities corresponding to the limiting positive and negative tensions to be P and N ($P > N$), and consider the case of two equal spheres of radius a , at so great a distance c apart that $\left(\frac{a}{c}\right)^3$ may be neglected, then by taking three consecutive images the reader will easily find that the charges which must be given to either ball in the case where both spheres are insulated and equally charged, and to the negative ball in the case where the positive ball is uninsulated, and to the positive ball when the negative ball is uninsulated, must be $(1 - 3\frac{a^2}{c^2})4\pi a^2 N$, $4\pi a^2 N$, $4\pi a^2 P$, respectively, in order to produce discharge. The discharge begins at the negative ball in the first two cases, and at the positive ball in the third, and the quantities are obviously in ascending order of magnitude when P is $> N$.

High
pres-
sures.

The dielectric strength goes on increasing when the pressure is raised above the atmospheric pressure. Cailletet¹ found that a powerful induction coil worked by eight large Bunsen cells was powerless to effect discharges across $\frac{1}{2}$ mm. of dry gas at a pressure of 40 or 50 atmospheres.

Minimum
strength.

On the other hand, however, the dielectric strength does not diminish indefinitely as the pressure decreases, but reaches a minimum.

Morren and De la Rive² have sought to determine this minimum dielectric strength by measuring by means of a galvanometer the mean intensity of the current sent through the gas by an inductorium so arranged that only the direct induction current passes; they thus obtain what they call a minimum resistance. Morren gives the pressures corresponding to this minimum for various gases; they lie between 0.1 and 3.0 mm. It may be questioned whether any very definite meaning can be attached to results of this kind; for the discharge is discontinuous, and resistance in the proper sense of the term cannot be spoken of.

Strength
of va-
cuum.

It is clear, however, that a minimum dielectric strength must exist; for, if we go on improving our vacuum, we find that our ordinary machinery fails to send electricity through any considerable length of the exhausted space.

Morgan³ seems to have been the first to discover that the electric spark would not pass in a vacuum. Having carefully boiled the mercury in a barometer tube, so as to remove the last traces of moisture, he found that the inductive discharge caused by electrifying a piece of tinfoil on the outside of the tube would no longer pass to the mercury, and cause the luminous phenomena usually seen under such circumstances. Masson repeated this experiment in a more satisfactory form. Gassiot⁴ greatly improved the exhaustion of vacuum tubes by filling them with CO_2 , pumping out as usual, and then absorbing the residual gas by fusing a piece of KHO previously inserted into the tube. He constructed tubes in

this way which had sufficient dielectric strength to insulate the pole of his great battery of more than 3500 Zn. Aq. Cu. cells. Hitroff and Geissler⁵ have constructed vacuum tubes (by pumping with a Geissler's pump, and heating the whole to 400° to 500°C.) in which the opposition to the discharge of an interval of $\frac{1}{4}$ mm. between two platinum electrodes was greater than that offered by 15 or 20 centimetres of ordinary air.

Different Forms of the Discharge in Gases.—We have said that the subsequent progress of the disruptive discharge when once begun is influenced by a great variety of circumstances. The beginning of the discharge evolves heat, which rarefies the neighbouring air, and therefore weakens its dielectric strength. Owing to this cause the discharge once started tends to go on. Again, if any considerable quantity of electricity escapes into the ruptured dielectric at the first burst, this relieves the tension at the surface of the conductor. On the other hand, the progress of part of the electricity towards the opposing conductor raises the tension at the surface of the latter, so that disruptive discharge is provoked or helped there. If the initial tension is considerable, or the quantity of electricity which passes to begin with very great, glowing metal particles are shot forth into the dielectric, causing a reduction of its strength, which will be very different in different directions. Motions of the air play a great if not a preponderating part in many forms of the discharge. The electrification, &c., of the walls of the tube, and the form of the electrodes and of the tube, both in the neighbourhood of the electrodes and at a distance from them, are as important in their influence on the continuance of the discharge as they are on its start. And, last but not least, much depends on the way the electricity which produces the discharge is furnished,—on the nature of the electromotor, in short. Although we have not yet exhausted the influencing conditions, we have probably said enough to convince the reader that little aid is to be hoped for in this matter from considerations *a priori*. There is a great deficiency even in proximate principles to guide us in the maze of experimental detail; and although most of the experiments are beautiful beyond all conception, yet the mere narration would scarcely interest the reader. Our description of the department will, therefore, consist simply in going round the boundary.

The luminous appearances may be roughly classed under the forms of spark, brush, glow and convective discharge, and dark discharge.

At the ordinary atmospheric pressure the disruptive discharge between two conductors at a moderate distance apart takes place in the form of a brilliant sharply-bounded streak of light, whose apparent breadth is in general small. For small distances the spark is straight, and has the appearance of being thicker, or at least more brilliant, at the ends than in the middle. When the distance is considerably increased the spark assumes the characteristic zig-zag form seen in forked lightning. It seems occasionally to be absolutely broken by perfectly dark spaces. The duration of the discharge in this form, more especially when the resistance of the discharging circuit is very small, as tested by a rotating mirror, appears to be exceedingly short.

We have taken photographs of the sparks of a Holtz's machine by simply moving the camera containing the sensitized plate vertically upwards past the electrodes of the machine. The result is a column of perfect photographs, quite unblurred by the jarring, &c., of the camera stand. Again, if a disc painted with white and black sectors be caused to rotate very rapidly, it appears in ordinary light to have a uniform grey colour; but when it is viewed by the light of an electric spark the sectors are seen exactly as if the disc were at rest, which proves that the illumination lasts for a very short time. Masson founded on this experiment a beautiful method for measuring the intensity of the light given out by the spark. A description of his apparatus, with an account of his results, will be found in Mascart.

The colour of the spark in air is bluish,⁶ but at the same

¹ Mascart, t. i. § 187

² Phil. Trans., 1786.

³ Wiedemann Bd. ii. § 952

⁴ Phil. Trans., 1859.

⁵ Pogg. Ann., 1869.

⁶ Faraday, Exp. Res., 1422.

time its great brilliancy gives an impression of whiteness. In nitrogen the appearance is much as in air, only the colour tends more to bluish purple, and the spark is more sonorous. In oxygen the spark is whiter and less brilliant than in air; in hydrogen crimson-coloured; in carbonic acid greenish; in hydrochloric acid white, and never broken by dark parts; in coal gas green or red, with occasional dark parts. If the spark be carefully examined, especially when the pressure is greater than an atmosphere, it will be seen that the central bright streak is surrounded by an envelope, of somewhat nebulous form, and of a lavender-blue colour. This envelope tends to spread over the negative electrode, where it is more conspicuous as compared with the central streak than elsewhere. This envelope appears to be due to the glowing metal particles torn from the electrodes. It has, unlike the central streak, a sensible duration, on account of which it happens in many cases that a much greater quantity of electricity passes through it than through the infinitely more brilliant but less enduring part of the discharge. The envelope can be actually separated from the streak by a current of air properly directed, or by the action of a magnet (*vide infra*, p. 74).

When the discharge in air at the atmospheric pressure takes place between a *salient* but *not pointed* part of one conductor and another conductor of *considerable surface* (e.g. between one sphere 2 cm. diameter and another 13 cm. diameter), the luminous appearance very often takes a characteristic form, which has been called the brush discharge. The name is to a considerable extent descriptive of the phenomenon; if the word broom had been applied it would have been even more appropriate, and a rough idea of the variety of forms the brush may assume will be obtained by thinking of the various forms of the domestic article in question. At the surface of the smaller conductor appears a short, straight, luminous stem differing in appearance very little except in brightness from a *spark*. From this radiate a series of twig-like branches of much inferior brilliancy, having a purplish-violet colour. These subdivide in many cases into still smaller ramifications, and are ultimately lost in the medium. When the large conductor is either altogether absent or very distant, the general tendency of the branches is to spread outwards more and more in all directions; but when the large conductor is brought nearer, the branches have a tendency to bend down towards it, so that the whole assumes an ovoid shape. The brush is generally accompanied by a crackling or hissing sound, or even a musical note. On approaching the hand or a conductor of extended surface, the pitch of this sound rises considerably. This at once suggests that the brush is an intermittent phenomenon. That this really is so was clearly proved by Wheatstone in one of the earlier applications of his rotating mirror.¹ Wheatstone saw in his mirror not one image of the brush, but several arranged in succession at regular intervals. Each of these images corresponds to a single discharge, and each appears less complicated than the brush as viewed by the unaided eye, which is, in reality, a superposition of a considerable number of brushes, the number depending on the time taken by a light impression to fade on the retina. At the same time each individual image is a little drawn out in the direction of motion of the mirror, which shows that the brush has a sensible duration. Faraday speculates very acutely concerning the nature of the brush discharge (see *Exp. Res.*, 1425 *seq.*). He finds that, although it is generally accompanied by a current of air, yet it is not always or necessarily so. He also carefully illustrates the difference between the positive and negative brush. If we have a small ball on the end of a

wire projecting freely into the air, the positive brushes² obtained from it are much larger and finer than the negative brushes so obtained. Again, if we charge a large metal ball positively, and bring an uninsulated metal point up to it, a star appears on the point, which gets brighter and brighter as the point approaches the sphere, but the form does not change until the distance is very small. If the sphere be charged negatively, the star appears as before when the distance is considerable, but at a moderate distance (1 to 2 inches) a brush forms, and when the distance is still farther reduced a spark passes. It seems, therefore, that the negative discharge keeps its form unchanged under considerable variety of influencing circumstances, whereas the form of the positive discharge is more readily affected. The explanation of these differences he finds in the fact, which he established by experiments already alluded to, that the limiting tension is smaller at positive than at negative surfaces; so that, *ceteris paribus*, the negative discharge occurs oftener than the positive discharge; but, on the other hand, when the latter does occur, more electricity passes. This, no doubt, accounts for the lower pitch of the sound of the negative brush, and the greater extent and brilliancy of the positive one. Faraday found great differences in the character of the brush in different gases; in none apparently does it reach the brilliancy attained in air or nitrogen. He also observed that rarefaction up to a certain point favoured the production of brushes.

When discharge takes place from the rounded end of a *Glow*. wire projecting freely into the air, the brush is very often replaced by a quiet phosphorescent glow, which covers a greater or less extent of the end of the wire. The noise which accompanies the brush is entirely absent in this form of the discharge, and the means by which the brush can be analysed into a series of successive discharges give no corresponding result for the glow. In the rotating mirror it simply stretches out into a uniform band of light. The glow is therefore either a continuous discharge or an intermittent discharge of incomparably shorter period than the brush. Diminishing the discharging surfaces favours the production of glow.³ Increase of power in the electric machine which is furnishing the electricity has a similar effect. Rarefaction of the air has also a great effect in facilitating the production of glow, especially in the case of negative glow, which is extremely hard to produce in air at common pressures. In Faraday's opinion, the star which is obtained with a positive sharp point is a positive glow; but he thinks it not improbable that the negative star is not a negative glow, but a small negative brush. The glow is invariably associated with a current of air to or from (generally both) the glowing conductor. Everything that favours this air-current increases the glow; e.g., a brush may sometimes be converted into a glow by properly directing an air-current near it. Again, everything that prevents or retards the formation of an air-current has a similar effect on the glow: a glow can be converted into a brush in this way. Lastly, everything which tends to prevent abrupt variation of the tension favours the glow, and everything having an opposite tendency is destructive of it. Faraday concludes, therefore, that the glow is due to a gradual discharge by convection, in which the agents are the particles of the gas. The order of the appearance of spark, brush, and glow at positive and negative surfaces is, in general, the same; but the gradation is different. Positive spark does not pass into brush so soon as negative spark does; but, on the other hand, positive brush turns to glow long before negative brush.

¹ *Phil. Trans.*, 1834, &c.

² By positive brush, of course, is meant brush emanating from a positively charged surface.

³ *Exp. Res.*, 1527.

Convec-
tive dis-
charge.

Intimately connected with the glow is the convective discharge, if indeed they are not degrees of the same phenomenon. "The electric glow is produced by the constant passage of electricity through a small portion of air in which the tension is very high, so as to charge the surrounding particles of air which are continually swept off by the electric wind, which is an essential part of the phenomenon."¹ Now there seems little reason to doubt that at lower tensions² discharge of this kind may occur without the luminous phenomenon at the surface of the conductor. If this be so, then the convective discharge is only a different degree of the glow discharge.

Discharge by convection plays a very important part in all electrostatical experimenting. The air in the neighbourhood of an electrified conductor gets charged, forming an electrical atmosphere, which surrounds the conductor, being more extensive in the neighbourhood of salient angles than elsewhere. Such electrical atmospheres are often a source of great inconvenience in the laboratory and lecture-room when delicate electrical experiments are in progress.

A curious little instrument, called the electrical tourniquet or windmill, depends for its action on the electrical wind which accompanies convective discharge. A small rectangular cross, with equal arms, is made of light wire; the extremities of the arms are bent through a right angle in the plane of the cross, so as to point all one way. The little cross thus made is poised, like a compass needle, on a vertical wire connected with an electrified conductor. Convective discharge takes place at the points, giving rise to an electrical wind, the reaction of which causes the little machine to revolve with great rapidity. If the experiment be conducted in the dark, a glow usually appears on the revolving points. The experiment also succeeds when the cross is immersed in a non-conducting liquid.

Dark
interval.

We have already alluded to the dark spaces that sometimes appear in the spark in gas at the atmospheric pressure. Faraday observed that a phenomenon of this kind was very common in coal gas. When the discharge takes place in highly rarefied gas, a dark space of this kind almost always separates the positive from the negative light, its situation having a certain degree of fixity with respect to the negative, but not to the positive electrode. It is very difficult to form an idea of the exact nature of the discharge which takes place in this space. Discharge there undoubtedly is of some kind; and pending further investigation, Faraday called it the dark discharge. The fact that its real nature is still undiscovered amply justifies the separate name. Faraday found that it occurred in discharges that pass almost instantaneously, and concluded that it could hardly be due to convection of the ordinary kind, which requires time. De la Rive and Hittorf have made out many peculiarities connected with its appearance in vacuum tubes, the phenomena in which we now attempt briefly to describe.

Pheno-
mena in
rarefied
gases.

A variety of forms may be given to the vessel in which the rarefied gas to be experimented on is inclosed.

One of the most common used to be the electric egg, which is simply an oval glass vessel furnished with two small metal spheres for electrodes; the stems which carry these electrodes pass air-tight through tubes cemented to the ends of the vessel; the stem which supports the whole is perforated and fitted with a stop-cock, so that the apparatus can be exhausted to any required extent and then temporarily closed. The commonest of all instruments of this kind now-a-days is the Geissler tube. This is simply a glass tube, into which are fused two electrodes of platinum or other metal; a capillary tube allows the apparatus to be connected with an air-pump, and exhausted; when this is done, the capillary tube is sealed up by means of a spirit-lamp. A very common form of such tube is the spectrum tube (see art. LIGHT), consisting of two wider parts, connected by a capillary part, in which the light of the discharge is much more intense than elsewhere. Complicated tubes of all kinds have also been constructed as electric toys.

The reader must not forget that the form of the tube exercises a great influence on the phenomena, whether at the positive or negative electrode. In the summary description that follows the

electric egg is referred to, unless it is otherwise stated. We further assume that the electromotor used gives currents in one direction only. A Holtz machine would satisfy this condition, within certain limits at least.

When the gas is rarefied to a considerable extent, the spark loses its sharp outline, becomes interspersed with nebulous portions, and by-and-by loses its characteristic form altogether. As the rarefaction goes on, the discharge ceases to reach from the positive to the negative electrode. The latter now displays a patch of lavender-blue light, separated from the positive light by a dark interval, the length of which depends on the distance between the electrodes. In certain cases the positive light terminates in a cup-shaped depression, whose concavity is turned towards the negative electrode. As the rarefaction is still further increased, the positive light tends more and more to fill the tube, although in general it recedes from the negative electrode, over which, on the other hand, the beautiful lavender glow spreads more and more, exhibiting at the same time a growing tendency to fill a limited space surrounding the electrode. At a still higher degree of rarefaction, the positive light, which now occupies a considerable space, and takes a shape more or less corresponding to that of the inclosing vessel, is divided transversely into a number of cup-shaped striæ, separated from each other by darker intervals. These striæ vary in form and appearance considerably, according to circumstances. In the neighbourhood of the positive electrode, their concavity is turned towards the positive electrode; but towards the other end of the positive light, the concavity may be turned the other way, especially in the electric egg. The positive light, in vacuum tubes, shows therefore the same remarkable variability, and the negative light the same measure of stability that Faraday remarked in gas at ordinary pressures. The colour of the positive light varies very much in different gases; in nitrogen and air its rosy-red colour contrasts very sharply with the blue of the negative light. The negative light is remarkable for its power of producing fluorescence. It is very dependent as to its extent on the form and size of the uncovered surface of the electrode; anything placed on the electrode cuts it off sharply, as if the light were projected from the electrode and stopped by the obstacle. Disintegration of the negative electrode also goes on very rapidly, so that, after a vacuum tube has been used for some time the glass all round the negative electrode is blackened, browned, &c., as the case may be, with a deposit of finely divided metal. The quantity as well as the quality of this deposit depends very much on the nature of the metal; it is smallest with aluminium, which is on that account much used for electrode terminals. The negative light occasionally shows one, two, or even three stratifications; but in this respect it never equals the positive light. When the rarefaction is carried to the utmost, both positive and negative lights fall off greatly in splendour. The negative light contracts more and more in upon the electrode, and confines itself even there to a small patch near the end, showing, however, a tendency to pass along the axis of the tube towards the positive electrode. The positive light, on the other hand, gradually draws inwards, till at last it is only a star on the end of the electrode, which now disintegrates, owing to the great tension.

The temperature at the two electrodes is, in general, very different. The true explanation of this difference has not been made out, although it is doubtless connected with the equally unexplained differences in the light phenomena. A general rule has been laid down, that the temperature of the negative electrode is always higher when the discharge takes place through the gas alone, and the tempera-

¹ Maxwell, *Electricity and Magnetism*, i. § 55.

² The reader will not forget the exact sense in which we use the word tension. Of course, low tension does not mean low potential.

ture of the positive electrode higher when the discharges pass mainly through particles of disintegrated metal. The former case is commoner in vacuum tubes, where the negative electrode may get white hot, and even melt, while the positive electrode remains quite dark. The latter case is exemplified in the voltaic arc, in which great disintegration of the positive electrode is accompanied by a higher temperature there. Attempts have been made to investigate the temperature in different parts of the tube, and it seems to have been made out that the temperature is lower in the dark intervals than elsewhere.

When the electromotor is an induction coil, which furnishes discharges alternately in opposite directions, there will be a mixture of positive and negative light at each electrode, unless the maximum tension corresponding to the inverse discharge be so small that the direct discharge alone can break through. If, however, the tube be examined by means of a rotating mirror, or if it be itself fastened to a rotating arm, the images of the different discharges will be separated, and it will be seen that the appearances at each electrode alternate.

Again, when a Leyden jar is discharged through a vacuum tube, the appearances at the two electrodes are often very much alike, particularly when the resistance of the discharging circuit is very small. When the resistance is increased by introducing a column of water or lengths of wetted string, the appearances are similar to those indicated in our summary description. The reason of this is fully explained by the observations of Feddersen. He examined the spark of a Leyden jar by means of a rotating concave mirror. The machine which drove the mirror had a contact-maker, which brought on the discharge when the mirror was at a definite position; the image of the spark was thus thrown by the mirror on a piece of ground glass or a photographic plate, properly placed to receive it. He found that the discharge assumed three distinct characters as the resistance of the discharging circuit was gradually decreased.

1. The discharge was *intermittent*, that is to say, consisted of a series of partial discharges all in the same direction, following each other at more or less irregular intervals.

2. When the resistance was reduced to a certain extent, the discharge became *continuous*. The image of the spark on the plate had then the form of an initial vertical strip, with two horizontal stripes extending from each end, and gradually thinning off to a point. The vertical strip indicates a single initial spark, and the horizontal bands the finite duration of the light from the glowing metal particles, &c., near the electrodes.

3. When the resistance is very small, the discharge is *oscillatory*, i.e., consists of a succession of discharges alternately in opposite directions. These oscillations are due to the self-induction of the discharging circuit; we shall examine the matter more carefully under Electromagnetic Induction.

It is obvious that when the discharge is either *intermittent* or *continuous*, the luminous phenomena will be of the normal form sketched above, but when the discharge is *oscillatory* there will be a mixture of positive and negative appearances at each electrode, the independent existence of which cannot be detected by the unaided eye.

This is the place to remark that it is rarely that the discharge is of the simple form (2), i.e., consists of a single continuous discharge; in by far the great majority of cases it consists of a series of partial discharges. With the inductorium, both varieties (1) and (3) may occur according to the length of the air space, the resistance of the whole secondary circuit, and so on. A number of very beautiful experiments have been made to illustrate these principles, which it would take us beyond our limits to describe. Good summaries of the results of Felici, Cazin and Lucas, Donders and Nyland, Ogden Rood and Alf. Mayer, will be found in Mascart and Wiedemann. Recent researches of a very important character have been made by Wallner¹ and Spottiswoode² on the discharge in vacuum tubes. They employ the rotating mirror. It would be premature to attempt to sum up or criticise their results, suffice it to say that they show an amount of agreement which augurs well for the future of this branch of electrical science. The *strise* seem, according to them,

to play a more essential part in the phenomenon than was perhaps previously expected. Spottiswoode, in fact, seems to incline to the view that all discharges having a dark interval are really stratified, although, owing to their rapid motion, the strata may not be distinguishable by the eye alone.

In connection with this subject it may be well to mention Wheatstone's early experiments of Wheatstone,³ to determine the so-called velocity of electricity in conducting circuits. Six balls, 1, 2, 3, 4, 5, 6, were arranged in a straight line on a board; 2 and 5 were connected with the coatings of a charged Leyden jar; discharge passed by spark from 2 to 1, then through a large metallic resistance to 3, thence by spark to 4, then through a large metallic resistance to 6, and thence by spark to 5. It was found, as Feddersen observed later, that the introduction of the metallic resistance increased the duration of the sparks at all the intervals, so that the images in the mirror were *lines* of small length; but, in addition, the spark between 3 and 4 began a little later than the sparks at 1, 2 and 5, 6, which were simultaneous. From this the velocity of electricity has been calculated, by taking the interval⁴ between the sparks to be the time which the electricity takes to travel through the metal wire between the intervals. Faraday long ago pointed out that this interval depends on the capacity of the wire, and may vary very much according to circumstances. It is very great in submarine telegraph wires for instance (*vide supra*, p. 36). Accordingly, the values of the so-called velocity of electricity, which have been found by different observers, differ extremely.

The sketch we have just given of the disruptive discharge in rarefied gases must be regarded as the merest outline. There are many points of great importance to which we have not even alluded. Hittorf's investigation on what has been called the "resistance" of different parts of a vacuum tube during the discharge has not been mentioned, although it led to results of much interest, which must come to be of great importance when the clue to an explanation of the whole phenomena has been found. The reader who desires to study the matter will find in Wiedemann an excellent account of Hittorf's work, with references to the original sources. We have not so much as raised the delicate and difficult questions concerning the spectroscopic characteristics of the discharge. A good part of this subject belongs indeed more properly to the science of Light.

Miscellaneous Effects, chiefly Mechanical.—Owing to the heat suddenly developed by the electric spark, and perhaps to a specific mechanical effect as well, there is a sudden dispersion in all directions of the particles of the dielectric. This commotion may be shown very well by means of Kinnersley's older form of the thermo-electrometer; or Gauss's instrument may be used if we replace the thin wire by a couple of spark terminals. When the spark passes, the liquid in the stem sinks suddenly through a considerable distance, even if the spark be of no great length (2 to 3 mm.).

Very curious effects are obtained when an electric spark is repeated *Strise* several times at a little distance above a plate strewed with finely from powdered chalk. After a time the chalk is seen to be divided by a con- network of fine lines, resembling the markings on shagreen. If a cushion. plate of glass be covered with powdered charcoal, and the spark passed through the powder, it arranges itself in a series of *strise* closely resembling those seen in a vacuum tube.

The power of the spark to induce chemical combination (in particular, combustion) is due no doubt mainly to its high temperature.

The discharge through non-conducting liquids may take place in the form of spark or brush. The brush, however, is poor compared with that obtained in air, and is very hard

¹ *Phil. Trans.*, 1834.

² A better statement would be "the time that elapses before sufficient electricity has reached 3 and 4 to raise the tension at their nearest points to the disruptive limit."

³ *Pogg. Ann.*, "Jubelbd.," 1874.

⁴ *Proc. R. S.*, 1875-6, 7.

to get. When the spark passes, pressure is suddenly transmitted through the fluid in all directions, and if it be inclosed in a tube the tube is generally broken, even when the spark is by no means long. When the surface of the liquid is free, a considerable portion is usually projected into the air. The convective discharge is very marked in liquids. If two small balls connected with the electrodes of a Holtz's machine in action be dipped in paraffin oil at a small distance apart, the whole liquid is thrown into violent motion by the convection currents, runs up the wires which lead to the balls, and spouts off in little jets.

There is also a distinct heaping up of the liquid between the balls, and if one of them be gradually withdrawn from the liquid, for a centimetre or so it raises a column after it, which adheres until the machine is stopped. It is very probable that other effects due to the alteration of the apparent surface tension, owing to the difference of electrical stress in the air and oil, are present in these phenomena, but this is hardly the place to discuss the matter.

The electric discharge passes with great facility through card-board and other bodies of loose texture. In all probability the air in such cases has quite as much to do with the resulting effects as the solid body.

Lullin's experiment.

A curious experiment of this kind is often made. Two points are arranged so as to touch the opposite sides of a piece of card-board. If the points be opposite each other, the discharge passes straight through, leaving in the case of small charges a tiny hole with burnt edges. If, however, the points be not opposite each other, the perforation occurs in the neighbourhood of the negative point. The peculiarity is no doubt connected with those differences between positive and negative discharges in air which we have several times noticed above. In fact, it is found that in an exhausted receiver the card is pierced at a spot very nearly equidistant from the two points.

Discharge in solids.

In other cases the main part of the dielectric strength depends on the solid material. The power of such bodies to sustain the electrical tension is often very considerable. Yet there is a limit at which they give way. A thickness of 6 centimetres of glass has been pierced by means of a powerful induction coil.

In such experiments special precautions have to be taken to prevent the spark from gliding over the surface of the glass instead of going through; this is managed in some cases by embedding the glass along with the terminals of the coil in an electrical cement of considerable insulating power; in ordinary experiments, however, it is in general sufficient to place a drop of olive oil round one of the terminals where it abuts on the glass. The appearance of the perforations depends considerably on the quantity of electricity that passes in the discharge. In some cases the glass cracks or even breaks in pieces. In some large blocks we have seen a perforation in the form of several independent threads, each of which had a sort of beaded structure, which may possibly be in some way analogous to the stratifications in vacuum tubes.

Surface electrification.

Discharge along the Surface of a Body, Dust Figures, and Dust Images.—The class of phenomena referred to under this head are remarkable for the methods by which they are usually demonstrated. They were at one time much studied on account of the light they were supposed to throw on the nature of the so-called electric fluid or fluids. Though no longer regarded in this light, they have reference to an extremely important and comparatively little studied subject, viz., the distribution of electricity over the surface of non-conductors. It is easy to see that the demonstration of surface electrification on insulators is beset with difficulties of a peculiar kind. A very convenient method is to project on the surface a powder electrified in a known way; this powder clings to the parts oppositely electrified to itself, and avoids those similarly electrified, so that the state of the surface is seen at once. Lycopodium seed and powdered resin have been used in this way; they are sifted through linen cloth, the lycopodium becoming thereby weakly positive, and the powdered resin strongly negative. If the lycopodium be used, it covers both positive and negatively electrified patches, only the latter more thickly than the former.

The powdered resin, on the other hand, covers the positive and avoids the negative regions. The most effective powder, however, is a mixture of flowers of sulphur¹ and red lead. In the process of sifting, the red lead powder becomes positively and the sulphur negatively electrified, and the powders separate themselves. The sulphur colours positive regions yellow, and the red lead colours negative regions red. The result is very striking; and the test is found to be very delicate.

The dust figures of Lichtenberg are one of the best known instances of the kind of experiment indicated above. A sharp-pointed needle is placed perpendicular to a non-conducting plate, with its point very near to or in contact with the plate. A Leyden jar is discharged into the needle, and the plate is then tested with the powder. If the electricity communicated to the needle was positive, a widely extending patch is seen on the plate, consisting of a dense nucleus, from which branches radiate in all directions. If negative electricity was used, the patch is much smaller and has a sharp circular boundary entirely devoid of branches. This difference between the positive and negative figures seems to depend on the presence of the air; for the difference tends to disappear when the experiment is conducted in vacuo. Riess explains it by the negative electrification of the plate caused by the friction of the water vapour, &c., driven along the surface by the explosion which accompanies the disruptive discharge at the point. This electrification would favour the spread of a positive, but hinder that of a negative discharge. There is, in all probability, a connection between this phenomenon and the peculiarities of positive and negative brush and other discharge in air; Riess, indeed, suggests an explanation of the latter somewhat similar to the above.

There is another class of figures, to which Riess gives the name of electric images, of which the following may be taken as a type. A signet or other engraved piece of metal is placed on a plate of insulating material, and steadily electrified by means of a dry pile or otherwise positively or negatively for half an hour or so. When the metal is removed and the plate dusted, an exact figure of the stamp appears, consisting of a red or yellow background on which the engraved lines stand out free from dust. There is no difference between positive and negative electricity here as far as form is concerned, and the colour of the figure indicates charge on the plate opposite to that on the metal. The phenomenon appears to be due simply to the electrification of parts of the non-conducting surface opposite the metal.

Another class of phenomena, to which Riess gives the name of secondary, depend, not on the electrification of the surface, but on of permanent alterations produced by the discharge, whether in the form of spark or otherwise. Sometimes these are directly visible to the eye or touch, e.g., the roughening and discoloration which mark the path of the spark over a polished glass surface. In some cases they are chemical alterations, which may be shown by means of the proper reagents, e.g., the separation of the potash in the spark traces on glass. In certain cases they become evident on breathing upon the glass; of this description are the images of Karsten. A piece of mirror glass is placed on an uninsulated metal plate, and on the glass is placed a coin or medal. Sparks are taken for some time between the coin and an electric machine, and then the glass plate is removed and breathed upon. A representation of the coin then appears on the glass, often complete to the smallest detail. The reader who is interested in these matters, historically or otherwise, will find a variety of information, with directions how to find more, in Riess's *Reibungselectricität*, Bd. ii. § 739 sqq.

*Electromagnetism and Electrodynamics.*²

Mention has already been made of the discovery of Oersted, that the electric current exerts a definite action on a magnetic needle placed in its neighbourhood. This dis-

¹ First used by Villarsy in 1788.

² Throughout this section the reader is supposed to be familiar with the experimental laws of magnetism (see art. MAGNETISM). If he desires fully to understand the mathematical developments that occur here and there, an occasional reference to the analysis used in the theory of magnetism will also be necessary, if he is not already familiar with it.

covery formed the starting-point of that division of electrical science with which we are now to deal. It was natural, once the action of a current¹ on a magnet was observed, to look for the reaction of the magnet on the current, and after seeing two currents act on the same magnet, it was reasonable to expect that the currents would act on each other. Yet it may be doubted whether the first of these results is a legitimate deduction from the discovery of Oersted, and the second certainly is not so. Before we can apply the principle of the equality of action and reaction we must be quite certain of the source of the *whole* of any action to which the principle is to be applied. Again, two bodies A and C may act on B owing to properties acquired by virtue of B's presence, so that in the absence of B they need not necessarily act on each other. A good example is the case of two pieces of perfectly soft iron, each of which will act on and be acted on by a magnet, but which will not act on each other when the magnet is not near them.

The questions thus raised by Oersted's discovery were experimentally settled by Ampère. He found that a magnet or the earth (which behaves as if it were a magnet) acts on the current, and the direction of these actions is found to be consistent with the principle of equality of action and reaction. As no experimental fact has yet been quoted against the application of this principle in such cases, we shall assume it henceforth. Ampère also discovered the action of one electric current on another, and thereby settled the second question. We may conclude, therefore, that the space surrounding an electric current is a field of magnetic force just as much as the space around a magnetized body.

The next step is to determine the distribution of magnetic force, or what amounts to the same thing, to find a distribution of magnetism which shall be equivalent in its magnetic action to the electric current. This also was completely accomplished by Ampère. In expounding his results we shall follow the order of ideas given by Maxwell,² which we think affords the simplest view of the matter, and is the best practical guide that we know of through the somewhat complicated relations to which the subject introduces us. We shall in addition give a sketch of the actual course which was followed by Ampère, and which is adhered to by the Continental writers of the present day.

It results alike from the fundamental experiments of Ampère and the elaborate researches of Weber, to both of which we shall afterwards allude, that an electric current circulating in a small plane closed circuit, acts and is acted upon magnetically exactly like a small magnet placed perpendicular to its plane at some point within it,³ provided the moment of the magnet be equal to the strength of the current multiplied by the area of the circuit,⁴ and its north pole be so placed that the direction of the axis of the magnet (from S-pole to N-pole), and the direction in which the current circulates are those of the translation and rotation of a right-handed (ordinary) screw which is being screwed in the direction of the axis. In this statement we have spoken of a *small* closed circuit. The word "small" means that the largest dimensions of the circuit must be infinitely smaller than its distance from the nearest magnet or electric current on which it acts, or by which it is acted on.

We may break up our small magnet into a number of similar magnets, and distribute them over the area of the small circuit, so that the sum of the moments of all the magnets on any portion ω of the area is $i\omega$, where i is constant. We thus replace the circuit by a "magnetic shell" of strength

i , which, if we choose, may be represented by two layers parallel to the area, one of north the other of south magnetism, the surface density of which is i/θ , where θ is the distance between the layers.⁵

Starting from the principle thus laid down we can derive the laws of the mutual action of magnets and electric currents.

Consider any finite circuit ABC (fig. 29). Imagine it filled with a surface of any form, and a network of lines drawn on the surface as in the figure, dividing it up into portions, such as $abcd$, so small that they may be regarded as plane. It is obvious that any current of strength i circulating in ABC may be replaced by a series of closed currents, each of strength i circulating in the meshes (such as $abcd$) of the network on the surface; for in each line such as bc we have two equal and opposite currents circulating whose action must be *nil*. Now, we may replace each of the small circuits by a magnet as above, or by a magnetic shell of strength i . The assemblage will constitute a magnetic shell of strength i filling up the circuit, whose magnetic action, at every point *external*⁶ to the shell will be the same as that of the current. The north side of the shell is derived from the direction of the current by the right-handed screw relation given above.

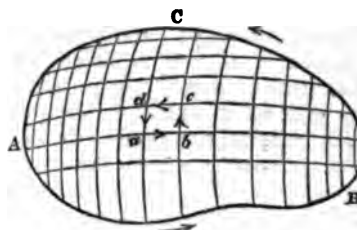


Fig. 29.

If dS be an element of the surface of a magnetic shell of strength i , D its distance from P , and θ the angle which the positive direction of magnetization (which is normal to dS) makes with D , then the magnetic potential⁷ at P is given by

$$v = i \iint \frac{\cos \theta}{D^2} dS \quad (1),$$

the integration extending all over S .

When properly interpreted this double integral is found to represent the "solid angle" subtended at P by the surface S , or, as it may also be put, by the circuit ABC which bounds it. Hence, solid angles subtended by the north side being taken as positive, and the usual conventions as to sign adhered to, we may write

$$V = i\omega, \quad (2),$$

where ω is the solid angle in question.

We see, therefore, that the potential of a magnetic shell at any point P is equal to the product of the strength of the shell into the solid angle subtended by its boundary at P . Now the potential of such a shell is continuous and single-valued at all points without it. (With points within it we are not now concerned, since the action of the current at such points is not the same as that of the shell.) If, therefore, a unit north pole start from any point P and return to the same, after describing any path which does not cut through the shell, i.e., does not embrace the current, the work done by it will be *nil*. Let us now examine what happens if the path cuts through the shell S . Take two points P and Q , infinitely near each other, but the one P on the positive side, the other Q on the negative side of the

⁵ The reader who finds difficulty with the magnetic shell may adhere to the small magnet; it will be found sufficient for most practical purposes.

⁶ This limitation is the equivalent of the limitation *small* applied to the elementary plane circuit, and follows therefrom.

⁷ We need scarcely remind the reader that all the definitions of potential, &c., in the theory of electrostatics apply here if we substitute + and - magnetism for + and - electricity. The unit of + magnetism is sometimes called a unit north pole.

¹ "Current" is used here and in corresponding cases as an abbreviation for the "the linear conductor conveying a current."

² *Electricity and Magnetism*, vol. ii. §§ 475, &c.

³ Naturally the centre of the area if it is symmetrical.

⁴ We shall see directly what system of units this statement presupposes.

shell. In passing from P to Q, without cutting the shell, the solid angle ω decreases by 4π infinitely nearly. Now, during the passage from Q to P we may not represent the action of the current by S, but nothing hinders us from representing its action by another shell S', which does not pass between Q and P, but is at a finite distance from either of them; for it will be remembered that the shell which represents the action of a current i is definite to this extent merely—that its strength is i , its boundary is the circuit, and it does not pass through the point at which the action is being considered. But infinitely little work, owing to the action of S', is done in passing from Q to P. Hence the work done by a unit pole in going once completely round any path which embraces the current once is $4\pi i$.

To reconcile this result with the continuity of the magnetic potential of a linear circuit, for the existence of which we have now furnished sufficient evidence, we must admit that the potential of a linear circuit at any point P is $V = i(\omega + 4n\pi)$, where n is any integer. In other words, V is a many-valued function differing from i times the solid angle subtended at P by a multiple of 4π . If we pass along any path from P and return thereto, the difference of the values of V , or the whole work done on the journey, is zero if the path does not embrace the circuit, $4\pi i$ if it embraces¹ it n times.

The considerations enable us to determine the action of any closed current on a magnetic pole, and consequently on any magnetic system. We have next to find the action on a linear circuit when placed in any given magnetic field, whether due to magnets or electric currents. This we do by replacing the circuit acted on by its equivalent magnetic shell.

If the potential at any point of the magnetic field be V , then the potential energy of a magnetic shell S, of strength i , placed in the field is given by

$$M = i \iint \left(l \frac{dV}{dx} + m \frac{dV}{dy} + n \frac{dV}{dz} \right) dS, \quad (3)$$

where (l, m, n) are the direction cosines of the positive direction (south to north) of the normal to the element dS . Since, so long as the magnetic force considered is not due to S itself, there is none of the magnetism to which V is due on S, we may write $-a, -b, -c$ for $\frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz}$, where a, b, c are the components of the magnetic induction.² Then, if $N = \iint (la + mb + nc) dS$ (i.e., —the surface integral of magnetic induction, or the number of lines of magnetic force which pass through the circuit), we may write

$$M = -iN \quad (4).$$

From this expression for the potential energy of the equivalent magnetic shell we can derive at once the force tending to produce any displacement of the circuit regarded as rigid.

Thus let ϕ be one of the variables which determine the position of the system, then the force Φ tending to produce a displacement $d\phi$ is given by $\Phi d\phi + dM = 0$, or

$$\Phi = -i \frac{dN}{d\phi} \quad (5).$$

Hence the work done during any displacement of a closed circuit, in which the current strength is i , is equal to i times the increase produced by the displacement in the number of lines of force passing through the circuit. The force tends, therefore, to produce the displacement or to resist it, according as the displacement tends to increase or to diminish the number of lines of force passing through the circuit. It is evident, therefore, that a position of stable equilibrium will be that in which the number of lines of magnetic force passing through the circuit is a

maximum. If that number is a minimum, we have a case of unstable equilibrium.

Maxwell³ has shown how we may deduce from the Action above theory the force exerted on any portion of the circuit which is flexible or otherwise capable of motion. "If a portion of the circuit be flexible so that it may be displaced independently of the rest, we may make the edge of the shell capable of the same kind of displacement by cutting up the surface of the shell into a sufficient number of portions connected by flexible joints. Hence we conclude that, if by displacement of any portion of the circuit in a given direction the number of lines of induction which pass through the circuit can be increased, this displacement will be aided by the electromagnetic force acting on the circuit."

From these considerations we may find the electromagnetic force acting on any element ds of the circuit. Let PQ (fig. 30) be the element ds belonging to the arc AB of any circuit. Let P \mathfrak{B} be the direction of the magnetic induction⁴ at P, and \mathfrak{B} its magnitude. It is obvious that no motion of PQ in the plane of PQ and P \mathfrak{B} will increase or diminish the number of lines of force passing through the circuit; consequently no work will be done in any such displacement. Hence the resultant electromagnetic force R must be perpendicular to the plane PQR. Let PR be a small displacement perpendicular to this plane, the work done in the displacement is R.PR, and the number of lines of force cut through is i times the rectangular area PQR multiplied by the component $\mathfrak{B} \sin \theta$ of the magnetic induction perpendicular to it. Hence we have

$$R \times PR = i ds \times PR \times \mathfrak{B} \sin \theta, \quad (6).$$

Hence the resultant electromagnetic force on the element ds may be determined as follows:—Take P \mathfrak{B} in the direction of the resultant magnetic induction (magnetic force) and proportional to $i\mathfrak{B}$, and take PQ in the direction of ds and proportional to it; the electromagnetic force⁵ on the element of the circuit is proportional to the area of the parallelogram whose adjacent sides are P \mathfrak{B} and PQ, and is perpendicular to it. The force in any direction making an angle ϕ with the direction of the resultant is of course $R \cos \phi$. The following consideration is convenient for determining which way the resultant force acts. It is obvious that the force on the element will be the same to whatever circuit we suppose it to belong, so long as the direction and strength of the current in it is the same. Take, then, a small circuit PQR perpendicular to the lines of magnetic induction (magnetic force) near PQ, in such a way that the direction of the current in PQR (as determined by the direction in PQ) is related to the direction of the magnetic induction in the same way as rotation and translation in right-handed screw motion; then the element PQ tends to move so that the number of lines of force passing through PQR increases.⁶

³ *Electricity and Magnetism*, vol. ii. § 490.

⁴ "Resultant magnetic force," if there is none of the magnetism producing it at P.

⁵ We need scarcely remind the reader that this is a ponderomotive force acting on the matter of the element of the circuit. There is no question of force acting on the current or the electricity in it.

⁶ From this may be derived the following, which is often very convenient. Stand with feet on PQ and body along the positive direction of the line of magnetic force and look in the direction of the current, then the force is towards the right hand.

¹ On the space relations involved here see Maxwell, vol. i. § 17, &c.

² Magnetic induction is used here in Maxwell's sense. It coincides in meaning with "magnetic force" at points where there is no magnetism. "Line of force" in Faraday's extended sense is synonymous with "line of induction" in Maxwell's sense.

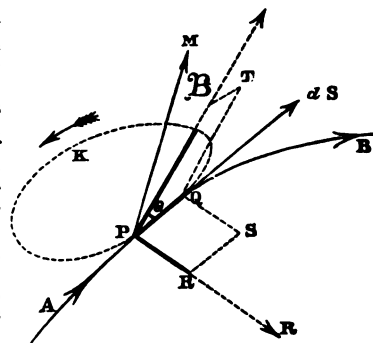


Fig. 30.

Several other ways of remembering this direction might be given. Although the above may sound arbitrary and look clumsy at first, yet we have found it more convenient in practice than some others we have tried.

We may extend what has been said above to the case where part of the magnetic force, it may be the whole of it, is due to the current in the circuit itself; for we might suppose the magnetic field to be that due to a shell whose boundary coincides infinitely nearly with the circuit. If the circuit is rigid, there will of course be no motion caused by its own action; but if it be flexible, there may be relative motions; in fact each portion will move until the number of lines of force that pass through the circuit is the greatest possible consistent with the geometrical conditions.

It is an obvious remark, after what has been said, that the potential energy of the magnetic shell which represents a current depends merely on its boundary, or, in other words, that the magnetic induction or the number of lines of magnetic force which pass through a circuit depends merely on its form. Hence we should expect to find some analytical expression for the surface integral of magnetic induction depending merely on the space relations of the circuit; in other words, we should expect to find a line integral to represent it. And when the field is that of another circuit, we should expect to find a double line integral for the mutual potential energy of the two representative shells.¹ We shall describe briefly how these expectations are realized.

In the first place, a vector may be found which has the property that its line integral taken round any circuit is equal to the surface integral of magnetic induction taken over any surface bounded by the circuit.² This vector has been called by Maxwell the "vector potential" (\mathbf{A}). Let its components be F, G, H . Then applying the definition to small areas $dydz, dzdx, dxdy$, at the point xyz perpendicular to the three axes,³ a, b, c being components of magnetic induction as before, we get

$$a = \frac{dH}{dy} - \frac{dG}{dz}, \quad b = \frac{dF}{dz} - \frac{dH}{dx}, \quad c = \frac{dG}{dx} - \frac{dF}{dy} \quad (7).$$

These equations might be used to determine F, G, H , and would lead to a much more general solution than is here required. The following synthetical solution is simpler.

Consider a magnetized particle m at O (fig. 31). Let the positive direction of its axis be OK , and let its moment be m . The resultant force due to m at any point P is in a plane passing through OK ; hence the vector potential \mathbf{A} at P must be perpendicular to this plane. Let its direction be taken so as to indicate a rotation round OK , which with translation along OK would give right-handed screw motion. Describe a sphere with O as centre and $OP (=D)$ as radius. Let PQ be a small circle of this sphere whose pole is K . Consider the line integral round PQ , and the surface integral over the spherical segment PKQ . Since \mathbf{A} is the same at all points of PQ by symmetry, the former is $2\pi D \sin \theta \mathbf{A}$, and the latter is

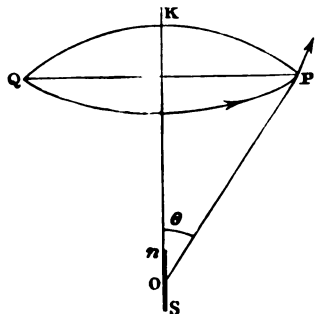


Fig. 31.

¹ It is important to remark here that we say "of the two representative shells," not "of the two circuits," or "of the two currents" (see below, p. 76).

² The mathematical idea concerned here seems to have been originally started by Prof. Stokes; it is deeply involved in the improvements effected in the theories of hydrodynamics, elasticity, electricity, &c., by Stokes, Thomson, Helmholtz, and Maxwell.

³ It is to be noted that the rectangular axes here used are drawn thus:— oz horizontal, or vertical (in plane of paper say), and oy from the reader; thus— $\begin{matrix} y \\ \swarrow \\ z \\ \searrow \\ x \end{matrix}$ In this way rotation from y to z and translation along oz give right-handed screw motion, and so on in cyclical order.

$\frac{2\pi m \sin \theta}{D}$. Equating these we get for vector potential of m at P

$$\mathbf{A} = \frac{m}{D^2} \sin \theta \quad (8),$$

its direction being that already indicated.

Suppose now the particle m placed at $Q(xyz)$ so that the direction cosines of m are λ, μ, ν . Let the coordinates of P be ξ, η, ζ ; also let $QP = D = \sqrt{(\xi-x)^2 + (\eta-y)^2 + (\zeta-z)^2}$. Then the direction cosines of QP are $D \frac{dx}{d\xi}, D \frac{dy}{d\eta}, D \frac{dz}{d\zeta}$, where $p = \frac{1}{D}$; and we get for the component of the vector potential at P

$$F = m \left(\mu \frac{dp}{d\xi} - \nu \frac{dp}{d\eta} \right) \quad (9),$$

and two similar expressions for G and H .

The vector potential of a magnetized body may be got by compounding the vector potentials of the different elements; hence, λ, μ, ν being the components of magnetization at any point of the body, we get

$$F = \iiint \lambda \left(\mu \frac{dp}{d\xi} - \nu \frac{dp}{d\eta} \right) dxdydz \quad (10),$$

and two similar expressions for G and H . The first part of our problem is thus solved.

Let us, in the second place, apply the above result (10) to the case of the two shells which are equivalent to two currents. In a lamellar distribution of magnetism $\frac{d(\lambda\mu)}{dx} = \frac{d(\lambda\nu)}{dy}$, &c.; hence the volume integral in (10) reduces to a surface integral, and

$$F = \iint \frac{1}{D} (\mu n - \nu m) dS \quad (11),$$

where λ, m, n are the direction cosines of the outward normal to dS .

Now the magnetic shell of thickness τ and strength i is a lamellar magnetized body of constant intensity i/τ . It may be looked upon as bounded by two parallel surfaces normal everywhere to the lines of magnetization, and by an edge generated by lines of magnetization. At every point on either of the parallel surfaces we have therefore $\lambda = \lambda, m = \mu, n = \nu$; and at the edge $\lambda = \nu \frac{dy}{ds} - \mu \frac{dz}{ds}$, and similarly for m and n . Hence every element of the double integral in (11) belonging to either of the parallel surfaces vanishes, and there remain only the parts on the edge which give

$$F = \frac{i\tau}{2} \left\{ \mu \left(\nu \frac{dx}{ds} - \lambda \frac{dy}{ds} \right) - \nu \left(\lambda \frac{dx}{ds} - \mu \frac{dy}{ds} \right) \right\} ds = i \int \frac{1}{D} dx ds \quad (12),$$

since $\lambda \frac{dx}{ds} + \mu \frac{dy}{ds} + \nu \frac{dz}{ds} = 0$. (12) gives the vector potential at $(\xi\eta\zeta)$ due to a magnetic shell S . Let $(\xi\eta\zeta)$ be any point on the boundary of another shell S' , of strength i' , and let $d\sigma$ be the element of arc of the boundary, then

$$-i' \int \left(F \frac{d\xi}{d\sigma} + G \frac{d\eta}{d\sigma} + H \frac{d\zeta}{d\sigma} \right) d\sigma \quad (13)$$

is the magnetic induction through S' due to S with the sign changed, in other words, the mutual potential energy M . Putting for F, G, H their values by (12), we have

$$M = -i' \iint \frac{1}{D} \left(\frac{dx}{ds} \frac{d\xi}{d\sigma} + \frac{dy}{ds} \frac{d\eta}{d\sigma} + \frac{dz}{ds} \frac{d\zeta}{d\sigma} \right) ds d\sigma$$

$$= -i' \iint \frac{\cos \epsilon}{D} ds d\sigma \quad (14),$$

Double line integral for M .

where ϵ is the angle between ds and $d\sigma$.

The result of (14) realizes the second of our expectations. The double integral arrived at is of great importance, not only in the theory of electrodynamics, but also as we shall see in the theory of the induction of electric currents.

Hitherto we have spoken only of closed circuits, and considered merely the action of a circuit regarded as a whole. When we did speak of the force on an element of a circuit, we deduced this force directly from the state of the magnetic field in its immediate neighbourhood. There is an order of ideas, however, in which the mutual action of two circuits is considered to be the sum of all the mutual actions of every element in one circuit on every element in the other. Now, we can easily show, by means of (14), that a system of elementary forces of this kind can be found which will lead to the same result for closed circuits as the theory given above.

Let the circuit S' be supposed rigid and fixed, and let the circuit S be movable in any way with respect to S' ; it may even be flexible.

Denote the angles between the positive directions of $d\sigma$ and ds and the direction of D from $d\sigma$ to ds by θ' and θ , then we have

$$\left. \begin{aligned} \cos \theta &= \frac{dD}{ds}, \quad \cos \theta' = -\frac{dD}{d\sigma}, \\ \cos \epsilon &= -\frac{dD}{ds} \frac{d\sigma}{ds} - D \frac{d^2 D}{ds d\sigma} \end{aligned} \right\} \dots (15).$$

By means of these we get

$$M = i' \iint \frac{1}{D} \frac{dD}{d\sigma} \frac{dD}{ds} d\sigma ds, \dots (16).$$

The part which is a complete differential has been left out, because it disappears when the integration is carried round closed circuits, as we always suppose it to be. Consider now the work done in a small displacement which alters D and S , $\frac{dD}{d\sigma}$, $\frac{dD}{ds}$, and ds , but not $d\sigma$; we have

$$\begin{aligned} \delta M &= -i' \iint \frac{1}{D^2} \frac{dD}{d\sigma} \frac{dD}{ds} \delta D d\sigma ds + i' \iint \frac{1}{D} \frac{d\delta D}{d\sigma} \frac{dD}{ds} d\sigma ds \\ &\quad + i' \iint \frac{1}{D} \frac{dD}{d\sigma} \left(\frac{d\delta D}{ds} - \frac{dD}{ds} \frac{d\delta s}{ds} \right) d\sigma ds \\ &\quad + i' \iint \frac{1}{D} \frac{dD}{d\sigma} \frac{dD}{ds} \frac{d\delta s}{ds} d\sigma ds. \end{aligned}$$

The parts containing δs disappear in this expression, and if the rest be arranged by integration by parts as usual, we get

$$\delta M - \iint R \delta D ds d\sigma = 0 \dots (17),$$

where $R = i' \frac{2 \cos \epsilon - 3 \cos \theta \cos \theta'}{D^2}$.

Hence the electro-dynamical action of the two circuits is completely accounted for by supposing every element $d\sigma$ to attract every element ds with a force

$$\frac{i' ds d\sigma}{D^2} (2 \cos \epsilon - 3 \cos \theta \cos \theta') \dots (18).$$

We may therefore use this elementary formula whenever it suits our convenience to do so.

It is very easy to obtain a similar elementary formula, which is very often useful, for the action of an element of a circuit on a unit north pole.

We have seen above how to find the action on an element PQ (ds) of a circuit in a given magnetic field. Let the field be that due to a unit north pole N (fig. 32). Then the magnetic induction at P is in the direction NPK , and is equal to $\frac{1}{D^2}$, if $NP = D$. Hence by (6) the force R on PQ is perpendicular to NP and PQ , is in the direction PM shown in the figure, and is equal to $\frac{id s \sin \theta}{D}$. Now, by the principle of "action and reaction," the force on N is R in the

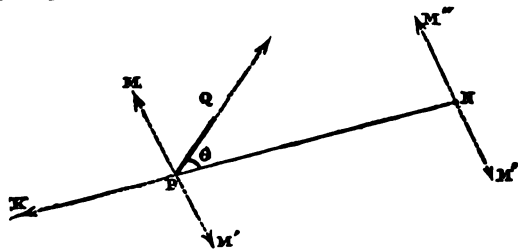


Fig. 32.

direction PM' opposite to PM , i.e. is equal to a force R acting at N in a direction NM' parallel to PM' , together with a couple whose moment is $R \times PN$, and whose axis is perpendicular to NP and in the plane NPQ . Now a simple calculation, which we leave to the reader, will show that for any closed circuit the resultant of all the couples thus introduced is $\pi i'$; hence, since we deal with closed circuits only, we may neglect the couple.

The force exerted by a closed circuit on a unit north pole may therefore be found by supposing each element ds to act on the pole with a force equal to

$$\frac{id s \sin \theta}{D^2} \dots (19),$$

whose direction is perpendicular to the plane containing the pole and the element, and such that it tends to cause rotation round the element related to the direction of the current in it by the right-handed screw relation.

¹ PQ is supposed to be drawn from the reader.

Comparison of Theory with Experiment.—The best verification of the theory which has just been laid down consists in its uniform accordance with experience. We proceed to give a few instances of its application, adopting now one, now another, of the equivalent principles deduced from it.

We have already remarked that the lines of magnetic force in an electric field due to an infinite straight current are circles having the current for axis. It is easy to deduce from the fact that there is a magnetic potential that the force must vary inversely as the distance from the current.

This may also be proved by means of the formula (19); in fact, the resultant force at P is given by

$$R = i \int \frac{\sin \theta}{D^2} ds = i \int_0^\pi \frac{\sin \theta}{d^2 \cos^2 \theta} \cos^2 \theta d\theta = \frac{2i}{d} \dots (20),$$

d being the distance of P from the current.

Let AB (fig. 33) be a very long straight current, and poq an element ds of a parallel current, having the same direction as AB . If we draw the line of force (a circle with C as centre) through O , the

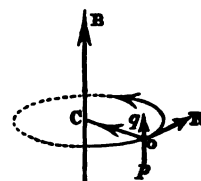


Fig. 33.

tangent OR is the direction of the force at O ; hence by (6) and (20), the force on poq is $\frac{2i}{d} ds$, and acts in the direction OC ; poq is therefore attracted. If the current in poq be reversed, the force will have the same numerical value, but will act in the direction CO . Hence two parallel straight conductors attract or repel each other according as the currents in them have the same or opposite directions.

Let AB (fig. 34) be an infinitely long (or very long) current, CD a portion of a current inclined to it, and passing very near it at O .

If the plane of the paper contain AB and CD , then at every point in OD the magnetic force is perpendicular to the plane of the paper and towards the reader, at every point in OC perpendicular to the plane of the paper and from the reader; hence at the elements P and Q the forces acting will be in the direction of the arrows in the figure, and CD will tend to place itself parallel to AB . If both the currents be reversed, the action will be unaltered; but if the current in CD alone be reversed, it will move so that the acute angle DOB increases.

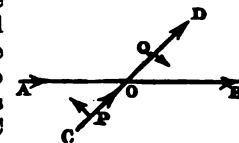


Fig. 34.

Hence it is often said that currents that meet at an angle attract each other, when both flow to or both flow from the angle, but repel when one flows to and the other flows from the angle.

These actions may be demonstrated in a great variety of ways.

Figure 35 shows an arrangement for demonstrating the attraction or repulsion of parallel currents, which is essentially that first used by Ampère. A is an upright consisting of a tube in good metallic connection with one of the binding screws t , and with a little cup p , containing a drop of mercury. A stout wire passes up the centre of the tube, and is insulated from it, but in metallic connection

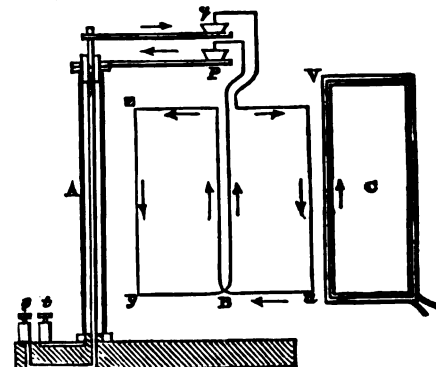


Fig. 35.

Ampère's apparatus.

tion with the screw s and the cup g . B is a light conductor,¹ consisting of two parallelograms of wire, in which the current circulates in opposite directions, the object of which is to eliminate the magnetic action of the earth. The conductor is hung in the cups

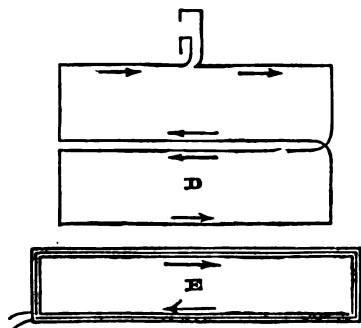


Fig. 36.

It is clear, therefore, that the action of C on uv will prevail and determine the motion.

The action of straight conductors, making an angle with each other, may be shown by means of the conductor D , represented in fig. 36, which may be fitted to the stand shown in fig. 35.

In a very large class of practical cases, circular circuits play an important part. The most convenient way of dealing with these, as a rule, is to replace them by the equivalent magnets or magnetic shells. The action of a circular circuit may be represented by two layers of north and south magnetism, whose surface densities are $\pm i/\tau$, where i is the strength of the current and τ the distance between the layers. For details concerning the calculations in a variety of cases, we refer the reader to Maxwell's *Electricity and Magnetism*, vol. ii. cap. xiv.

We may calculate the force exerted (see fig. 37) by a circular current AB on a unit north pole at its centre C , as follows. Replace the current by two discs AB and $A'B'$, of north and south magnetism, the distance between which is τ ; the surface densities are $+i/\tau$ and $-i/\tau$. The first of these exerts a repulsive force $2\pi i/\tau$, the second an attractive force

$$2\pi i/\tau(1 - \cos \frac{1}{2} A'CB);$$

hence the resultant repulsive force is

$$2\pi i \cos \frac{1}{2} A'CB + \tau = 2\pi i/\tau,$$

r being the radius of the disc. Hence a unit of length of the current exerts a force i/τ^2 at the distance r .



Fig. 37.

It follows therefore that the statement of our fundamental principle (p. 67) involves a unit of current strength such that unit length of the unit current, formed into an arc whose radius is the unit of length, exerts a unit of force on a unit pole placed at the centre of the arc. From this statement and the definition of a unit negative pole it follows at once that the dimension of the unit of current is $[L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}]$.

One arrangement of circular currents has become famous from the part it plays in Ampère's theory of magnetism. A wire wound into a cylindrical helix, such as that represented in figure 38, the ends of the wire being returned parallel to the axis of the helix, and bent into pivots, so that it can be hung upon Ampère's stand (fig. 35), is called a solenoid. The conductor thus formed is obviously equivalent to a series of circular currents disposed in a uniform manner perpendicular to a common axis. In the case represented in figure 38, this axis is straight; but the name solenoid is not restricted to this particular case,

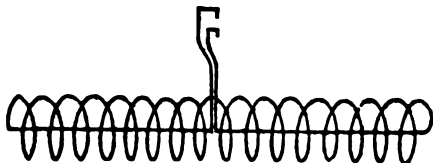


Fig. 38.

The conductor thus formed is obviously equivalent to a series of circular currents disposed in a uniform manner perpendicular to a common axis. In the case represented in figure 38, this axis is straight; but the name solenoid is not restricted to this particular case,

and what we are about to advance will apply to a solenoid whose axis is a curve of any form.

Let there be nds of the circular currents (each of area λ) in the arc ds of the axis of the solenoid. As we suppose the distribution to be uniform, n is constant. We may suppose each current to be placed at the middle of a length $\frac{1}{n}$ of the axis, which it occupies for itself. Hence, if each circular current be replaced by a shell of thickness $\frac{1}{n}$, the surface densities of the magnetism on each of these shells will be $\pm ni$, and the north magnetism of each shell will coincide with the south magnetism of the next; so that the whole action at points external to the solenoid reduces to the action of a quantity $ni\lambda$ of magnetism spread over one end of the solenoid, and a quantity $-ni\lambda$ spread over the other. The positive or north end of the solenoid is obtained, as usual, from the direction of the current, by means of the right-handed screw relation. If λ be very small, or if the system acting on, or acted upon by, the solenoid is at a distance very great compared with the dimensions of λ , then we may suppose the representative magnetism concentrated at the ends of the axis of the solenoid.

Hence the particular arrangement of electric currents, which we have called a solenoid, acts and is acted on exactly like an ideal linear magnet (whose poles coincide with the ends of its axis).

Thus the north pole of a magnet or solenoid repels the north end and attracts the south end of a solenoid; a solenoid tends to set under the action of the earth, its north end behaving like a magnetic north pole, and so on.

In a cylindrical bobbin wound to a uniform depth with silk-covered wire we have an arrangement which is equivalent to a drical number of solenoids all having a common axis. Each of these bobbin solenoids may be replaced by the equivalent terminal discs of positive and negative magnetism, and the external action of the whole thus calculated. The magnetic disc at each end will, of course, not be of uniform density,² but if the points acted on be at a distance which is infinitely great compared with the lateral dimensions of the bobbin, we may collect the magnetism at the ends of the axis; the quantities will be

$$\pm mn\pi \frac{\pi}{3} (a^2 + ab + b^2),$$

where a and b are the outer and inner radii of the shell of wire, m the number of layers in the depth, and n the number of turns per unit of length of each layer. The magnetic moment of the bobbin is therefore

$$mp\pi \frac{\pi}{3} (a^2 + ab + b^2),$$

where p denotes the number of turns in each layer, and mp the whole number of turns on the bobbin.

The above is a simple case of the kind of calculation Weber's on which Weber founded his verification of Ampère's experiments. He did not, however, replace the circular currents by the equivalent magnetic distributions, but calculated directly from Ampère's formula (18).

The instrument (electrodynamometer) which he used in his experiments was invented by himself. It consists essentially of a fixed coil and a movable coil, usually suspended in the bifilar manner, and furnished with a mirror, so that its motions about a vertical axis can be read off in the subjective manner (see art. GALVANOMETER) by means of a scale and telescope. Two varieties of the instrument were used by Weber. In one of these (A), the movable coil was suspended within the fixed coil; in the other (B), the movable coil was ring-shaped, and embraced the fixed coil, which, however, was so supported that it could be arranged either inside the movable coil or outside it at any distance and in any relative position with respect

¹ Aluminium is often used.

² The reader will easily find the law for himself.

to it. We do not propose to go into detail respecting Weber's experiments, but merely to indicate their general character and give some of the results. Those desiring further information will find it in §§ 1-9 of the *Electrodynamische Maassbestimmungen*.

Weber first showed that the electrodynamic action between two parts of a piece of apparatus traversed by the same current varies as the square of the current. Apparatus A was arranged with the plane of its fixed coil in the magnetic meridian. The movable coil was concentric with the fixed one, but its plane was perpendicular to the magnetic meridian. The current of 1, 2, or 3 Grove's cells was sent through the fixed coil and through the suspended coil; but as the deflection with this arrangement was too great, the latter was shunted by connecting its terminals by a wire of small but known resistance. A measurement of the *first power* of the strength of the current was found by observing the deflection produced by the current in the fixed coil on a magnet suspended in its plane at a convenient distance north of it. After the necessary corrections were applied, the following results were obtained:—

n	D	M	M'	Diff.
3	440.038	108.426	108.144	-0.282
2	198.255	72.398	72.589	+0.191
1	50.915	36.332	36.786	+0.454

where n is the number of cells, D the electrodynamic force on the suspended coil, expressed in an arbitrary unit, M the force on the magnet, M' the force on the magnet calculated from \sqrt{D} by means of a constant multiplier. The agreement between M and M' is within the limits of experimental error.

In another series of experiments Weber used the apparatus B described above. The suspended coil was arranged with its axis in the magnetic meridian, and the fixed coil set up with its axis perpendicular to the magnetic meridian. Experiments were made with the centres of the two coils coincident, and with the centres in the same horizontal plane, at distances of 300, 400, 500, and 600 millimetres, the fixed coil being, in one set of experiments, east or west from the suspended coil; in another set, north or south. In the present series of experiments the strength of the current was measured by means of a magnet acted on, not by the fixed coil, but by another coil in circuit with it. After proper corrections, the following results were arrived at:—

d	P	P'	Q	Q'
0	22960	22680	22960	22680
300	189.93	189.03	77.11	77.17
400	77.45	77.79	34.77	34.74
500	39.27	39.37	18.24	18.31
600	22.46	22.64

where d is the distance between the centres of the coils, P the couple¹ exerted on the movable coil when the direction of that distance is perpendicular to the meridian, Q the couple when it is in the meridian. P' and Q' are the values of the same couples calculated from the theory of Ampère. The agreement here again is as near as could be expected.

Weber further showed that the deflections (v , w) of the suspended coil, calculated by means of the formulae

$$\tan v = ad^{-3} + \beta d^{-4}$$

$$\tan w = \frac{1}{2}ad^{-3} + \gamma d^{-4}$$

in the two cases where the centres of the coils were at a considerable distance apart, agreed with observation within the limits of experimental error. Now these formulae are identical with those established by Gauss for two magnets with their axes placed like the axes of the coils. This agreement therefore is an experimental proof that the coils are replaceable by magnets.

On the whole, therefore, the experiments of Weber² confirm the theory of Ampère, as far as experiment can test it. They form, therefore, a sufficient basis for the proposition on which we founded our theory; for this proposition leads to the same result for closed circuits as the theory of Ampère.

The action of any current on a magnetic pole, and hence on any magnet, may be calculated either by replacing the circuit by an equivalent shell or by means of formula (19). We have already found this action in the particular case of an infinitely long straight current. This result was originally found experimentally by Biot

¹ Reduced to a standard current strength by means of the magnet deflections.

² For another verification by Casin, see Wiedemann, *Gale*, Bd. ii. § 44.

and Savart, and Laplace showed that it followed from their result that the force exerted by an element of the current varies inversely as the square of the distance. The fact that a circular current acts on a magnetic pole at its centre in the same way as a zig-zag current which is everywhere very nearly coincident with it, leads, when properly interpreted, to the conclusion that the force varies as $\sin \theta$. In this way formula (16) was originally arrived at, independently of Ampère's theory.

A great variety of instances might be given of the action of a Earth's magnet on a current. The earth, for instance, acts on a circular action current, hung up on Ampère's stand: the current, being movable about a vertical axis, will turn until the maximum number of the earth's lines of magnetic force pass through it—i.e., it will set with its plane perpendicular to the magnetic meridian, in such a way that the current, looked at from the north side, goes round in the opposite direction to the hands of a watch.

A very simple way of showing the action between magnets and De la currents was devised by De la Rive. A small plate of copper and a Rive's small plate of zinc are connected together by a wire passing through floating a cork and making a circuit of several turns; the cork is placed in a vessel containing dilute sulphuric acid, and floats on the surface, carrying the little circuit about with it. Such a circuit will set under the earth's action, and may be chased and turned about, &c., by a magnet. After what has been already said, however, such experiments offer no new point of interest.

Electromagnetic Rotations.—It is obvious that no On rotations is general
invariable system of electric currents can produce continuous rotation of a magnetized body. For, suppose an elementary magnet, whose action may be represented by two poles of strengths $\pm m$, to describe any path and to return exactly to its former position; either it has or has not embraced the circuit in its path; if it has not, no work has been done on either pole; if it has embraced the circuit n times, an amount of work $4\pi n m r i$ has been done on the north pole, and an amount $-4\pi n m r i$ on the south; on the whole, therefore, no work has been done on the magnet. As any magnetized body may be conceived to be made up of such elementary magnets, it is obvious that it is impossible for such a body to rotate continuously, doing work against friction,³ &c.

The same is obviously true if we replace the magnet by an invariable system of electric currents.

If, however, part of the electric circuit is movable with respect to the rest, and communicates therewith by means of sliding contacts or the like, continuous rotation of part of the circuit may occur. Again, if by any artifice the magnet can be transferred every revolution from one side of the current to the other, continuous rotation of the magnet may result. Lastly, if the direction of the current in some part of the apparatus be always reversed at a certain stage of the revolution, continuous motion may ensue.

Rotations of the first and second class were first discovered by Faraday, and the ground principle of most of the pieces of apparatus used in demonstrating them is that originally used by him.

One of the simplest cases is the rotation under the action of the vertical component of the earth's magnetic force. Let ABC (fig. 39) be a horizontal circular conductor, OP a conductor pivoted at O, having sliding contact at P with ABC. Let a current i enter ABC at A, and leave it at P, flowing through PO to O and thence to the battery again. The magnetic force at any element dr of OP is perpendicular to OP and to the plane of ABC, hence the electromagnetic force on the element will be in the plane of ABC, in the direction of the arrow p ,⁴ and will be equal to $iRdr$ (R =vertical component of earth's force). Hence the moment about O of the forces acting on OP is $\int iRrdr$, i.e. $\frac{1}{2}OP^2Ri$, which is independent of the position of OP. OP will therefore rotate about O, with an angular velocity which will go on increasing until the work lost by friction, &c., during each revolution is equal to πOP^2Ri .

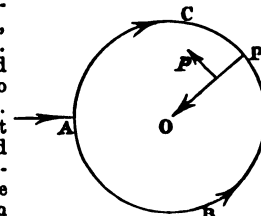


Fig. 39.

³ Maxwell, vol. i., §§ 486 and 491.

⁴ We are here supposed to be in southern latitudes.

m-
re's
theory.

Ampère has given a general theory of the rotation of a circuit under the action of a magnet. Let AB (fig. 40) be any circuit, which we may suppose connected with the axis of the magnet, but free to rotate about it. We suppose the magnet replaced by quantities $\pm m$ of magnetism at its poles. Take the axis of the magnet for axis of z , and the other axes as in the figure, O being the centre of the magnet, and let $ON=OS=c$. Let PQ be any arc ds of AB , and let the coordinates of P be x, y, z ; then if l, m, n be the direction cosines of NP , and $NP=D$, we have $Dl=x, Dm=y, Dn=z-c$; also the direction cosines of Pp , which is perpendicular to NP and PQ , and is the direction of the

Fig. 40.

force exerted by the pole N on P , are $\left(n \frac{dy}{ds} - m \frac{dz}{ds}\right) \div \sin QPK$, &c.

Hence by formula (6) the components of the force acting on PQ are

$$\frac{m}{D^2} \left(n \frac{dy}{ds} - m \frac{dz}{ds} \right) ds, \text{ \&c.}$$

Hence, if K denote the moment of these forces about OZ , we have from the north pole alone

$$K = m \int \frac{ds}{D^2} \left\{ \left(l \frac{dz}{ds} - n \frac{dx}{ds} \right) x - \left(n \frac{dy}{ds} - m \frac{dz}{ds} \right) y \right\}.$$

If we substitute the values of l, m, n this reduces to

$$K = m \int ds \frac{d}{ds} \left(\frac{z-c}{D} \right) = m \int dn.$$

If therefore $\beta_1, \alpha_1, \beta_2, \alpha_2$ denote the angles BNZ, ANZ, BSZ, ASZ , we have, adding the results from both poles,

$$K = m \{ (\cos \beta_1 - \cos \alpha_1 - \cos \beta_2 + \cos \alpha_2) \}. \quad (21).$$

It follows from this remarkable formula that the couple K tending to turn a part AB of an electric circuit about the axis of a magnet depends merely on the position of the ends A and B .

In particular, if A coincide with B , i.e. if AB form a closed circuit, or if A and B both lie on parts of the axis not included between N and S ,¹ the couple will be nil , and there will be no rotation.

The application of this formula to cases where there are sliding contacts at A and B not lying on the axis presents no difficulty; we leave it to the reader.

Several of these rotations may be exhibited by means of the apparatus represented in figure 41. ABC is a horizontal coil of wire

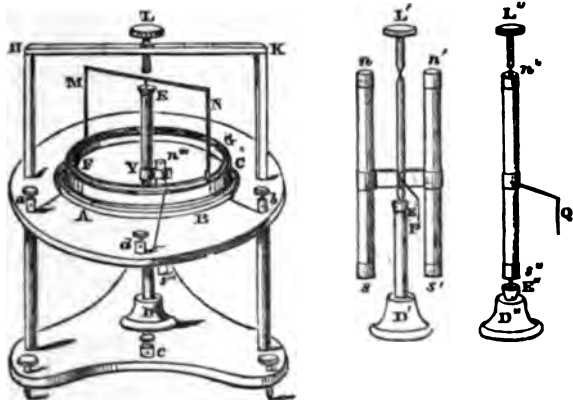


Fig. 41.

¹ We might consider what would happen if A or B lay on NS , but the case never arises in practice, for all magnets have a finite thickness (see on this subject Wiedemann, *Bil. ii.* § 119).

terminating at the binding screws a, b . FG is a ring-shaped trough of mercury for the sliding contacts. A wire connects the mercury with the binding screw d . DE is an upright support screwed into a metal base D in connection with the binding screw c , and terminating above in a mercury cup E . When required, DE can be replaced by the shorter supports $D'E'$ and $D''E''$. HLK is a support for a screw L , which carries an adjustable centre.

1. Poise in the cup E the wire stirrup MN , so that the ends just dip in the mercury trough. Then, if a strong current be sent from c to d , MN will rotate (in northern latitudes) in a direction opposite to the hands of a watch.

2. If we fix a vertical magnet $n''s''$ to DE by means of a clip at Y , then the rotation will take place with a weaker current in the same direction as before, if the north pole of the magnet be upwards (as shown in figure), but in the opposite direction if the magnet be reversed.

3. Reversing the current alone in either of the last two cases causes the direction of rotation to be reversed.

4. The magnet may be removed and a current sent from a to b round ABC in the direction opposite to the hands of a watch. The result is the same as for the magnet with its north pole upwards. If the current in ABC is reversed, the rotation is reversed; and so on.

5. The support $D'E'$ with the two magnets $ns, n's'$ may be screwed into D instead of DE , the wire P now dipping into the mercury. If the current be sent from c to d , the vertical current in $D'E'$ will act on s and s' , and cause the magnet to rotate in the direction of the hands of a watch. This rotation is reversed if the current go from d to c .

6. We may consider any magnet of finite size as made up of a series of magnets like ns and $n's'$ arranged about an axis. Hence, if we replace $D'E'$ and the magnets $D'E'$ by the single magnet supported by means of the pivot L' , there will still be rotation.

Figure 42 represents a very elegant piece of apparatus devised by Faraday, to show the rotation at once of a magnet and of a movable conductor. The rotating pieces are the magnet sm , which is tied to the copper peg at the bottom of G by means of a piece of string, and swims round the vertical current buoyed up by the mercury in G , and the wire DE , which is hinged to D by a thin flexible wire, and swims round the pole of the vertical magnet $n's'$.

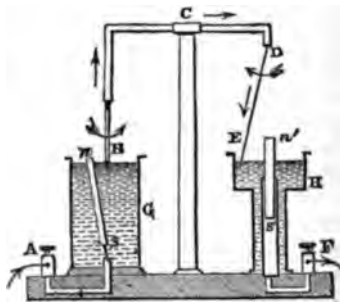


Fig. 42.

Another apparatus invented by Barlow, and known by the name of Barlow's wheel, is represented in figure 43. A current is caused to pass from the mercury trough C along the radius of the disc A through the field of magnetic force due to the

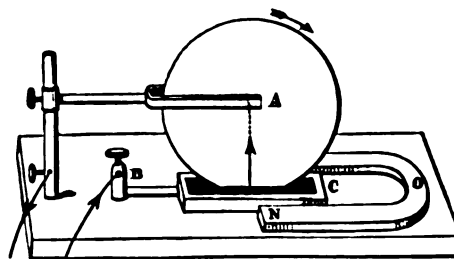


Fig. 43.

horseshoe magnet NO . The result is that the wheel rotates in the direction indicated by the arrow.

Fluid conductors may also be caused to rotate under the action of a magnet. We mentioned in our historical sketch the experiment by which Davy demonstrated this rotation in the case of mercury. A variety of such experiments have been since devised. The following is a simple one. Fill a small cylindrical copper vessel with dilute sulphuric acid and set it upon the north pole of a powerful electromagnet. If a thick zinc wire be connected by a piece of copper wire to the copper vessel, and then immersed in the acid so as to be in the axis of the vessel, a current is set up in the liquid which flows radially from the zinc to the copper across the lines of force. The

liquid therefore rotates in the direction of the hands of a watch.

Action of magnet on electric discharge.

Magnetic Action on the Electric Discharge in Gases.—A large number of very interesting results have been obtained concerning the behaviour of the electric discharge in a field of magnetic force. We can only make a brief allusion to the matter here. The key to the phenomena lies in the remark that the electric discharge in vacuum tubes may be regarded as an electric current in a very flexible elastic conductor. It is clear that such a conductor would be in equilibrium if it lay in a line of magnetic force passing through both its fixed ends. Again, if the flexible conductor be constrained to remain on a given surface, it will not be in equilibrium until it has so arranged itself that the resultant electromagnetic force at each point is *perpendicular* to the surface. At each point, therefore, the magnetic force must be tangential to the surface.¹

A perfectly flexible but inextensible conductor, two points of which are fixed, will take such a form that the electromagnetic force at each point is balanced by the tension. Le Roux fastened a thin platinum wire to two stout copper terminals, and caused it to glow by passing a current through it. When the terminals were placed equatorially between the flat poles of an electromagnet, the wire bent into the form of a circular arc joining the terminals. When the terminals were placed axially, it assumed a helical form. (See also Spottiswoode and Stokes, *Proc. R. S.*, 1875.)

Rotation of electric discharge.

The behaviour of the light emanating from the positive pole may be explained in general as lying between the two cases which we have just discussed. One of the most remarkable of these phenomena is the rotation of the discharge discovered by Walker, and much experimented on by De la Rive. This may be exhibited by means of the apparatus shown in fig. 44, consisting essentially of an exhausted vessel, one of the electrodes in which is ring-shaped; a bar of soft iron, covered with some insulating material, is passed through the ring and fixed to the stand. When this apparatus is placed on the pole of a powerful magnet, the discharge rotates as a wire hinged to the upper electrode would do.



Fig. 44.

Plücker's experiments.

Owing to the distinct character of the negative light, the action of the magnet on it is different from that on the positive light. Plücker found that the general character of the phenomena may be thus described:—The negative light is bounded by magnetic curves that issue from the electrode and cut the walls of the tube.

The two diagrams in fig. 45 will convey an idea of the

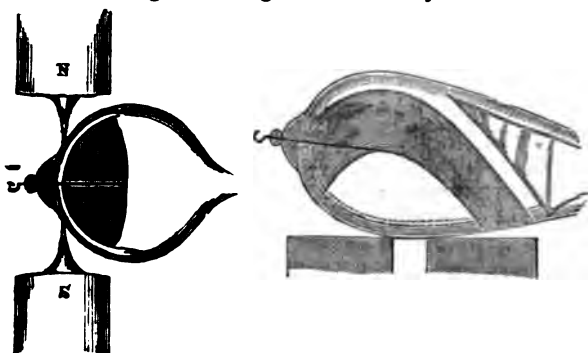


Fig. 45.

appearance of the phenomenon. Although much tempted

¹ Loci having this property were called by Plücker epipolar curves.

to follow the subject further, we must be content to refer the reader to the interesting papers of Plücker² and Hittorf.³ An excellent summary will be found in Wiedemann.

Ampère's Method.—Before quitting the subject of electromagnetism, it will be useful, for the sake of comparison, to give a brief sketch of the method of Ampère, or rather of that modification of the original method now commonly found in Continental books, which was suggested by Ampère himself, in a note to the *Théorie des Phénomènes Électrodynamiques*. Ampère starts with the idea that the electrodynamic action of two circuits is the sum of the actions at a distance between every pair of their elements. He supposes, as the simplest and most natural assumption, that the force between two elements is in the line joining them. Besides this assumption, his theory rests on four experiments.⁴ The first of these shows that, when a wire is doubled on itself, the electrodynamic action of any current in it is *nil*. The second experiment shows that this is also true, even if one of the halves of the wire be bent or twisted in any way, so as never to be far removed from the other. The third experiment proves that the action of any closed circuit on an element of another circuit is perpendicular to the element. In the fourth experiment it is shown that the force between two conductors remains the same when all the lines in the system are increased in the same ratio, the currents remaining the same. From the assumption, together with the first experiment, it follows that the force between two elements is proportional to the product of the lengths of the elements, multiplied by the product of the strengths of the currents and by some function of the mutual distance and of the angles which determine their relative position. Hence it may be shown, from the fourth experiment, that the force between the elements must vary inversely as the square of the distance between them. The second experiment shows that we may replace any element of a circuit by the projections of the element on three rectangular axes.

From these results it is found that the force between ds and $d\sigma$ must be

$$\frac{A i i' ds d\sigma}{D^2} (\cos \epsilon - k \cos \theta \cos \theta').$$

The constant k is then determined from the result of the third experiment; and it is found that k must be equal to $\frac{2}{3}$. The formula is thus completely determined, with the exception of A , which depends on the unit of current which is chosen. The action of a closed circuit on an element is then calculated, and a vector found, which Ampère calls the "directrix," from which this action can be found in exactly the same way as we derived this same action from the magnetic induction. The theory is then applied to small plane circuits, solenoids, and so on.

As was remarked in the historical sketch, a variety of other elementary laws may be substituted for that of Ampère, all of which lead to the same result for closed circuits.

Maxwell has presented Ampère's theory in a more general form, in which the assumption about the direction of the elementary action is not made. Neglecting couples, he finds for the most general form of the components of the force exerted by $d\sigma$ on ds ,

$$\left. \begin{aligned} R &= \frac{1}{D^2} \left(\frac{dD}{ds} \frac{dD}{d\sigma} - 2D \frac{d^2 D}{ds d\sigma} \right) i i' ds d\sigma + D \frac{d^2 Q}{ds d\sigma} i i' ds d\sigma \\ \text{in the direction of } D, \\ \text{and} \quad S &= - \frac{dQ}{d\sigma} i i' ds d\sigma, \quad S' = \frac{dQ}{ds} i i' ds d\sigma \\ \text{in the direction of } ds \text{ and } d\sigma \text{ respectively.} \end{aligned} \right\} \text{ (22).}$$

² *Pogg. Ann.*, ciii., clv., cvii., cxiii., 1858, &c.

³ *Pogg. Ann.*, cxxxvi., 1869.

⁴ Details respecting these experiments, and other matter connected

In these expressions Q is a function to be determined only by further assumption. $Q = \text{constant}$ gives Ampère's formula; $Q = -\frac{1}{2r}$ gives the formula of Grassmann, and so on. We may in fact construct an infinite variety of different elementary formulæ. The reader interested in this subject may consult Wiedemann, Bd. ii. §§ 26, 27, 45-54, &c., and Tait, *Proc. R.S.E.*, 1873.

In our account of the magnetic action of electric currents no mention has been made of the effect of the proximity of soft iron. Under the magnetic action of the electric circuit soft iron is magnetized inductively. The distribution of the lines of force is in general greatly affected thereby. The general feature of the phenomenon is a concentration of the lines upon the iron. By the proper use of this effect electromagnetic forces of great power may be developed. It is not easy to give a mathematically accurate account of the action, owing to our ignorance of the exact law of magnetic induction in powerfully paramagnetic bodies. The discussion of this subject, however, belongs to MAGNETISM (which see).

The Induction of Electric Currents.

A brief account has already been given (see Historical Sketch, p. 11) of Faraday's discovery¹ of the induction of electric currents. The results he arrived at may be summed up as follows.

Let there be two linear circuits, ABKE (the primary) and CDG (the secondary), two portions of which, AB and CD, are parallel, and near each other.

I. When a current is started in AB, a transient current flows through CD in the opposite direction to the current in AB; when the current in AB is steady, no current in CD can be detected; when the current in AB is stopped, a transient current flows through CD in the same direction as the current in AB. These currents in CD are said to be induced, and may be called inverse and direct currents respectively, the reference being to the direction of the primary. Both inverse and direct currents last for a very short time, and the quantity of electricity which passes in each of them is the same.

II. If the circuit AB, in which a steady current is flowing, be caused to approach CD, an inverse current is thereby induced in CD; when the circuit AB, under similar circumstances, recedes from CD, a direct current is induced in CD. We have already mentioned that when AB is at rest, and the current in it does not vary, there is no current in CD. AB has been supposed to approach and recede from CD, but the same statement applies when CD approaches and recedes from AB.

III. When a magnet is magnetized or demagnetized in the neighbourhood of a circuit, or approaches or recedes from the circuit, the effect is the same as if an equivalent² current approached or receded from the circuit. For example, imagine a small circular circuit placed horizontally, and a vertical bar magnet lowered in the axis of the circuit with its north pole pointing down upon the circuit, the magnet may be replaced by a series of coaxial circular currents (see above, p. 71), and the motion will induce a current passing round the circuit against the hands of a watch.

Faraday showed how the direction of the induced current can be predicted when the variation of the magnetic field or the motion of the conductor in it is known, and he gave, in his own manner, indications how the magnitude of the current could be inferred.

Maxwell has thrown the law of Faraday into the following form:—"The total electromotive force acting round a circuit at any instant is measured by the rate of decrease of the number of lines of magnetic force which pass through it."

Or, integrating with respect to the time:—"The time integral of the total electromotive force acting round any

circuit, together with the number of lines of magnetic force which pass through the circuit, is a constant quantity."

For "number of lines of force" may of course be substituted the equivalent expressions, "induction through the circuit," or "surface integral of magnetic induction," taken over any surface bounded by the circuit.

Some care must be taken in determining the positive direction round the circuit. The following is a correct process:—Assume one direction (D, fig. 46) through the circuit as positive, then the positive direction round (R) is determined by the right-handed screw relation; if the number of lines of force reckoned positive in direction D is decreasing, then the electromotive force is in direction R; if that number is increasing, the electromotive force is in the opposite direction.

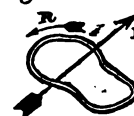


Fig. 46.

This will be clearer if we consider the following simple example. Let ABCD (fig. 47) be a horizontal rectangular circuit (AB next the reader). In a northern latitude, the vertical component Z of the earth's magnetic force is downwards; if, therefore, the positive direction through the circuit be taken downwards, the positive direction round is ADCB, and the number of lines of force through it is Z.AB.BC. If BC slide on DC and AB parallel to itself through a small distance BB' in time τ , Z.AB.BC increases by Z.CB.BB'; hence the electromotive force is Z.BC.BB' $\div \tau$, and acts in the direction ABCD. If v be the velocity of BC, we may write for the electromotive force Z.BC.v. That is, the electromotive force at any instant is proportional to the velocity.

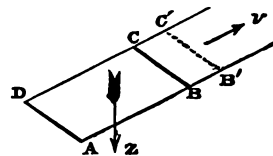


Fig. 47.

The law of Faraday leads to a complete determination of the induced current in all cases. We may regard it as resting on the experiments of Faraday, and of those who followed out his results.

Another view of the matter of great importance was enunciated independently and about the same time by Helmholtz³ and Sir William Thomson.⁴

Let a circuit carrying a current i move in an invariable magnetic field, so that the number of lines of magnetic force passing through it is increased by dN , then the work⁵ done by the electromagnetic forces on the circuit is by Ampère's theory $i dN$; also, if R be the resistance of the circuit, $R i^2 dt$ is the heat generated in time dt . Now if E be the electromotive force of the battery which maintains the current i , the whole energy supplied is $E i dt$; hence we must have

$$i dN + R i^2 dt = E i dt$$

$$\text{and } i = \left(E - \frac{dN}{dt} \right) \frac{1}{R} \dots (23).$$

Hence there is an electromotive force $-\frac{dN}{dt}$ in the moving circuit.

Now $\frac{dN}{dt}$ is the rate of increase of the number of lines of force passing through the circuit.

We have therefore deduced the law stated above from Ampère's theory and the principle of the conservation of energy; at least we have done so for the case of induction by permanent magnets, and the same reasoning will also apply to the case where the alteration of the magnetic field, owing to the induced current in the primary circuit, is so small that it may be neglected.

We have now the means of stating in a convenient form the electromagnetic unit of electromotive force. It is the unit of electromotive force of induction in a circuit the number of lines of magnetic force through which is increasing at the rate of one per second.

³ *Ueber die Erhaltung der Kraft*, 1847.

⁴ *Rep. Brit. Ass.*, 1848, and *Phil. Mag.*, 1851.

⁵ All the quantities are supposed to be measured in electromagnetic absolute units.

⁶ We may suppose this work spent in raising a weight, &c.

with Ampère's theory, may be found in Maxwell, vol. ii. § 502, &c., and in almost any Continental work on experimental physics.

¹ *Exp. Res.*, ser. I, ii. (ix.), xxviii., xxix., 1831-32, 1851. The general statement in the text is given for the reader's convenience, and is not meant to be historical.

² Equivalent in the sense of producing the same magnetic field.

In the case where the field is due to a current i' , we have by formulæ (4) and (14) of last division

$$N = i' M \dots \dots \dots (24),$$

where M now stands for $\iint \frac{\cos \epsilon}{D} ds ds'$ extended all over the two circuits. M , which depends merely on the configuration and relative position of the two circuits, is called the coefficient of mutual induction.

An application of the principle of the conservation of energy of great importance was made by Sir William Thomson to the case of two electric circuits of any form, in which the currents are kept constant.

Let two such circuits, the currents in which are i, i' , be displaced so that the coefficient of mutual induction M increases by dM . Let us suppose that the currents i and i' are maintained by two constant batteries of electromotive forces E and E' , and that the motion takes place so slowly that the currents may be regarded as constant throughout. If R and R' be the resistances of the circuits, Hdt the mechanical equivalent of the whole heat generated, and Kdt the whole expenditure of chemical energy in the batteries in time dt ,

$$H = Ri^2 + R'i'^2, \text{ and } K = Ei + E'i', \\ K - H = i(E - Ri) + i'(E' - R'i').$$

Now, applying (23),

$$Ri = E - i' \frac{dM}{dt}, \text{ and } R'i' = E' - i \frac{dM}{dt};$$

whence $K - H = 2i' \frac{dM}{dt},$

or, as we may write it,

$$(K - H)dt = 2i'dM \dots \dots \dots (25).$$

Now $i'dM$ is the work done by the electromagnetic forces during the displacement which we may suppose spent in lifting a weight.

Hence, when two electric currents are allowed slowly to approach each other, being kept constant and doing work the while, over and above the work which is spent in generating heat in the conductors, an amount of energy is drawn from the batteries equivalent to twice the work done by the electromagnetic forces.

There remains therefore an amount of work as yet unaccounted for. What becomes of it? The answer is, that the energy, or, as Sir W. Thomson calls it, the "mechanical value," of the current is increased. But how increased? When a material system (and we may consider the two circuits, the batteries, the lifted weight, &c., as such) is left to itself, it moves so that its *potential* energy decreases. In this case, therefore, there must have been an increase of *kinetic* energy somewhere. This energy may be called the electrokinetic energy of the system; according to Maxwell's theory, this kinetic energy has its seat in the medium surrounding the wire. The energy thus stored up is accounted for in the increased development of heat, &c., when the two currents are broken in succession.

Case of
two cir-
cuits.

Returning now to our general law of induction, let us write down in the most general form the equations which determine the course of the currents in two circuits (A, B), in which the form and relative positions of the circuits, as well as the current strengths, are variable. The number of lines of force which pass through a circuit depends partly on neighbouring circuits, partly on the circuit itself. Retaining the notation used above, we may, in the case of two circuits, write the first part Mi' , and the second part Li ; where L is a double integral of the same form as M , only both elements ds and ds' now belong to the same circuit. We have, therefore, for the whole number of lines of force passing through the circuit A, $Mi' + Li$. Similarly we have for B, $Mi + N'i'$. We have therefore by our general law,

$$\left. \begin{aligned} E - \frac{d}{dt}(Mi' + Li) &= Ri \\ E' - \frac{d}{dt}(Mi + N'i') &= R'i' \end{aligned} \right\} \dots \dots (26).$$

These are the general equations for the induction of two circuits. The electromotive force of induction in A can be

divided into two parts: one of these, viz., $\frac{d}{dt}(Mi')$ is due

to the circuit B, the other $\frac{d}{dt}(Li)$ is due to the circuit A itself, and is called the electromotive force of self-induction. L is called the coefficient of self-induction for A. Similarly $\frac{d}{dt}(N'i')$ is the electromotive force, and N the coefficient force of self-induction for B.

If we have only one circuit then $M = 0$, and the equation for the course of the current is

$$E - \frac{d}{dt}(Li) = Ri;$$

here there is *only* self-induction.

F. E. Neumann, to whom belongs the honour of first stating with mathematical accuracy the laws of induction, adopted a foundation for his theory very different from the one chosen above. His method was based on the law of Lenz¹, enunciated very soon after the great discovery of Faraday, which lays down that, in all cases of induction by the motion of magnets or currents, the induced current has a direction such that its electromagnetic action on the inducing system tends to oppose the motion producing it.

Besides its historical importance, this law affords a very convenient guide in many practical applications of the theory of induction. The reader will find no difficulty in verifying it on the elementary cases given at the beginning of this division. It can be deduced at once from our general law. Consider any circuit in which a current i is flowing, and let the direction of the current be the positive direction round the circuit. Suppose the circuit to move so that the number of lines of force passing through it increases, this is the way the circuit would tend to move under the electromagnetic forces when traversed by a current i ; but the electromotive force of induction is in the negative direction round the circuit by the general law, and would therefore produce a current opposite in direction to i . The electromagnetic action on this current would be opposite to that on i , that is, would tend to hinder the displacement. It is a curious fact that a law exactly like this had been announced shortly before Lenz by Ritchie, only with the direction of the action reversed in every case.

The results of Neumann are identical with those given above. The double integral M , which is here called the coefficient of mutual induction of two circuits, Neumann calls the mutual *potential* of the two circuits, and what has been called above the coefficient of self-induction of a circuit he calls the *potential of the circuit on itself*. Accounts of his theory will be found in Wiedemann's *Galvanismus*, and in most Continental works on electricity.

Experimental Verification of the Laws of Mutual Induction.

—It will be observed that, in the law of induction for linear circuits, no statement is made respecting the material or thickness of the circuit in which the electromotive force of induction acts, or of the non-conducting medium across which induction takes place.

Faraday showed that the material of the circuit has no effect.² Experiment. He found, for instance, that when two wires of different metals were joined and twisted up together, as in fig. 48, so as to be insulated from each other, no induced current could be obtained by passing the arrangement between the poles of a powerful magnet. The same result was obtained when one of the branches of the circuit was an electrolyte. Lenz³ connected two spirals of wire in circuit with each other, and placed first one then the other, on the soft iron keeper of a horse-shoe magnet; so long as the number of turns on each spiral was the same, the induced



Fig. 48.

¹ Pogg. Ann., 1834.

² Exp. Res., 193, &c., 1832; also 3143, &c., 1851.

³ Pogg. Ann., 1835.

current was the same, no matter what the material or thickness of the wire in each spiral. Since in this case the whole resistance of the circuit was always the same, the electromotive force of induction must have been the same.

We conclude, therefore, that the electromotive force¹ of induction is independent of the material, and also of the thickness of the wire, so long as the latter is so small that we may consider the wire as a linear circuit.

Lenz made quantitative determinations of the induced current by means of the above arrangement.

The soft iron keeper, with a coil of n windings, was rapidly detached from the magnet, and the first swing a of a galvanometer in circuit with the coil was measured. The quantity of electricity which passes in the induced current is measured by $\sin \frac{1}{2}a$, provided the whole duration of the current is small compared with the time of oscillation of the galvanometer needle (see art. GALVANOMETER). Again, when the keeper is attached to the magnet, very nearly all the lines of magnetic induction² pass through the keeper; hence the number of lines of induction which pass through the coil is very nearly proportional to the number of windings, and therefore, if the resistance of the circuit be kept the same, the whole amount of electricity which passes will be proportional to n . In the actual experiment the wire was wound and unwound from the keeper, so that the whole resistance did remain the same. The following is a set of Lenz's results:—³

No. of Windings.	2	4	8	12	16	20
$\sin \frac{1}{2}a$	0.0491	0.1045	0.2156	0.3319	0.4470	0.5594
$\sin \frac{1}{2}a + n$	0.0245	0.0261	0.0270	0.0276	0.0279	0.0280

The value of $\sin \frac{1}{2}a + n$ is very nearly constant. It increases a little as the number of windings increases, as ought to be the case, for, although most of the lines of induction pass through the keeper, yet all do not, and a few more are included when the number of turns is increased.

Faraday made special investigations in search of the effect of the medium across which induction is exerted. He found⁴ that no effect on the integral current was produced by inserting shellac, sulphur, copper, &c. between the primary and secondary coils. The insertion of iron or any strongly magnetic body, of course, produces an effect, because the distribution of the lines of magnetic force is thereby altered, and therefore, by our general law, the electromotive force of induction will be correspondingly affected. We conclude, therefore, that the electromotive force of induction is independent of the medium across which it is exerted.⁵

It must be remarked, however, that in the case of conducting media, the statement is subject to a certain limitation, the nature of which follows from the law of induction itself. For there will be induced currents in the intervening medium if it be a conductor, and these currents will disturb the lines of force while they continue to flow. These currents are *transient*, however, so that their integral effect on the number of lines of force passing through the secondary is zero. It is obvious, therefore, that, if we replace "electromotive force" by "time integral of electromotive force extended over the whole time that the induction currents last," the statement will still be true. The only effect, therefore, of interposed conducting media is on the time which the induced currents take to rise and fall.

Weber⁶ applied his electro-dynamometer to test the laws of induction.

The suspended coil was caused to oscillate when there was no current either in it or in the fixed coil, and the logarithmic decre-

ment⁷ of its oscillations carefully determined. This decrement, due to the friction of the air, &c., was found to be constant for different lengths of the arc of oscillation. The terminals of the suspended coil were next connected so that it formed a closed circuit, and a constant current was sent through the fixed coil. Induction currents were now generated in the suspended coil, whose electrodynamic action constantly opposed its motion. It was found that the logarithmic decrement was still constant, but greater than before. Weber therefore concluded that the induced current at each instant was proportional to the velocity of the coil. Since the resistance does not vary, this is in accordance with the general law.

Weber further showed that the induced current is the same whether it is produced by a current in the fixed coil or by a magnet, which exercises the same electromagnetic action as that current on the suspended coil, when the latter is traversed by a current of unit strength.

The electro-dynamometer may also be used to demonstrate the equality of the whole amounts of electricity which pass in the direct and inverse currents. If the induced currents from a secondary coil whose primary is being "made and broken" be passed through both coils of the instrument, there will be a deflection, since the action depends on the square of the current; but if the induced current be sent through the suspended coil alone, and a constant current be sent through the fixed coil, there will be no deflection, which shows that the quantities of electricity passing in the alternate currents of the secondary coil are equal and of opposite sign.

Felici (1852 and 1859) made an extended series of Felici's experiments on the laws of induction. He used null methods, and his experiments bear a resemblance in some respects to the electro-dynamical experiments of Ampère. Maxwell⁸ has given a summary of Felici's results.

It is found, for instance, that the electromotive force of induction of a circuit A on another B is independent of the material or section of the conductors, that it is proportional to the current in A and to the number of windings in B. The induction of A on B is the same as that of B on A, when the inducing current i is the same in both cases. Any portion of A or B may be replaced by a zig-zag portion, which nowhere deviates far from it. In pairs of circuits geometrically similar, the electromotive force of induction is proportional to the linear dimensions, and so on.

If B be so situated with respect to A that starting or stopping a current in A produces no induced current in B, B is said to be conjugate to A. There are an infinite number of such conjugate positions of B; and Felici shows that, if B be moved from one of these P_1 into another P_2 very quickly, no effect is produced on the galvanometer. If B be moved from P_1 to any position P (not a conjugate position), the effect on the galvanometer is the same as if the current i were suddenly started in A, B being in the position P.

All these results are direct consequences of our general law, and indeed might be used as a foundation for it.⁹

In his later researches on electromagnetic induction Faraday (series xxvii. and xxix.), Faraday develops in considerable detail his ideas on the connection between the lines of magnetic force and the induced current, and gives increased precision to the experimental methods that flow therefrom. He points out the great value of methods, such as the use of iron filings, for exhibiting in a visible form the course of the lines of magnetic force. He also insists on the great use of a small moving circuit, which can be used to explore the magnetic field under circumstances which render the application of other methods impossible.

The direction of a line of force may be determined in various ways by means of the moving conductor. Maxwell¹⁰ gives four such ways:—(1) if a conductor be moved along a line of force parallel to itself, it will experience no electromotive force; (2) if a conductor carrying a current be free to move along a line of force, it will show no tendency to do so; (3) if a linear conductor coincide with a line of force and be moved parallel to itself in any direction, it will experience no electromotive force in the direction of its length; (4) if a linear conductor carrying an electric current coincide in direction with a line of magnetic induction, it will not experience any mechanical force.

In these researches Faraday treats at considerable length Unipolar a case of the induction of electric currents, to which Continental writers have given the somewhat mysterious name of "unipolar induction." It belongs to a class of cases on

¹ Of course, the same is not true of the current of induction, which depends on the resistance of the circuit.

² In Maxwell's sense; we might say "lines of magnetic force" in Faraday's sense; see art. MAGNETISM.

³ Wiedemann, Bd. ii. § 706.

⁴ *Exp. Res.*, 1709, &c., 1838.

⁵ Other investigators have sought for such effects, and some have affirmed their existence; but there is no body of concurrent testimony on the point.

⁶ *Maassbestimm.*, §§ 10 and 11, 1846.

⁷ See art. GALVANOMETER.

⁸ Vol. ii. § 536; see also Wiedemann, Bd. ii. § 709.

⁹ See Maxwell, *l.c.*

¹⁰ Vol. ii. § 597.

which they have rightly dwelt as being in a sense the reverse of the electromagnetic rotations. The following theory of the phenomenon will make this clearer:—

Referring back to figure 40, let AB be part of a conducting circuit arranged as there described, and let it be caused to move in the direction Pp . Then if E be the electromotive force in the circuit in the direction AB, N the number of lines of force passing through the circuit, ϕ the angle through which AB moves (from X to Y) about OZ, we have, by our general law,

$$E = - \frac{dN}{dt} = - \frac{dN}{d\phi} \frac{d\phi}{dt}.$$

Now, by Ampère's theory, $K = \frac{idN}{d\phi}$, hence (p. 73)

$$E = - \frac{K}{i} \frac{d\phi}{dt} \\ = - m (\cos \beta_1 - \cos \alpha_1 - \cos \beta_2 + \cos \alpha_2) \frac{d\phi}{dt}. \quad (27).$$

Hence, if the conductor AB be caused to move with given angular velocity about the magnet SN, in that direction which it would take under the action of the magnet if it carried a current i , then there will be an electromotive force of induction along the circuit of which AB forms part, whose direction is opposite to that of i , and whose magnitude is found by dividing the couple acting on AB (when traversed by i) by i , and multiplying it by the given angular velocity. This result is a beautiful instance of the law of Lenz.

A great variety of experimental arrangements may be imagined to realize the case thus described. Every apparatus devised to produce an electromagnetic rotation may be used to illustrate it.

The following case may be taken as typical. SN (fig. 49) is a bar magnet whose action may be represented by two poles, N and S. At the middle point of its axis is fixed a disc BA, against which presses the terminal of a wire CA in metallic connection with the axis through the pivot at S. If CA be caused to rotate in the direction of the arrow p , the disc standing still, there will be an induced current in CAB in the direction of the arrow q . If CA and the disc revolve together, there will be no current. If CA stand still, and the disc rotate in the direction of the arrow, there will be a current in the opposite direction; for this is clearly the same as if the disc stood still, and CA rotated in the opposite direction.¹ The electromotive force in each case is independent of the form of CA, and is given by $2m(1 - \cos \alpha)\omega$, where m is the strength of the pole N, α the angle ANB, and ω the angular velocity.

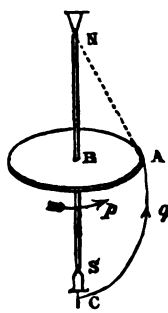


Fig. 49.

It is well to remind the reader that the lines of force are closed curves, every one of which passes up the axis of the magnet from S to N, and back through the outside medium to S. If this be forgotten, and an attempt be made to determine the electromotive force of induction by considering the motion of the disc, an error will easily be made. If we take the simpler course above, and consider the motion of the conductor, there is then no danger of mistake.

Coils with iron core.

In most of the experiments we have hitherto been describing, the object has been to obtain indications of the direction of the currents of induction, or to measure the electromotive force of induction under definite circumstances; if, however, we desire to exhibit the effects of induction in a striking manner, in order to convey belief to the spectator, or to serve some practical purpose, recourse is had to a different kind of apparatus. We may wind our primary and secondary coils on bobbins, and insert the former within the latter, so as to get the greatest possible

¹ If the reader wish for a proximate rule for the direction of the electromotive force of induction, the following will serve. Stand with the body in the line of magnetic force with the head pointing in the positive direction, look in the direction in which the part of the circuit on which the feet are is moving; the E. M. F. along the circuit is towards the right hand.

number of turns of wire into proximity. The number of turns on the primary is usually made small, in order that the current in it may not be weakened by a large resistance, and that its coefficient of self-induction (see below) may be small. Mention has already been made of the effect of soft iron in increasing the number of lines of force that pass through a circuit. It is easy to see that it will produce a corresponding effect in strengthening induction. The precise amount of it is very hard to calculate, owing to the irregularities in the magnetization and demagnetization that arise from residual magnetism. The question belongs, however, to magnetism. The effect can be demonstrated practically by observing the alteration in the inductive action produced by inserting a bundle of iron wires² into our primary coil.

The physiological effects of induced currents are very striking; indeed, the nerve and muscle preparation of the logical physiologist affords a very delicate method for detecting effects. If the human body form part of the circuit of the secondary coil of such an induction apparatus as we have just indicated, and the primary current be stopped and started in rapid succession, say by stripping one terminal of the circuit on a toothed wheel attached to the other, a sensation is experienced which, with a moderately powerful apparatus furnished with a core, is so painful and peculiar that the patient is not likely to forget either it or its cause. The tetanic muscular contractions produced in this way have formed the subject of much physiological investigation, of which an account will be found in the proper place (see article PHYSIOLOGY).

The flat spirals of Henry, formed of flat bands of copper insulated from each other with silk ribbon, are also very convenient for demonstrating the existence of induced currents.

The most powerful inductive apparatus for furnishing large quantities of electricity are the various magneto-electric machines which have now been brought to great perfection (see Historical Sketch).

By means of these and similar appliances, all the effects of the electric current and the electric discharge may be shown in the greatest perfection.

Induction by Discharge of Static Electricity.—The phenomena of induction can be exhibited with the transient current of electricity in the discharge of a Leyden jar or other accumulator of static electricity. There is a difficulty in exhibiting the effect, owing to the great differences of potential between different parts of the circuit, which render the application of a coil of silk-covered wire useless. A common way of getting over the difficulty consists in cutting two spiral grooves in two flat ebonite discs. Wires are embedded in these, and they are then put together with a thin plate of glass between, so that the spirals are opposite each other. When a jar is discharged through one spiral, an induction current passes in the other, and may be indicated by a galvanometer, or, better still, by a frog preparation. The induced current is, however, in general a complicated phenomenon, owing to the oscillatory nature of the discharge (see above, p. 65).

It would lead us too far to go into these and kindred subjects: the reader who desires to pursue the matter will find excellent accounts in Mascart, t. ii. §§ 611–825, and Riess, Bd. ii. §§ 780–906. Particularly interesting are the researches of Verdet, an account of which will be found in his works, along with many indications of what others have done in the same field.

Induced Currents of Higher Orders.—Induced currents may in their turn induce other currents, and these again

² The iron is broken up into wires to prevent the formation of induced currents in the body of the metal. These currents retard the rise of the induced currents.

ad others, and so on.¹ This can be brought about by forming its part of the secondary circuit of one inductive apparatus her into the primary of the next, and so on. As may be supposed, the successive induced currents diminish very rapidly in strength, and require special means for their detection. But the phenomenon also goes on increasing in complicity. Suppose we start the current in the first primary, there is a single inverse current of the "first order" which rises and then falls; there will, therefore, be two currents of the "second order"—first a direct, then an inverse; each of these rising and falling causes two currents of the third order, and so on in geometric progression. These currents have been detected in certain cases by means of their physiological action and their magnetizing powers. The latter effects present some points of interest in connection with magnetism, but we cannot spare space for the matter here.

Self-Induction.—The existence of self-induction has been deduced as a theoretical consequence of the general law of induction. It was not so discovered, however. It was first arrived at by Faraday² from experimental considerations. The observation from which he started was the following fact communicated to him by Mr Jenkin, who had shortly before discovered it:—Although it is impossible with a short circuit of wire and a single battery cell to obtain a shock by making and breaking contact, yet a very powerful shock is obtained if the coil of an electromagnet be included in the circuit. This may be shown thus:—Let ZC (fig. 50) be a battery of a single cell, CABDEF a circuit with a cross branch BF, in which at G the human body, &c., may be inserted. Contacts can be made and broken at A, very rapidly if need be, by means of a toothed wheel. When BDEF consists of a short single wire, nothing particular is felt at G, but when the coil of an electromagnet is inserted in DE, the patient at G experiences a series of powerful shocks comparable to that obtained from the secondary coil of an inductive apparatus in the manner already described.

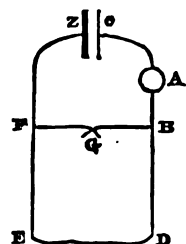


Fig. 50.

If the cross circuit be done away with, a powerful spark is obtained at A on breaking contact, but none on making. This spark is particularly bright if a mercury contact be used, owing to the combustion of the mercury. If, however, the electromagnet be removed from DE, and a short wire substituted, the spark becomes quite insignificant, although the whole circuit may be now very hot, owing to the increased current. Faraday found that the same effect, only smaller, was produced when a simple helix without a core was substituted for the electromagnet; and a similar effect, only still smaller, was obtained when a very long straight wire was used. Faraday soon recognized that these effects are consequences of the laws at which he had arrived in his first series of researches on induction. When the current is rising in a circuit, the number of lines of magnetic force passing through it is on the increase, hence an electromotive force is generated which opposes that of the battery, and causes the current to rise slowly; again, when the current begins to decrease the number of lines of force begins to decrease, and an electromotive force of induction is called forth which tends to prolong the current. We have, therefore, a weakening of the electromotive force at starting and an exaltation at stopping, which accounts for the absence of the spark or shock at make, and the presence of one or other at break. Such

¹ Some physicists have called these currents induced currents of the second and third orders, &c.

² *Exp. Res.*, 1048, &c., 1834.

inductive effects are obviously heightened when the current is wound into a spiral form; if, however, the spiral were wound double, and the current sent through the two wires in opposite directions, the inductive effects would annul each other, and, in fact, with this arrangement the spark and shock are extremely small.

Faraday demonstrated the existence of these electromotive forces by means of the currents which they produce in derived circuits,³ when the battery contact is broken or made.

He used the arrangement given in figure 50. A galvanometer was inserted at G, and the needle stopped by pins properly placed from deviating as urged by the branch of the battery current from B to F, but left free to move in the opposite direction. It was found that the needle deviated sharply when contact was broken at A, in a direction indicating a current from F to B. Again the contact was made, and the needle stopped at the deviation due to the current from B to F, so that it could not return to zero. The contact was then broken and made again, and it was found that at the make the needle tended to go beyond the position due to the steady current in BF. Faraday also arranged a platinum wire at G, so that it did not glow under the steady current in BF, but immediately ignited when the contact at A was broken. Chemical action was produced in a similar manner. In fact we may, by taking advantage of the self-induction, cause a single cell to produce decomposition of water and evolution of gas, which it could not do alone consistently with the conservation of energy. This may be managed⁴ by inserting at A (fig. 50), instead of the contact breaker, the coil of an electromagnet, and placing the decomposing cell in DE. Let contact be made and broken at G (say by an automatic break); when the contact is made the current flows through the coil and through BF, when it is broken the electromotive force of induction added to that of the battery enables the current to pass through the cell and liberate the ions. At the make there is no such effect; there results therefore continued chemical decomposition.

Edlund⁵ investigated the integral electromotive forces of self-induction at the opening and closing of a circuit, and showed that they are equal. His experimental arrangement is very ingenious:—

G (fig. 51) is a differential galvanometer, A a coil whose self-induction is to be examined, C a wire wound in a zig-zag⁶ so as to have no self-induction. The battery E is connected at B and D with the circuit composed of G, A, and C, so that the currents in BcdCD and BbaAD pass round the coils of G in opposite directions. The resistance C is so arranged that there is no deflection of the needle in G. If now the current be stopped by breaking the circuit EB at K, the electromotive force due to the self-induction of A causes an extra current to flow round the circuit AabBcdCD, traversing the coils of G in the same direction. We therefore get a deflection D₁. In a similar manner if we make contact at K we get another deflection D₂, due to the starting of the current in A. There is no difficulty in showing that, if E₁, E₂ be the time integrals of the electromotive force in the two cases, then

$$\frac{E_1}{E_2} = \frac{D_1}{D_2}.$$

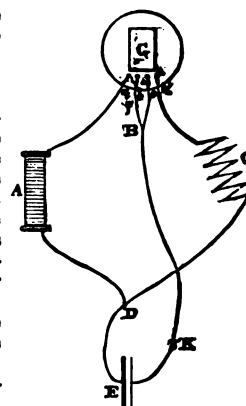


Fig. 51.

One of the difficulties encountered in such experiments is the increase of the electromotive force of the battery E when it is left open for a time; this causes the extra current at make to be greater than that at break. Rijke, who made experiments similar to those of Edlund, avoids this difficulty by circuiting the battery, when BK is broken,

³ These currents are sometimes called extra currents, and the name is applied even when there is no alternative circuit. The extra currents are then the defect or excess of the currents at the make and break, considered with reference to the steady current.

⁴ De la Rive, Wiedemann, Bd. ii. § 740.

⁵ *Pogg. Ann.*, 1849.

⁶ The best arrangement would be to use insulated wire and double it on itself.

through a resistance equal to the effective resistance from E to K. Further details concerning the method and results of these experiments may be found in Wiedemann, Bd. ii. § 744, &c.

Max-
well's
method.

A very convenient method for exhibiting and measuring the extra current is obtained by using a Wheatstone's bridge instead of a differential galvanometer. Let the bridge be balanced as usual, so that when the battery circuit is made, and the galvanometer circuit made afterwards, there is no deflection. If one of the resistances be wound so as to have a large coefficient of self-induction, and the galvanometer circuit be completed before the battery is thrown on, then, owing to the self-induction, the galvanometer needle will be suddenly deflected.

Let AC, CD, DB, BA be four conductors of resistance S, Q, P, R, arranged as a Wheatstone's bridge (see fig. 22), with a battery between A and D, and a galvanometer G between B and C. Let L be the coefficient of self-induction of the coil S. Then, A, C, &c., denoting the potentials at A, C, &c., x and y the currents in AC and AB, and z the current in G, we have

$A - C = Sy + L \frac{dy}{dt}$, $A - B = Rx$, $C - D = Q(y + z)$,
&c. Eliminating, as in Maxwell, vol. i. p. 399, or above, p. 43, we get

$$z = \frac{PS - QR + PL \frac{d}{dt}}{D'} E \quad (28),$$

where $\frac{d}{dt}$ is a separated symbol, and D' is the determinant of the system of resistances with $S + \frac{d}{dt}$ written for S. We may therefore write

$$D' = D + HL \frac{d}{dt},$$

D being the ordinary determinant, and H a function of PQR, &c., which we need not determine. Equation (28) may therefore be written

$$Dz + HL \frac{dz}{dt} = PL \frac{dE}{dt} \quad (29),$$

provided the bridge be balanced, i.e. if $PS - QR$ be zero. Suppose now the galvanometer circuit is closed, and then the battery circuit closed; then, integrating equation (29), from the instant before the battery is thrown in up to a time τ when all the currents have become steady and no further current flows through the galvanometer, we get

$$D \int_0^\tau z dt = PLE,$$

$$\text{or } z_1 = \frac{PLE}{D} \quad (30),$$

where z_1 denotes $\int_0^\tau z dt$, i.e. the whole amount of electricity that flows through the galvanometer owing to the induced current. If now we derange the balance in the bridge by increasing S by a small quantity x , and decreasing Q by as much, we get a steady current through the galvanometer given by

$$z = \frac{(P+R)x E}{D}$$

Hence

$$\frac{z_1}{z} = \frac{PL}{(P+R)x} \quad (31).$$

Now, if β be the first swing of the galvanometer needle, owing to the induction current, α the deflection under the steady current, and T the time of oscillation of the needle under the earth's force alone (T is supposed to be so large that the duration of the induced current is very small compared with it); then it may be shown that

$$\frac{z_1}{z} = \frac{T \sin \frac{1}{2} \beta}{\pi \tan \alpha} \quad (32).^1$$

Hence

$$L = \frac{(P+R)x T \sin \frac{1}{2} \beta}{\pi \tan \alpha} \quad (33).$$

We thus get L in terms of quantities which can be easily measured. This method of finding L is due to Maxwell.

¹ Certain corrections would in general be necessary in practice, but we need not discuss them here.

The application of the equations (26) to determine the march of the current in certain simple cases leads to results of great interest.

Suppose that an electromotive force E begins to act in a circuit of resistance R and coefficient of self-induction L . The equation for the current strength i at any time t after it has begun to act, is

$$L \frac{di}{dt} + Ri = E \quad (34).$$

The integral of this is $i = \frac{E}{R} (1 - e^{-\frac{R}{L}t})$ (35),

the constant of integration being determined by the condition $i = \frac{E}{R}$ (steady current) when $t = \infty$.

Hence the current starts with the value zero, and increases continuously till it reaches the steady value $\frac{E}{R}$.

The part $\frac{E}{R} e^{-\frac{R}{L}t}$ is due to self-induction, and is called the extra current. The whole amount of electricity passing in this part of the current is

$$-\frac{EL}{R^2} \quad (36).$$

The quantity $\frac{L}{R}$ is of the same dimension as t , and is called the time constant of the coil. According as the time constant is greater or less, the longer or shorter time will the current take to rise to a given fraction of its steady value. If we desire therefore to prolong the induction and to increase it as well, we must make L large and R small, two conditions which in the extremes are inconsistent. Calculations of the form of coil for maximum inductive effects might be made, but this is not the place to enter on them.

Next, let the electromotive force E suddenly cease to act, the resistance of the circuit being unchanged. This may be realized experimentally within certain limits by throwing the battery out of the circuit, and at the same time substituting for it a wire of equal resistance. It is easy to show as above that the extra current at a time t after E ceases to act is

$$+\frac{E}{R} e^{-\frac{R}{L}t},$$

and the whole amount of electricity which passes is $+\frac{EL}{R^2}$.

Helmholtz,² who was the first to treat this subject both experimentally and mathematically, operated as follows:—

- (1) The battery was thrown into the circuit, and after a time t the circuit was broken.
- (2) The battery was thrown in, and after a time t replaced by a circuit of equal resistance.

These changes were effected by means of a system of levers, which it is not necessary to describe here. An account of the apparatus will be found in the paper quoted.

The amount of electricity which passes through the circuit is measured by a galvanometer whose time of oscillation is long compared with t . In the first case the amount is

$$A = \frac{Et}{R} - \frac{EL}{R} \left(1 - e^{-\frac{R}{L}t} \right);$$

in the second

$$B = \frac{Et}{R},$$

because here the two extra currents just counterbalance each other. The observed value of B in each case enables us to calculate t . E and R being found by separate observations, one observed value of A enables us to calculate L . Using these values of E , R and L , a series of values of t , and hence A , can be calculated from the observed values of B , and the result compared with the observed value of A . The agreement between theory and experiment was sufficiently close to justify the application of the principles from which the above formulæ were deduced. Among these principles may be mentioned the validity of Ohm's law for transient currents.

The reader will find in the original paper details concerning the above and other similar results arrived at by Helmholtz.

fixed The case of two circuits of invariable form and position **is** of great interest, from the light it throws on the action of the induction coil. We shall suppose that we have no soft iron core, and that the break in the primary is instantaneous. The latter condition is very far from being realized in practice, even with the best arrangements, so that our case is an ideal one.

Let i and j be the current strengths in primary and secondary, R and S the resistances, L , M , N the coefficients of induction, E the electromotive force in the primary. The equations are

$$L \frac{di}{dt} + M \frac{dj}{dt} + Ri = E \quad (37),$$

$$M \frac{di}{dt} + N \frac{dj}{dt} + Sj = 0 \quad (38).$$

It is easy, in the first place, to show that the whole amounts of electricity which traverse the secondary at make and break of the primary are equal but of opposite signs. In fact, if we integrate (38) from the instant before make to a time when the induction currents both in primary and secondary have subsided, we get

$$\int j dt = -\frac{M}{S} I - \frac{ME}{SR} \quad (39).$$

where I denotes the steady current in the primary. Similarly integrating over the break, we get

$$\int j dt = +\frac{M}{S} I - \frac{ME}{SR}.$$

In the second place, if we assume the break instantaneous, we can find the initial value of the direct current in S . Thus integrate¹ (38) from the instant before break to the time τ after it, τ being infinitely short compared with the duration of the induction currents, then

$$-MI + Nj_0 + S \int j dt = 0.$$

Now the last term may be neglected, because τ is infinitely small and j is not infinite, hence we have, for the initial value of j ,

$$j_0 = \frac{M}{N} I = \frac{ME}{NR} \quad (40).$$

It is very easy now to determine the farther course of the current in S . The equation for j reduces to

$$N \frac{dj}{dt} + Sj = 0;$$

and we get, using (40),

$$j = \frac{ME}{NR} e^{-\frac{S}{N}t} \quad (41).$$

The direct induced current (current at break), therefore, starts in our ideal case with an intensity which is to the intensity of the steady current in the primary as the coefficient of mutual induction of the coils is to the coefficient of self-induction of the secondary, and then dies away in a continuous manner like any other current left to itself in a circuit of given resistance and self-induction.

Since we have already given enough of these calculations to serve as a specimen, we content ourselves with stating the result for the current at make. Owing to the self-induction of R , &c., the current in R rises continuously from zero to the value I ; the induced current in S therefore begins also from zero, rises to a maximum, and then dies away. The mathematical expression for it contains, as might be expected, two exponential terms.

electro- static It is instructive, in connection with what has already been said concerning the electrokinetic energy of two moving circuits, to examine what becomes of the energy in the case of two fixed circuits of invariable form.

Equations (37) and (38) may be used if, for generality, F be written instead of 0 in (38), so that there is electromotive force (say of constant batteries) in both circuits. Multiplying (37) by i and (38) by j , adding, and integrating from the time before E and F begin to act to a time τ when the currents have all become steady, we get

¹ The reader might suppose that this process of integration might be equally applied to (37). This is not so, however, owing to the variability of R at the break.

$$\int_0^\tau (Ei - Ri^2) dt + \int_0^\tau (Fj - Sj^2) dt = \frac{1}{2}(Li^2 + 2Mij + Nj^2) \quad (42).$$

In words, the excess of the chemical energy exhausted in the batteries over the amount of energy which appears as heat in the circuits is $\frac{1}{2}(Li^2 + 2Mij + Nj^2)$, which we denote by K . Similar remarks to those made at p. 76 apply here. K is the amount of electrokinetic energy stored up in the medium surrounding the circuits during the time that E and F are raising the currents against self and mutual induction.

If we integrate similarly over the break of both currents, we find the defect of the chemical energy exhausted under the heat evolved in the circuit to be again K . Much of the energy thus discharged from the system at break usually appears in the spark.

Electrical Oscillations.—Helmholtz² seems to have been the first to conceive that the discharge of a condenser might consist of a backward and forward motion of the electricity between the coatings, or of a series of electric currents alternately in opposite directions. Sir William Thomson³ took up the subject independently, and investigated mathematically the conditions of the phenomenon.

Let q be the charge of the condenser at time t , C its capacity, E the difference of potentials between the armatures, i the current in the wire connecting the armatures, R its resistance, L the coefficient of self-induction. Then we have

$$q = CE, \quad i = -\frac{dq}{dt},$$

$$\text{and} \quad L \frac{di}{dt} + Ri = E,$$

$$\text{i.e.,} \quad \frac{d^2q}{dt^2} + \frac{R}{L} \frac{dq}{dt} + \frac{1}{LC} q = 0 \quad (43).$$

The solution of this equation is

$$q = e^{-mt} (Ae^{nt} + Be^{-nt}), \quad (44),$$

where

$$m = \frac{R}{2L}, \quad n = \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}.$$

A and B are constants to be determined by the conditions $q = Q$ and $\frac{dq}{dt} = 0$ when $t = 0$.

Two distinct cases arise.

(1.) Let R be greater than $\sqrt{\frac{4L}{C}}$; then the exponentials in (44)

are real, the discharge is continuous, all in one direction, and involves no essentially new features.

(2.) Let R be less than $\sqrt{\frac{4L}{C}}$; then the appropriate form of the solution is

$$q = e^{-mt} (A \cos nt + B \sin nt),$$

where m has the same meaning as before, but n stands now for $\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$. If we determine A and B by the initial conditions,

$$\text{we get} \quad q = e^{-mt} \left(\cos nt + \frac{m}{n} \sin nt \right) Q \quad (45).$$

The current is given by

$$i = \frac{Q}{NLC} e^{-mt} \sin nt, \quad (46).$$

It follows from these equations that, when $R < \sqrt{\frac{4L}{C}}$, the charge of one armature of the condenser passes through a series of oscillations. The different maxima are

$$Q, \quad -Qe^{-\frac{m\pi}{n}}, \quad Qe^{-\frac{2m\pi}{n}}, \quad -Qe^{-\frac{3m\pi}{n}}, \quad \&c.,$$

occurring at times

$$0, \quad \frac{\pi}{n}, \quad \frac{2\pi}{n}, \quad \frac{3\pi}{n}, \quad \&c.$$

² *Die Erhaltung der Kraft*, 1847.

³ *Phil. Mag.*, 1855. This paper is a very remarkable one in many respects. The methods used in the beginning to arrive at the equation (43) are well worth the reader's study.

⁴ R here must be understood to represent the mean resistance of the circuit during the discharge.

When the charge has any of these maximum values, the current is zero. The current maxima form a similar descending geometric series, the times of occurrence being

$$\frac{\theta}{n}, \frac{\theta + \pi}{n}, \frac{\theta + 2\pi}{n}, \frac{\theta + 3\pi}{n}, \&c.,$$

where θ is the acute angle $\tan^{-1} \frac{n}{m}$.

The interval between any positive and the next negative maximum, whether of charge or current, is therefore $\frac{\pi}{n}$.

We need not insist on the evident importance of this result. Thomson, in his original paper, points out the various applications of which it is capable. He predicts the phenomena afterwards observed by Feddersen; in fact, he suggests the use of Wheatstone's mirror to detect it. Its bearing on the anomalous magnetization of steel needles by jar discharges, and on the anomalous evolution of gas by static discharges, when electrodes of small surface are used (in Wollaston's manner), are also dwelt upon.

Experiments of Feddersen, &c.

Several physicists have taken up the experimental investigation of this matter. Feddersen's experiments realize the case above discussed, if we abstract the disturbance owing to the air interval, of the effect of which it is not easy to give an accurate account. Feddersen's results are in good general agreement with theory. He finds, for instance, that the critical resistance at which the discharge begins to assume the oscillatory character varies inversely as the square root of the capacity of the battery from which the discharge is taken. A good account of the researches of Paalzow,¹ Bernstein,² and Blaserna, and of the older researches of Helmholtz,³ remarkable for the use of his pendulum interruptor, will be found in Wiedemann, §§ 801, &c. Schiller, in a very interesting paper,⁴ describes a variety of measurements of the period of oscillation, and the damping of the alternating currents in a secondary coil, when the current of the primary is broken. By means of the pendulum interruptor of Helmholtz (for description of which see his paper) the primary is broken, and at a measured interval thereafter the secondary circuit, which contains a condenser and a Thomson's electrometer, is also broken. The deflection of the electrometer indicates the charge of the condenser at the instant when the secondary is broken. The interval between two null points separated by a whole number of oscillations can thus be found, and hence the time of oscillation of the coil calculated. The agreement of Schiller's results with calculation is very remarkable, and must be regarded as a highly satisfactory proof of the validity of the theoretical principles involved.

Induction in masses of metal.

Induction in Masses of Metal and Magnetism of Rotation.—Hitherto we have dealt only with linear circuits; but it is obvious that currents will also be induced in a mass of metal present in a varying magnetic field. If the variation of the field be due to relative motion between the mass of metal and the system to which the field is due, the electromagnetic action of the induced currents will oppose the motion. Many instances might be given of this principle. If a magnet be suspended over a copper disc, or, better still, in a small cavity inside a mass of copper, its vibrations are opposed by a force due to the induced currents which for small motions varies as the angular velocity of the needle. Accordingly, it comes much sooner to rest than it would do if suspended in the air at a distance from conducting masses; it moves beside the copper as if it were immersed in a viscous fluid.

Experiments of Plücker and Foucault.

Plücker suspended a cube of copper between the poles of a powerful electromagnet, and set it spinning about a vertical axis; directly the magnet was excited it stopped dead. Foucault arranged a flat copper disc between the

flat poles of an electromagnet placed at such a distance apart as just to admit it between them. The disc was set in rotation by means of a driving gear. So long as the magnet was not excited, the driver had comparatively little work to do; but as soon as the magnet was excited, the work required to keep up any considerable velocity greatly increased. The additional work thus expended appears in the heat developed in the disc by the induced currents. Tyndall demonstrates this very neatly by causing a small cylindrical vessel of thin copper filled with fusible metal to rotate between the poles of an electromagnet, when enough heat is quickly developed to melt the metal.

On the other hand, when a mass of metal moves in the Arago's neighbourhood of a magnet, the electromagnetic action of experiment the induced currents will cause the magnet to move, if it be free to do so. Thus, if we suspend a magnet with its axis horizontal over a disc which can be set in rotation about a vertical axis, owing to the electromagnetic action of the induced current, the needle will tend to rotate in the same direction as the disc. If the velocity be great enough, or the needle be rendered astatic, it may be carried round and round continuously. This action was discovered by Arago, and excited the attention of many philosophers, till it was at last explained by Faraday (see Faraday's Historical Sketch). Many of the observations made by Faraday's predecessors, and some made by himself, are at once seen to support the conclusion that the phenomenon is simply a case of Lenz's law. Thus Snow Harris found that the deflecting couple on a suspended needle varied approximately as the velocity of the disc directly, and as the square of the distance of the needle from the disc inversely. It was also found that the action of the disc was directly proportional to the conductivity of the metal of which it was made, an exception occurring in the case of iron, whose action was disproportionately great. Cutting radial slits in the disc diminished the action very much.

Besides the component tangential to the disc, it is found that there is a repulsive normal action on the pole of the magnet, and also a radial action, which may be towards or from the centre of the disc, according as the pole is nearer or farther from the centre of the disc. These actions look at first sight somewhat more difficult to explain; but a little consideration will show that the laws of induction account for these also.

Let us first suppose the induced currents to appear and die away instantly after the small motion of the disc which produces them (we may suppose the motion of the disc to take place by an infinite number of small jumps). Thus the currents of induction are obviously symmetrical with respect to the diameter through the foot of the perpendicular from the magnetic pole on the disc, and we may roughly represent the electromagnetic action by a magnet placed perpendicular to the diameter at a certain distance from the centre of the disc, its south pole pointing in the direction of the disc's motion if the inducing pole be a north pole. Let OX (fig. 52) be the direction of the diameter in the same vertical plane as the pole, NS the representative magnet, OY being the direction of motion. By our present supposition the inducing pole M lies in the plane of ZOY, in which case it is obvious that the resultant action reduces to a tangential component T parallel to OY.

But, owing to the inductive action on each other of the currents in the disc, the induced currents do not rise and fall instantaneously, but endure for a sensible time. We may roughly represent

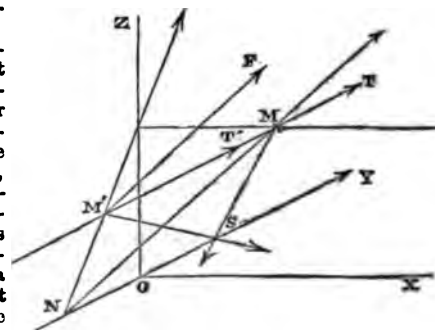


Fig. 52.

¹ Pogg. Ann., 1861.

² Monatsber. der Ber. Akad., 1874.

³ Pogg. Ann., 1871.

⁴ Pogg. Ann., 1874.

the effect of this by supposing the representative magnet NS carried onwards a little with the disc, or, which amounts to the same thing, we may suppose the pole M to lag a little behind at M' (lying, say, on MM' perpendicular to ZOY.) The action of N will now preponderate, and the resultant force on M' will be in the direction M'F. This force, when resolved parallel to OY, OZ, OX, gives a tangential component as before, a repulsive normal component, and a radial component, which will be directed to or from the centre of the disc, according as the representative magnet lies farther from or nearer to the centre of the disc than the foot of the perpendicular from M'.

The original explanation of rotation magnetism (Faraday, *Exp. Res.*, 81, &c.) should be consulted by the reader who wishes to pursue the subject. An account of the researches of Nobili, Matteucci, and others will be found in Wiedemann, *Bd. ii. § 860, &c.* The mathematical theory has been treated by Jochmann, who neglected the inductive action of the currents on each other (*Crelle's Journ.*, 1864; *Pogg. Ann.*, 1864; also Wiedemann, *l.c.*). A complete theory of the induction of currents in a plane conducting sheet has been arrived at by Maxwell by means of an extremely elegant application of the method of images (*Proc. R.S.*, 1872; also *Electricity and Magnetism*, vol. ii. §§ 668, 669).

On the Origin of Electromotive Force.

It remains for us now to view the transformations of energy which take place in the voltaic circuit from the other side, and to inquire whence comes the energy that is evolved in so many different forms by the electric current. Two distinct questions are here involved. First—What form of energy is being absorbed, and at what part of the circuit does the absorption take place? Secondly—Where is the electromotive force which drives the current situated?

To the first of these questions experiment has given, on the whole, a very satisfactory answer. The electric circuit is, indeed, one of the best instances of the great law of conservation, which states that the appearance of energy anywhere is always accompanied by the disappearance somewhere of energy to an equal amount. No general discussion of this first question is necessary; it will be sufficient to indicate the application of the general principle when we deal with particular instances.

Unfortunately the answers, both experimental and theoretical, that have been at different times given to our second question, are not so concordant as could be desired. The reader is, therefore, cautioned against accepting without due examination¹ anything that may be here advanced.

Perhaps the most general principle concerning the origin of electromotive force recognized by physicists of the present day is the following:—

When two different substances are in contact, there exists in general an electromotive force at the surface of separation, tending to displace electricity across that surface.

This electromotive force is commonly called the "electromotive force of contact," or simply the "contact force." In the particular case of two conductors in contact, the effect of this force would simply be to maintain a certain difference of potential between them.

Although the earliest known case of electrification—viz. amber rubbed with woollen cloth—is an instance in point, and although many experiments on electrification by the friction of different substances were made, yet this principle was not recognized fully till the experiments of Galvani and Volta directed the attention of men of science to the matter.

Volta was the first to demonstrate clearly the existence of the contact force in the case of metals. A simplified form of his fundamental experiment is the following. The

¹ This applies particularly to any indications of the views of living physicists.

upper and lower plates of a condensing electroscope (see above, p. 34) are made of different metals, say copper and zinc. Let the upper plate be laid upon the lower, and the metallic contact ensured by connecting them for an instant by means of a wire. If the upper plate be now lifted vertically upwards, the gold leaves of the electroscope diverge, indicating that the zinc plate is now positively electrified to a considerable potential. This is explained as being due to the contact force at the junction of the copper wire with the zinc plate, by virtue of which the zinc is at a higher potential than the copper. Suppose the upper plate to be connected with the earth, then if E be the contact force, the potential of the zinc plate is E. Now E is very small, but as soon as the upper plate is raised the potential of the lower plate is increased in the same ratio as its capacity is diminished; hence the divergence of the leaves. Volta Law of found that he could arrange the metals in series, thus— Volta.

Zn.....	0	Fe.....	1
Pb.....	5	Cu.....	2
Sn.....	1	Ag.....	3

such that, when any metal is placed in contact with one below it in the series, it takes a higher potential; and he found that the electromotive force between any two metals in the series is the sum of the electromotive forces between every adjacent intervening pair. Thus, if Zn|Pb denote the electromotive force from lead to zinc, we get from the above table,

$$\begin{aligned} \text{Zn|Pb} &= 5, \text{ Pb|Sn} = 1, \\ \text{Zn|Sn} &= \text{Zn|Pb} + \text{Pb|Sn} = 6, \\ \text{Pb|Cu} &= \text{Pb|Sn} + \text{Sn|Fe} + \text{Fe|Cu} = 6, \end{aligned}$$

and so on.

It follows from Volta's law that, if a number of metals be connected up in series, the difference of potentials between the extreme metals is independent of the intermediate metals, and, in particular, is zero if the extreme metals be the same. We cannot, therefore, have a resultant electromotive force in a closed circuit consisting of metals merely. This is entirely in accordance with experiment, provided the temperature be the same everywhere.

While one party of physicists neglected or attempted to explain away Volta's contact force, another took up the investigation, and endeavoured to obtain precise measurements of it in different cases. Careful experiments of this kind were made by Kohlrausch² and Gerland,³ by a method due to the former.

A condenser is used whose plates are made of the metals to be tested, say zinc and platinum. The plates are first placed parallel to each other at a very small distance apart, and touched simultaneously with a wire (say of platinum). A difference of potentials is thereby established, so that if the potential of the Pt be zero that of the Zn is Zn|Pt. (Here we neglect the contact force between air and zinc and between air and platinum. No experimental proof that we know of has been given in support of this, see below, p. 85). In consequence of this difference of potentials the Zn plate becomes positively charged. The wire is now removed, the plates of the condenser separated to a considerable distance, and the Zn plate connected with one electrode of a Dellmann's electrometer, the other electrode of which is connected to earth. The reading is proportional to the potential difference Zn|Pt increased in the ratio in which the capacity of the Zn plate has been decreased by the separation. Hence, if A be the reading,

$$\text{Zn|Pt} = \lambda A \quad \dots \dots (1).$$

The condenser plates are now brought into their former position, and connected through a Daniell's cell, consisting

² *Pogg. Ann.*, 1853.

³ *Pogg. Ann.*, 1863.

of a strip of zinc immersed in a porous vessel filled with zinc sulphate, which is itself immersed in a vessel containing copper sulphate, into which dips a strip of copper. In the first instance, the copper strip is connected with the zinc plate, and the zinc strip with the copper plate of the condenser. The difference between the potentials of the condenser plates is easily found by an application of Volta's law¹ to be $D + \text{Zn|Pt}$, where D denotes the difference between the potentials of the two pieces of copper forming the terminals of a Daniell's cell; hence if B be the electrometer reading, after removing the Daniell and separating the plates as before, we have

$$D + \text{Zn|Pt} = \lambda B \quad \dots \dots (2).$$

If we connect up the Daniell the opposite way with the condenser, then we get a reading C , such that

$$D - \text{Zn|Pt} = \lambda C \quad \dots \dots (3).$$

From (2) and (3) we get

$$\text{Zn|Pt} = \frac{B-C}{B+C} D \quad \dots \dots (4),$$

which gives the contact force Zn|Pt in terms of the electromotive force of a Daniell. From (1), (2), (3) we get

$$B - C = 2A,$$

an identical relation which the observations ought to satisfy, and which, therefore, affords the means of testing their accuracy.

In this way Kohlrausch found for Zn|Cu the value $\cdot 48D$, or in other words, that the contact force from copper to zinc is about equal to half the electromotive force of a Daniell's cell. As an instance of the general nature of the results, we give two series of numbers from the observations of Kohlrausch. The contact force is between zinc and the metal mentioned in first column in each case, and Zn|Cu is taken = 100.

Cu.....	100	100
Au.....	112	115
Ag.....	105	109
Pt.....	106	123
Fe.....	75	—

In the second set of experiments the metals were carefully cleaned, whereas in the first set they may have been a little oxidized. This may very well account for the differences, for Kohlrausch found oxidized zinc strongly negative² to freshly cleaned zinc. In fact, he found $\text{Zn|ZnO} = \text{about } \cdot 4\text{Zn|Cu}$.

In order to test Volta's law, a further series of observations was made, giving the contact force between iron and several metals. The following table gives the results observed directly and calculated from the table last given:—

	Observed.	Calculated.
Cu.....	31.9	25.3
Pt.....	32.3	32.3
Au.....	39.7	38.0
Ag.....	29.8	30.9

It will be seen that, with the exception of the values for Fe|Cu , the agreement is very fair.

It is not necessary to give here the results of Gerland and Hankel.³ The latter made a great number of very careful experiments. He showed that the results depend

¹ The truth of which, therefore, is assumed. The assumption of course is justified *a posteriori*.

² A metal is said to be negative to another when it assumes the lower potential in contact, and positive when it assumes the higher potential.

³ *Abh. der Königl. Sächs. Gesellschaft*, 1861, 1865.

on the nature of the surface of the bodies, being different when the surface is filed and when it is polished with rouge or other powder. His tables also show the gradual change effected in the contact force when the plates are exposed to the action of the air.

According to Volta, the contact forces between metals^{Liq and met} and liquids are either very small, and do not follow the same law as the contact forces between metals, or else are absolutely non-existent. Subsequent observers, however, demonstrated the existence of contact forces in this case also, but showed that they do not obey Volta's law like the contact forces in the case of metals.

Becquerel⁴ placed the fluid to be examined in a capsule^{Bec} of the metal, say copper. The capsule was placed on the upper plate of a condenser consisting of two copper plates in connection with a gold-leaf electroscope. The fluid and the lower plate of the condenser were touched each with a finger for a short time, and then the upper plate was removed. The divergence of the gold leaves was taken to indicate the contact force. In this way Becquerel found that zinc, copper, and platinum were mostly negative to alkaline solutions; but the metals were in general positive to concentrated sulphuric acid. It is obvious, however, that the result of the experiment is complicated by the contact of the hand with the liquid and with the metal of the condenser.

Similar objections apply to the results of Pfaff⁵ and Peclet.⁶

Buff⁷ made the lower plate of his condenser of the metal to be examined, of zinc for example; upon this was laid a thin glass plate on which was spread a thin layer of the liquid to be examined, or a piece of filter paper soaked with it. A zinc wire was used to bring the liquid and the lower plate of the condenser into communication; this wire was then removed and the glass plate with the liquid lifted. The divergence of the leaves was taken to indicate the contact force between zinc and the liquid. Although this method is an improvement on the methods of Becquerel and Peclet, it is still unsatisfactory, owing to the presence of the glass.

The most extensive and at the same time most careful^{Hank} experiments at present on record are those of Hankel.⁸ ^{meth}

The fluid (L) to be examined was placed in a wide-mouthed funnel. The condenser was formed by the surface of the liquid and a copper plate, which could be placed parallel to it at a very short distance apart, and raised as usual. The stem of the funnel was bent at a right angle twice, and ended in a wider portion, into which dipped a strip of the metal (M) to be examined. M was connected for a moment by a platinum wire with the copper plate and also with the earth. The wire was then removed, the plate was raised, and its potential determined by means of Hankel's dry pile electroscope. The reading (A) is proportional to $\text{Cu|Pt} + \text{Pt|M} + \text{M|L}$, or, by Volta's law, to $\text{Cu|M} + \text{M|L}$. Hence

$$\text{Cu|M} + \text{M|L} = \lambda A.$$

In the next place, the funnel is emptied and a plate of the metal M placed on its mouth. The copper plate is lowered so that it is at the same distance as before, contact established by means of the platinum wire, and so on. The reading being B , we have

$$\text{Cu|M} = \lambda B.$$

The plate of M is replaced by a plate of zinc, and the experiment repeated, and we have, C being the third reading,

$$\text{Cu|Zn} = \lambda C.$$

⁴ *Ann. de Chim. et de Phys.*, 1824.

⁵ *Pogg. Ann.*, 1840.

⁶ *Ann. de Chim. et de Phys.*, 1841.

⁷ *Ann. der Chem. u. Pharm.*, 1842.

⁸ *Abh. der Königl. Sächs. Gesellschaft*, 1865.

From these three results we get

$$M|L = \frac{A-B}{C} \text{ Cu|Zn.}$$

It is not necessary to quote Hankel's results here. The reader may refer to Wiedemann's *Galvanismus*, or to Hankel's paper.

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Sir William Thomson has given a new proof of the existence of Volta's contact force as follows.¹ A ring is formed, one-half of which is copper the other half zinc. This ring is placed horizontally, and a needle made of thin sheet metal is so balanced as to form a radius of the ring. If when the needle is unelectrified it be adjusted so as to be over the junction of the two metals, then, when it is positively electrified, it will deviate towards the copper, and when negatively electrified, towards the zinc. Again, if a whole, instead of a half needle as above, be suspended over a disc made of alternate quadrants of zinc and copper, or, better still, inside a flat cylindrical box constructed in a similar way, so that when the needle is unelectrified its axis coincides with one of the diameters in which the disc is divided, then when the needle is positively electrified it will take up a position such that its axis bisects the copper quadrants; if it be negatively electrified, its axis will bisect the zinc quadrants.

Thomson has also given an elegant demonstration of the contact force between copper and zinc by means of an apparatus which is a modification of his water-dropping apparatus.² A copper funnel is placed in a cylinder of zinc, and drops copper filings at a point near the centre of the cylinder. The filings are charged negatively by induction as they fall, owing to the excess of the potential of the zinc cylinder over that of the copper. If therefore the filings be caught in an insulated metal can, they will communicate to it a continually increasing negative charge, while the zinc cylinder and the copper funnel will become charged more and more positively.

Thomson finds, in agreement with Kohlrausch, that, when the copper and zinc are bright, the electromotive force of contact is about half that of a Daniell's cell. When the copper is oxidized by heating in air, the contact force is equal to a Daniell or more. He has gone a step farther, and shown that when two bright pieces of copper and zinc are connected by a drop of distilled water, their potentials are as nearly as can be observed the same.³

m. The subject of contact electricity has been taken up quite recently by Clifton.⁴ He experiments on the contact force between a metal and a liquid by a method which is a simplification of Hankel's.

Two horizontal plates are used of the metal M; the liquid L is placed in a glass vessel on the lower plate and connected with the lower plate by a strip of the same metal which dips into it. The upper plate is lowered to a distance of 0.1 or 0.2 mm. from the surface of the liquid, which acts as the lower surface of the condenser, and the upper plate and lower plate are connected by a copper wire. The difference of potential between the two surfaces of the condenser is therefore $M|L$. The copper wire is then removed, the upper plate raised, and its potential measured with a Thomson's electrometer. In this way a contact force equal to the thousandth part of $Zn|Cu$ can be detected.

Clifton finds zinc and copper to be both positive to water to about the same degree, and both very slightly negative to dilute sulphuric acid. He concludes therefore that zinc and copper dipping in water will be at the same potential. This he verifies directly, finding that any difference of

potential, if it exist, must be less than .00079 of the electromotive force of a Daniell. The result of Sir William Thomson is therefore confirmed.

There are many other points of interest in Clifton's paper, but, as the results are given in most instances as preliminary, we need not discuss the matter farther.

Before leaving this subject, it may be well to notice that Source of there is one point which is not touched by all these experiments, viz., the question whether there is or is not a contact force between metals or even liquids and air. It has not yet been shown that the results of the experiments which are supposed to demonstrate that $Zn|Cu$ is about half the electromotive force of a Daniell could not be equally well explained by supposing the difference of potential to be $Cu|A + A|Zn + Cu|Zn$, whence $Cu|Zn$ is very small compared with $Cu|A$ and $A|Zn$. This supposition would not invalidate Volta's law; nor would it contradict Clifton's results, for we have, in accordance with his experiments, on the new hypothesis,

$$Aq|A + Cu|Aq + Aq|Cu = Aq|A + Zn|Aq + A|Zn,$$

therefore, transposing,

$$Zn|Aq + Aq|Cu + Cu|A + A|Zn = 0,$$

which, according to the new hypothesis, means that copper and zinc immersed in water are at the same potential. In this view, the important part of the contact force usually observed between zinc and copper would be $Cu|A + A|Zn$,⁵ which must therefore, by Sir Wm. Thomson's result, be the same as $Cu|Aq + Aq|Zn$.

It is not very easy to see how this point is to be settled by direct measurements of electromotive force. Supposing, however, that it were settled, and that the contact force between two given metals A and B, and between each of them and a given liquid L, were known, then the difference of potentials between the two metals when immersed in any liquid could be predicted in all cases, and also the initial electromotive force tending to send a current through a circuit made by connecting the metals together and dipping them into the liquid.

A number of cases of this kind have been investigated Gerland's by Gerland;⁷ but satisfactory agreement between theory and observation has been attained in but a few cases. Researches of this kind are beset with a double array of almost insuperable difficulties.

The direction of the initial resultant electromotive force in any circuit of two metals and one fluid may be found by observing the first swing of a galvanometer through which the circuit is suddenly connected. Considerable precautions must be taken to obtain consistent results, and when all care has been taken, it is not found that the results of one observer always agree with those of another. This is scarcely to be wondered at, when we consider the difficulty of making sure that in two different experiments we are operating with absolutely the same metals and fluid in absolutely the same conditions as to surface.

When the current tends to pass from one metal to another through the liquid in which they are immersed, the former metal is said to be positive to the latter. If the whole electromotive force in the circuit be the sum of all the contact forces at the various junctions, then it follows easily that we ought to be able to arrange the metals in a series, such that any one in the series is positive to any following one and negative to any preceding when both are dipped in the same liquid. It does not follow that the order of the series is the same for different liquids; this would be so if the metallic contacts alone were operative.

Many electromotive series of this kind have been given by different experimenters; but they have lost much of their

¹ *Proc. Lit. and Phil. Soc. of Manchester*, 1862, or *Reprint*, p. 319.

² *Reprint*, p. 324.

³ *Jenkin, Electr. and Mag.*, § 22.

⁴ *Proc. R. S.*, June 1877.

⁵ A stands for air.

⁶ See Maxwell's *Electricity*, vol. I. § 249.

⁷ See Wiedemann, *Bd. I.* § 36.

Electromotive series.
Two metals and one liquid.

interest now that the theory of metallic contact, pure and simple, is given up. The following set is given by Faraday:—¹

HNO ₃ (dil.)	H ₂ SO ₄ (dil.)	HCl	HNO ₃ (strong.)	KHO.	KHS.	K ₂ S ₂
Ag	Ag	Sb	Ni	Ag	Fe	Fe
Cu	Cu	Ag	Ag	Ni	Ni	Ni
Sb	Sb	Ni	Sb	Cu	Bi	Bi
Bi	Bi	Bi	Cu	Fe	Pb	Sb
Ni	Ni	Cu	Bi	Bi	Ag	Pb
Fe	Fe	Fe	Fe	Pb	Sb	Ag
Sn	Pb	Pb	Sn	Sb	Sn	Sn
Pb	Sn	Sn	Pb	Cd	Cu	Cd
Cd	Cd	Cd	Zn	Sn	Zn	Cu
Zn	Zn	Zn	Cd	Zn	Cd	Zn

It will be seen that the order of the metals is not the same for different liquids.

Contact of two liquids.

Just as between different metals and between metals and liquids there is a contact force, so there is a contact force between different liquids. The direct observations of this contact force are very few and uncertain. One thing, however, is settled, viz., that the contact forces between liquids do not universally at least obey the law of Volta, or, at all events, do not form a consistent series with the metals; for a great variety of circuits of one metal and two solutions have been discovered in which the resultant initial electromotive force is not zero. Faraday² has even found cases of this kind with one metal and two different strengths of the same solution.

The cell of Becquerel is a favourite illustration of such a circuit. It consists of a porous vessel filled with a solution of potash and immersed in a beaker containing nitric acid; two strips of platinum immersed in the potash and nitric acid respectively form the plates. The current goes in the cell from the potash to the nitric acid. The following additional examples are taken from Wiedemann.³

One metal and two liquids.

The current flows from the metal through the liquids in the order named to the same metal again. For brevity, the metal is named only once.

Metal.	1st Fluid.	2d Fluid.
Pt	KHO	Acids
Pt	CuSO ₄	Dil. H ₂ SO ₄
Pt	NaCl	ZnCl ₂
Pt	NH ₃	CuSO ₄
L	CaCl ₂	Dil. HNO ₃
M	Conc. H ₂ SO ₄	HNO ₃
R	KCy	HNO ₃

L stands for Zn, Cu, or Pt.

M " Cu, Fe, Pb, Sn, or Ag.

R " Ni, Bi, Pt, Hg, Pd, Sb, Fe, C, Ag, Zn, Cu, Cd, or Sn.

Two metals and two liquids.

A great variety of active voltaic circuits have been formed with two liquids and two metals. The best known class of cases is that in which the metals are in contact, as in the two-fluid batteries of Daniell, Grove, and Bunsen. But Faraday⁴ gives a list of some thirty cases in which the fluids and metals are placed alternately, so that there is no metallic contact. He marks the following combinations as powerful:—

Iron	Diluted nitric acid.	Platinum	Green nitrous acid.
Do.	Hydrochloric acid.	Do.	Do. do.
Do.	Solution of com. salt.	Do.	Do. do.
Copper	Potassium sulphide.	Iron	Dil. nitric acid.
Do.	Strong nitric acid.	Do.	Do. do.

It must be carefully noticed that the galvanometer indi-

cation in the first instant only is to be considered. After the first rush of electricity the direction even of the current may alter. Above all, no conclusion concerning the value of the initial electromotive force is to be drawn from measurements of the subsequent current. Quantitative determinations of the electromotive force in many of the above cases have been made by various methods, of which an account will be found in Wiedemann's *Galvanismus*, Bd. i. § 230. The most convenient plan is to use Thomson's quadrant electrometer, Lippmann's capillary electrometer, or some other instrument which allows us to measure the electromotive force while no current is passing through the cell. The galvanometer may also be used as in Latimer Clark's modification of Poggendorff's compensation method.

The apparatus may be arranged according to the scheme in fig. 53. ABC denotes part of the resistance in the circuit of the battery K; the circuits ApELB, AqFMC each contain a galvanometer, a cell, and a key. The cells E and F are so arranged as to tend to send currents in the same directions as K, but the resistances AB, AC are so adjusted that when the key L or the key M is depressed, no current is indicated by p or q. When this is so, we must obviously have $E = V_A - V_B$, $F = V_A - V_C$, &c., V_A , V_B , V_C denoting the potentials at A, B, and C. Hence, if P, Q, R denote the resistances in AB, AC, and in the whole circuit of K,

$$E = \frac{P}{R} K, \quad F = \frac{Q}{R} K.$$

If K were a constant battery, and its internal resistance were either known or else so small as to form only an inappreciable fraction of R, then each of the equations just written might be used singly, and we might operate with one cell and one galvanometer, comparing the electromotive force of the cell with K. In general, however, this will not be possible, and then we have, eliminating K and R,

$$\frac{E}{F} = \frac{P}{Q},$$

from which we get the ratio of E to F independent of the variation of K and R. We can by this method therefore find the ratio of the electromotive force of a given combination to that of a standard cell, when no current is passing through either. The process would be perfect in practice if a standard cell could be constructed whose absolute constancy could be relied on.

Contact force from polarization.—The flow of electricity through the cell is accompanied by a deposition of the products of chemical decomposition on the plates, which alters the surface contact forces. This constitutes the phenomenon of polarization, which we have already partially studied. It will be useful to consider a little more in detail some of the forms in which it is met with.

The products of electrolysis which accumulate at the electrodes may be simply held in solution or precipitated, or they may combine chemically with the solution; they may be deposited as a crust on the electrode, or they may enter into more or less intimate connection with it. Several of these different effects may occur together; but in almost all cases the result is the same, viz. a great weakening of the current after the first instant or so. This weakening of the current might be due either to a transition resistance caused by the alterations at the electrodes, or to an op-

¹ *Exp. Res.*, 2012.

² *Galvanismus*, Bd. i. § 63.

³ *Exp. Res.*, 1975.

⁴ *Exp. Res.*, 2020.

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posing electromotive force arising from the alteration of the contacts. The former was the explanation adopted in the earlier researches of Poggendorff and Fechner; but there can be no doubt about the existence of the latter effect, and Lenz showed that it was sufficient to account for the facts observed. It has been usual, therefore, to neglect the transition resistance altogether in the great majority of cases. It is certain, however, that it really exists in some instances. Consider the case of two electrolytic cells containing concentrated sulphuric acid, the electrodes being in the one copper plates, in the other platinum plates. Either of these cells inserted into a voltaic current will weaken the current, but for different reasons. In both cases hydrogen is freed at the negative electrode, and reduces sulphur from the strong acid, the effect of which is not great either way, for if the negative electrode be replaced by a fresh plate the weakening of the current remains. At the positive electrode oxygen is evolved, which oxidizes the copper in the one case and is deposited on the platinum in the other,—in both cases replacing the positive electrode by a fresh plate will cause momentary increase in the current; but the copper plate which served as positive electrode if tested against a fresh copper plate gives very little return current, whereas the positive platinum plate similarly tested gives a very powerful one. In the one cell the weakening of the current was due to the resistance of a crust of non-conducting copper oxide, in the other it was due to the contact force at the surface of the oxygenated platinum.¹

The polarization which arises from the deposition of gas on the electrodes is, if we except the case where peroxides are formed, by far the most powerful form, and has been much studied ever since voltaic batteries were used. Experiments like the one just quoted prove decisively that the electromotive force has its seat at the surface of the electrode itself, and is due to local alterations there. The certainty of this fact gives the study of the phenomenon great theoretical importance, since we may hope thereby to arrive at some idea of the nature of the contact force.

It is also certain that in most cases each electrode contributes separately to the whole electromotive force, for if we remove the polarized plates the adhering gas goes with them, and each plate is found to be electrically different when tested against a fresh or unpolarized plate. We may also take measures to remove the deposited gas by washing the plates with water, potash, or nitric acid, or by igniting them; and it is found that the more energetic the chemical agency thus employed the more thoroughly is the polarization destroyed.

It seems clear, therefore, that it is the mere fact of the presence of the gas on the electrode in some form or other which causes the electrical difference. We may go still further and produce the phenomenon by depositing gas by means other than electrolytic. If a piece of platinum foil be immersed in hydrogen it absorbs the gas, as has been shown by Graham. A piece of foil thus treated is positive to a piece of freshly ignited foil when both are placed in dilute sulphuric acid. The same result is obtained by saturating the liquid in the neighbourhood of the foil with hydrogen.² The activity thus conferred on the plate may be again destroyed by immersing it in chlorine or bromine, or even in oxygen, by igniting it, and so on. Similarly, if we dip a fresh piece of foil into chlorine or bromine, it will become negative to a fresh plate. The effect obtained by dipping the foil in oxygen in the ordinary state is very small, the oxygen deposited by electrolysis must therefore be in a different state to that condensed during mere im-

mersion in the gas. This is probably due to the fact that electrolytically generated oxygen contains ozone (see art. ELECTROLYSIS); and in accordance with this we find that a platinum foil ozonized by being held in the electric brush proceeding from a charged conductor, or rubbed with phosphorus, is negative to a fresh plate in dilute sulphuric acid.

The gas battery of Grove is a remarkable instance of the electro-^{Grove's} motive properties of gas-coated metals. Two long glass tubes A and B are arranged in the two necks of a Woulfe's bottle. The upper ends of the tubes are closed, but pierced by two platinum wires, to which are fastened two long strips of platinum foil (which are sometimes platinized)³ reaching to very near the lower ends of the tubes. The bottle and part of the tubes are filled with some liquid, say dilute sulphuric acid, and hydrogen introduced into B and oxygen into A. This may be very conveniently done by sending an electric current from A to B and decomposing the dilute acid, but it may be done in other ways as well. This arrangement has an electromotive force comparable with that of a Daniell's cell (see below, p. 88), and if the original volume of hydrogen in B be twice that of the oxygen in A it will continue to send the current through a closed circuit, the gas gradually disappearing in the tubes until none is left, when the current stops. These gas elements may be connected up in series, &c., and used like ordinary battery cells.

If the tube B be filled with ordinary hydrogen and A with liquid only, a current in the same direction as before is observed; but the liquid in A is decomposed and hydrogen evolved, which produces an opposing electromotive force and stops the current. If A contain oxygen and B fluid only, the current lasts for a very short time unless the oxygen contain ozone. This is in accordance with what we have already seen.

Grove⁴ has given an electromotive series for the different gases and metals as follows:—

Chlorine.	Metals which do	Alcohol.
Bromine.	not decompose	Sulphur.
Iodine.	water.	Phosphorus.
Nitric oxide.	Camphor.	Carbonic oxide.
Carbonic acid.	Volatile oils.	Hydrogen.
Nitrogen.	Olefiant gas.	Metals which de-
	Ether.	compose water.

In this series any member is positive to any preceding member.

We have already remarked that the polarization in many Electro-^{lytic} cases comes on very rapidly. In cases where the electro- motive force of the battery is not sufficient to produce a continuous evolution of gas, the current after the first rush dies away very rapidly. There are cases, however, in which a small current continues to flow for a very long time. Such currents are not accompanied by any visible evolution of gas, and it is clear that they could not be so accompanied, for, if the electromotive force of the battery be under a certain limit, the amount of chemical energy absorbed by the current per unit of time is less than the intrinsic energy of the ions liberated in a unit of time. It was originally supposed, therefore, that, besides this electrolytic conduction proper, fluids conducted to a slight extent like metals. But Helmholtz⁵ has shown that no such assumption is necessary, and has pointed out that when the fluid holds gas in solution a sort of electrolytic conduction may take place which involves, it is true, liberation of the ions, or at least of an ion, but so that the final result does not imply absorption of more energy than the battery can furnish per unit of time in accordance with Faraday's law of electrolytic conduction.

Suppose, for instance, that the dilute sulphuric acid in an ordinary voltmeter held H in solution. When the current passes, O appears at the anode and unites with the H in solution to form water; a corresponding quantity of H is thereby liberated at the cathode, which is either absorbed by the platinum electrode or diffuses towards the anode, to combine in its turn with the appearing oxygen. It is obvious that the liberation of the ion in such a case does not involve absorption of energy to the extent necessary when both H and O are disengaged from water. A current might therefore be kept up under such circumstances for a long time by an electro-

¹ Wiedemann, Bd. i. § 455.

² See Macaluso's experiments, Wiedemann, *Nachträge*, § 53.

³ This is best accomplished by washing the foils in hot nitric acid, and then using them to decompose a solution of platonic chloride with the current of two cells of Grove.

⁴ *Phil. Trans.*, 1845.

⁵ *Pogg. Ann.*, 1873.

motive force under that of a cell of Daniell. Helmholtz has given the name of electrolytic convection to this species of electrolytic conduction. He has shown that the phenomenon comes to an end when the liquid is perfectly freed from gas. The absorption of the gases by the electrodes plays a great part here, and gives rise in gas-free liquids to a phenomenon analogous to the residual discharge. When the battery is first turned on, a rush of electricity takes place, then there is a small current which gradually dies away. The first rush is like the instantaneous charge of a condenser; the small current which arises from the slow penetration of the ions into the platinum corresponds to the formation of latent charge. When the voltmeter is disconnected from the battery and discharged through a galvanometer, we have a first rush due to the part of the ions accumulated on the surface of the platinum, and then a gradually decreasing current due to the emergence of the gas which had penetrated into the metal. When the electromotive force which presses the gas into the electrode is removed, the absorbed gas will move very nearly in accordance with the ordinary law of diffusion, and the rate of its reappearance will depend very little¹ on the electromotive force at the surface of the electrode. Consequently the residual current furnished by such an apparatus will not depend on its external resistance. A sudden increase of the external resistance will simply cause the current to diminish until sufficient surface density has been attained to raise the electromotive force to the value required to keep up the same current as before through the increased resistance; and the converse will happen if the external resistance be decreased.

This passage of the gas into the substance of the electrode has, at the instance of Helmholtz, lately been investigated by Root.² He finds that in certain cases, when only one side of a platinum foil is exposed to electrolysis, the deposited gas, whether H or O, actually passes through and produces the corresponding polarization on the other side of the foil.

Maximum of polarization.

It might at first sight be expected that in all cases where the electromotive force in the circuit is sufficient to produce continual evolution of the ions the polarization would be the same. This is not by any means the case, however, owing to the fact that the final state of the liberated ions varies with the strength of the current, or, more correctly speaking, with the *current density*, i.e., the amount of electricity which crosses unit section of the electrode in unit time. When H₂ and O are being liberated from dilute H₂SO₄, this depends mainly on the formation of variable quantities of ozone and H₂O₂. This variation of the physical state and intrinsic energy of the liberated ions, is a fact of the greatest importance in the art of electro-metallurgy. A better instance could not be given than Gore's electrolytic modification of antimony, whose intrinsic energy is strikingly manifested by its explosive properties.

The effect of enlarging the surface of the electrode in diminishing the polarization in the case where the maximum polarization is not reached was noticed above (p. 48). It has also the effect of diminishing the *maximum* of polarization in the case where the ions are completely set free. Platinizing has the same effect. Thus Poggendorff³ found for the maximum polarization 2.12 to 2.32⁴ for bright platinum plates, while it was only 1.83 to 1.85 for platinized plates. The effect of platinizing on the hydrogen and oxygen polarization was about equal for strong currents, but greater on the hydrogen polarization when the current was weaker. On the other hand, by using small platinum points to decompose water in Wollaston's manner, Buff⁵ found the polarization as high as 3.31.

Poggendorff⁶ and Crova⁷ have investigated the dependence of the maximum of polarization on the current density. It follows from their researches that it can be represented by $A - B^{-a}$, I being the current density.

It would appear that the maximum of polarization is decreased by increasing the temperature of the cell. The

effect, however, in *some* cases at least which have been adduced to prove this, might be explained by the decrease of the internal resistance of the cell.

Agitating the electrodes, stirring the liquid in their neighbourhood, or any other process which tends to dissipate the deposit on the electrode, leads, as might be expected, to a diminution of the polarization. The effect of increased pressure in retarding or helping electrolysis might be appreciable in certain cases. Suppose that an electrochemical equivalent of the ions, during liberation under a pressure p , increases in volume by v , then during the passage of a unit of electricity work to the extent pv is done; the electromotive force required to free the ions must therefore have a part equal to pv which may increase or decrease as the process goes on. If the ions be gases which obey Boyle's law very nearly, then pv is constant, so long as the temperature remains the same; so that we cannot expect, within reasonable limits, to check the electrolysis of dilute sulphuric acid by conducting it in a closed vessel.⁸

We have repeatedly drawn attention to the rapidity with which the polarization decays in the first few instants after the plates are connected through a circuit of moderate resistance. Direct proofs of this have been given by Beetz⁹ and Edlund¹⁰. The former shows that the oxygen polarization decays much more rapidly than the hydrogen polarization, which is not to be wondered at, considering the greater readiness of platinum to absorb hydrogen; with palladium electrodes the difference would doubtless be still more marked. The reader may also consult an interesting paper on this subject by Bernstein¹¹ who concludes that in a certain case the polarization diminished by $\frac{1}{3}$ to $\frac{1}{10}$ of its value in about $\frac{1}{1000}$ of a second.

There seems to be little reason to doubt the substantial accuracy of the facts just mentioned; and the reader will not fail to see the application to the theory and practice of the measurement of the electromotive forces of inconstant electromotors, a category under which, unfortunately for the electrician, all known voltaic batteries must be classed. The remark applies with double force to the measurement of the electromotive force of polarization. Many measurements of the latter have been made. We quote a few, to give the reader a general idea of the magnitudes involved; into a discussion of the methods we cannot enter here.

Hydrogen and Oxygen Polarization of bright Platinum Plates.¹²

Whole Polarization.	H Polarization.	O Polarization.	Observer.	Numerical results.
2.33	Wheatstone.	
2.56	Buff.	
2.31	1.15	1.16	Svanberg.	
2.33	1.16	1.16	Poggendorff.	
...	1.15	...	Beetz.	

Polarization of Platinum Plates with different Gases compared with the Electromotive Force of Platinum Plates with the same Gases against a fresh Platinum Plate in Grove's Gas Battery.¹³

Gas	Polarization.	Pt.G/Pt.
I	.171	.161
Br	.329	.323
Cl	.505	.466
H	.910	.814
Cl and H	1.375	1.335

¹ Within certain limits, of course.

² Pogg. Ann., 1876.

³ Wiedemann, Bd. i. § 480; Pogg. Ann., 1847.

⁴ Unless otherwise stated, our unit of the electromotive force is for the present the electromotive force of a Daniell's cell.

⁵ Wiedemann, Bd. i. § 473; Pogg. Ann., 1867.

⁶ Pogg. Ann., 1864.

⁷ Ann. de Chim. et de Phys., 1863.

⁸ Maxwell, vol. i. § 263. Other matters of great interest are stated there. See also the instructive analysis of the phenomena of polarization in §§ 294-271.

⁹ Pogg. Ann., 1850.

¹⁰ Pogg. Ann., 1852; see also Wiedemann, Bd. i. § 495; &c.

¹¹ Pogg. Ann., 1875.

¹² From Wiedemann, Bd. i. § 478.

¹³ Beetz, quoted in Wiedemann; Pogg. Ann., 1853.

Polarisation of various Metals measured with Thomson's Quadrant Electrometer.¹

Oxygen Plate.	Hydrogen Plate.	Polarization.	No. of Cells in Polarizing Battery.
Freshly ignited Pt.	Pt	1.64-2.30	1-8
Pt	Pd	1.50-1.85	1-4
Pd	Pt	1.60-1.91	1-4
Pt	Fe	2.16	3
Fe	Pt	0	3
Fe	Fe	0	3
Al	Al	1.09-5.20	1-6

Although the polarization by gas deposits has absorbed so much of the attention of physicists, it is by no means a solitary instance. The phenomenon is universal. It appears even with zinc plates in zinc sulphate, and copper plates in copper sulphate. The nearest approach to unpolarizable electrodes is the case of *amalgamated* zinc plates in zinc sulphate, originally discovered by Du Bois Reymond. When the sulphate solution is neutral, the polarization, as may be shown by immersing a large number of plates in series in the solution, is extremely small.

For an account of polarization at the surface of two liquids observed by Du Bois Reymond, and other kindred matters, and for many other facts which we have passed over in silence, the reader may consult Wiedemann's *Galvanismus*. Some account will be found in the article *ELECTROLYSIS* of the remarkable phenomenon of the "passivity of iron, and of the powerful polarization arising from the formation of superoxides, on which depends the action of the secondary pile of Planté."

Application of the Laws of Energy to the Voltaic Circuit.—In the classical series of researches by which Joule laid the foundations of the laws of energy, a considerable share of attention is devoted to the energetics of the electric current. Guided by the great idea which he was gradually developing, Joule made experimental determinations of the amount of energy of various kinds evolved in the electric circuit. We have already seen how he measured the quantity of heat developed in a metallic conductor, and in an electrolyte.² This quantity was found to vary as the product of the resistance of the conductor into the square of the current strength, account being taken of disturbances at the electrodes in the case of electrolytes.

These disturbances were considered in the first memoir and allowed for. The accuracy of the view taken of them, to which Joule was led by the opinion of Faraday, that the solution of the oxide in the voltaic cell had no active share in producing the electric current, was justly questioned, implicitly by Sir Wm. Thomson³ in 1851, and explicitly by Bosscha⁴ in 1859.

In a later memoir, however,⁵ Joule made a direct experimental investigation of these secondary effects, and shows how they can be accounted for. His results have not been shaken by subsequent investigators; and the general conclusions to be drawn from them are not in the least affected by the theory of secondary action, which is suggested in the paper. These, so far as we are now concerned with them, are as follows:—

"1st. In an electrolytic cell there are three distinct obstacles to the voltaic current. The first is *resistance to conduction*; the second is *resistance to electrolysis without chemical change*⁶ [arising simply from the presence of

chemical repulsion];⁸ and the third is *resistance to electrolysis, accompanied by chemical changes*.

"2d. By the first of these (the resistance to conduction) heat is evolved exactly as it is by a wire, according to the resistance and the square of the current; and it is thus that a part of the heat belonging to the chemical actions of the battery is evolved. By the second a reaction on the *intensity*⁹ of the battery occurs, and wherever it exists heat is evolved exactly equivalent to the loss of heating power in the battery arising from its diminished intensity. But the third resistance differs from the second, inasmuch as the heat due to its reaction is rendered latent, and thus lost to the circuit.

"3d. Hence it is that, however we arrange the voltaic apparatus, and whatever cells of electrolysis we include in the circuit, the whole caloric of the circuit is exactly accounted for by the whole of the chemical changes.

"4th. As was discovered by Faraday, the *quantity* of current electricity¹⁰ depends upon the number of atoms which suffer electrolysis in each cell; and the intensity depends on the sum of chemical affinities. Now both the mechanical and heating powers of a current are (per equivalent of electrolysis in any one of the battery cells) proportional to its intensity. Therefore the mechanical and heating powers of the current are proportional to each other.

"5th. The magnetic electrical machine enables us to convert mechanical power into heat by means of the electric currents which are induced by it; and I have little doubt that, by interposing an electromagnetic engine in the circuit of a battery, a diminution of the heat evolved per equivalent of chemical change would be the consequence, and this in proportion to the mechanical powers obtained."¹¹

The above statement of Joule's contains, in a form which seems to us neither ambiguous nor *obscure*,¹² an exposition of the leading experimental principles of the energetics of the electric circuit. Besides the papers of Joule we have mentioned, two others on the electrical origin of the heat of chemical combination ought to be read in connection with this subject.¹³ The now famous tract of Helmholtz, "Ueber die Erhaltung der Kraft," which appeared in 1847, shortly after these papers of Joule, did much, by its able statement of the issues, to advance this branch of electrical science, and should be consulted by every thorough student.

An extremely important contribution to the experimental evidence for the law of energy in the case of electric currents was furnished by the researches of Favre.¹⁴ He uses a calorimeter, which is virtually a mercury thermometer with an enormous bulb, into which are inserted a number of test-tube shaped vessels all opening outwards. When a heated body is placed in one of these vessels its heat is quickly communicated to the mercury in the calorimeter, and the amount of heat thus communicated is measured by the expansion of the mercury, which is measured as usual by noting the displacement along a capillary tube. Into one of the recesses of the bulb of this calorimeter containing a quantity of dilute sulphuric acid was introduced 33 grm. of granulated zinc. The heat evolved during its dissolution was 18682 units (gramme-degrees C.). Five of the recesses were then furnished with dilute sulphuric acid of the same strength as before, and into them were put five elements of Smee (amalgamated zinc and

¹ Tait, *Phil. Mag.*, 1869. This method is in some respects one of the best for measurements of the kind.

² *Phil. Mag.*, 1841. ³ *Phil. Mag.*, 1851 (2), p. 554.

⁴ *Pogg. Ann.*, cviii. p. 319.

⁵ *Mém. Lit. and Phil. Soc. Manchester*, 2d ser. vii., 1843.

⁶ This resistance is, in more modern language, an "opposing electromotive force."

⁷ The meaning of "without chemical change" will be seen directly.

⁸ The brackets here are ours; they contain Joule's theoretical view with which we are not now concerned.

⁹ In modern phrase, "electromotive force."

¹⁰ That is, current strength.

¹¹ This he experimentally verified, *Phil. Mag.*, 1843.

¹² Cf. Verlet, *Théorie Mécanique de la Chaleur*, § 327.

¹³ *Phil. Mag.*, 1842 (1), and 1843 (1).

¹⁴ *Ann. de Chim. et de Phys.*, 1854.

platinized copper). These were joined up in circuit by means of very thick copper wire, and the heat developed during the solution of 33 grm. of zinc observed as before. The result was 18674 units, i.e., almost exactly the same as before. A small electromagnetic engine was next introduced into the circuit, and the heat observed, first, when it was at rest; secondly, when it was in motion, but consuming all its energy in heat owing to friction, &c.; and thirdly, when it was doing work in raising a weight. The quantities of heat in the three cases were 18667, 18657, and 18374 units respectively. In the first four experiments, therefore, the heat developed in the circuit is sensibly the same, the mean being 18670; the heat developed in the last case is less than this by 296, which is the equivalent of the potential energy conferred on the raised weight. From this result the value of the mechanical equivalent of heat ought to be 443. This differs considerably from the best value (423 to 425), but not more so than might be expected from experimental errors.

Theory
of Sir
Wm.
Thom-
son.

Dynamical Theory of the Electromotive Force of the Battery.—In two very important papers published in the *Philosophical Magazine* for 1851, Sir William Thomson laid the foundations of the Dynamical Theory of Electrolysis, one of the objects of which, to use very nearly his own words, is to investigate, for any circuit, the relation between the electromotive force, the electrochemical equivalents of the substances operated on, and the dynamical equivalent of the chemical effect produced in the consumption of a given amount of the materials, and by means of this relation to determine in absolute measure from experimental data the electromotive force of a single cell of Daniell's battery, and the electromotive force required for the electrolysis of water.

Thom-
son's
law.

The relation sought for is found as follows. Let E be the electromotive force¹ of the battery. Then, by the definition of electromotive force, E is the whole work done in the circuit by a unit current during a unit of time. This work may appear as heat developed in the conductors or at the junctions according to the laws of Joule and Peltier, as the intrinsic energy of liberated or deposited ions, as work done by electromagnetic forces, as "local heat" in the battery (see below, p. 91), or otherwise. Let e be the electrochemical equivalent of any one of the elements which take part in the chemical action from which the energy of the current is derived, i.e., the number of grammes of that element which enter into the chemical action during the passage of unit current for a second; and let θ be the thermal equivalent of that amount of chemical action in the battery into which exactly a grammes of the element in question would enter,—in other words, the amount of heat that would be developed were the whole energy of the amount of chemical action just indicated spent in heat. Then it is obvious that the energy of the chemical action that takes place in the battery during the passage of unit current for a unit of time is $Je\theta$, where J is Joule's equivalent. Hence, by the principle of conservation, we must have

$$E = Je\theta;$$

or, in words, *the electromotive force of any electrochemical apparatus is, in absolute measure, equal to the dynamical equivalent of the chemical action that takes place during the passage of unit current for a unit of time.*

The value of J is known, being 4156×10^4 in absolute units. The value of e has been found by various experimenters (see below, p. 104), the most accurate result being probably that deduced from the experiments of Kohlrausch, viz. $e = .003411$ for zinc.

¹ All our quantities are measured, of course, in C. G. S. absolute units.

We may find θ by direct calorimetric experiments on Calcu the heat developed in the circuit. In this way Joule tion fr found for the thermal equivalent of the chemical action of heat d a Daniell's cell during the solution of 65 grammes of zinc velops in the circuit 47670 (grm. deg. C.), and Raoult², by a somewhat similar process, obtained the number 47800. These give for the heat equivalent of the chemical action during the solution of 1 grm. zinc 733 and 735 respectively. Substituting these values in our formula, we get for the electromotive force of Daniell's cell in absolute C. G. S. units 1.039×10^8 or 1.042×10^8 .

But we may proceed in a totally different way to find the value of θ . Direct observations have been made on the heat evolved in a great variety of chemical actions, and from these experiments we can calculate, with a considerable degree of accuracy, the value of θ , and thus deduce the electromotive force of a battery from purely chemical data. This method of procedure must of course be adopted if we wish to test the dynamical theory. Now, neglecting refinements concerning the state in which the copper is deposited, the state of concentration of the solution, &c., the chemical action in a Daniell's cell may be stated to be the removal of the Cu from CuSO_4 in solution, and the substitution of Zn in its place. Now, Favre and Silbermann have found for the heat developed in this chemical action 714 (grm. deg. C.) per grm. of zinc. This will give, by the above formula, for the electromotive force of Daniell's element 1.012×10^8 . The chemical action might also be stated as the combination of zinc with oxygen, and the solution of the zinc oxide thus formed in sulphuric acid, the separation of copper oxide from sulphuric acid, and of the copper from the oxygen. The quantity of heat evolved in the first two actions per grm. of zinc is, according to Andrews, $1301 + 369 = 1670$ (grm. deg. C.), and that absorbed in the last two actions $588 + 293 = 881$. Hence $\theta = 789$; this gives 1.118×10^8 . Professor G. C. Foster³ has calculated from Julius Thomson's experiments values 805, 1387, and 617 of θ for the cells of Daniell, Grove, and Smee respectively; the values of the electromotive forces corresponding to these are 1.141×10^8 , 1.966×10^8 , and $.875 \times 10^8$. These results are in fair agreement with the different values of the electromotive force obtained from direct experiment.

It follows from Thomson's theory that a certain minimum electromotive force is necessary to decompose water; and he calculated from the data of Joule that this minimum was 1.318 times the electromotive force of a Daniell's cell. Favre and Silbermann found for the heat developed in the formation of H_2O 68920, from which we conclude that the minimum electromotive force required to electrolyse water is $68920 \div 47800$, i.e., 1.44 times that of a Daniell's cell.⁴

Development of Heat at the Electrodes.—In a remarkable paper,⁵ which we have already quoted, Joule investigated directly the disturbing effect of the electrodes on the heat

² Wiedemann, Bd. ii. 2, § 1118.

³ Everett, *Illustrations of C. G. S. System of Units*, p. 77. No reference to the source is given.

⁴ Verdet (*Théorie Méc. de la Chaleur*, tom. ii. p. 207) states that Favre was the first to point this out, but gives no citation. It seems unlikely that Favre considered the matter so early as 1851. (See Violle's bibliography at the end of Verdet's volume.)

⁵ *Mem. Lit. and Phil. Soc. Manchester*, ser. 2, vol. vii., 1843. This paper seems to have been in a great measure lost sight of. In his earlier papers (*Pogg. Ann.*, ciii. § 504, 1858) Bosscha says he had not seen it. Poggendorff, in a note, p. 504, appreciates it in a manner which appears to us unjust. This may have arisen from misunderstanding of Joule's terminology, however. Verdet (*Chaleur*, tom. ii. p. 204) does not seem to have been acquainted with it. It is mentioned in the bibliography by M. J. Violle, however, under 1846, which is the date of the volume of the *Memoirs* in which it was published. The paper was actually read Jan. 1843.

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developed in an electrolyte. His method was as follows. A coil of wire whose resistance was known in terms of a certain standard was inserted in the circuit of six Daniell's elements, and the heat evolved in it carefully measured by immersing it in a calorimeter. The resistance of the rest of the circuit, including that of the battery, was found by interpolating a known resistance in the circuit and observing, by means of a tangent galvanometer, the ratio in which the current was reduced. (The assumption is here made that the electromotive force of a Daniell's cell is constant for different currents.) Knowing the heat evolved in a part of the circuit of known resistance, and knowing the resistance of the whole circuit, the heat evolved according to Joule's law in the whole circuit during the oxidation of 65 grammes of zinc can be calculated from the indications of the tangent galvanometer previously compared with a voltameter. Hence the thermal equivalent Θ of the work done by the electromotive force of a Daniell's cell during the solution of 65 gm. zinc can be found. The value of Θ arrived at by Joule in this way is 47670 (grm. deg. C.).

Electrolytic cells of various construction were then inserted into the circuit. The electromotive force resisting the passage of the current through the cells was found by taking the number of battery cells just sufficient to produce electrolysis, observing the current, then increasing the number of cells by one and observing the current again. If i be the current in the first case, corrected to bring it to the value it would have had if the resistance of the whole circuit had been the same as in the second case, and j the current in the second case, then, E being the number of battery cells used in the first case, the electromotive force Z opposing the current is given by

$$Z = E - \frac{i}{i-j},$$

the unit being the electromotive force of a Daniell's cell. Z being known and assumed constant for different currents within certain limits, the resistance of the whole circuit, electrolyte included, can be found by Ohm's method as above. The amount of heat H which ought to be developed in the electrolyte by Joule's law can then be calculated. The amount of heat H' actually developed was observed. It was found that H' is in general greater than H , the difference per electrolysis of 65 gm. zinc with various electrodes is shown in the following table:—

Electrode.		Z	H' - H	L	Z - L
+	-				
Pt	Amg. Zn	2.81	66800	1.39	1.42
Pt	Pt	2.47	53000	1.11	1.36
Ag ¹	Ag ¹	1.75	16400	.34	1.40
Pt ¹	Pt ¹	1.90	28800	.60	1.29
Pt ¹	Pt ¹	1.90	26700	.56	1.34

¹ Platinized.

The electrolyte in all these cases was dil. H_2SO_4 , excepting the last case, where it was a solution of potash of sp. g. 1.063. In all the cases the chemical process is finally the same or very nearly so, viz., the freeing of the elements of water, hydrogen and oxygen, in the ordinary gaseous¹ state, and the transference of a certain quantity of H_2SO_4 from the negative to the positive electrode, or of alkali in the opposite direction. Yet the values of $H' - H$ (which we may call the local heat) are very different. It will be seen, however, that the values of $H' - H$ and Z rise and fall together; and, if we calculate the electromotive forces (L) corresponding to the values of $H' - H$, by dividing by 47670, which was found for the thermal equivalent of the electro-

motive force of a Daniell's cell, and subtract the values of L thus found from Z , we get results which are not far from constant. The mean of the values of $Z - L$ is 1.36, the thermal equivalent of which is 64800, which is not very different from 68900, the heat evolved in the combination of 2 grm. of H with 16 grm. of O to form water. It appears, therefore, that the local heat corresponds to the excess of the electromotive force of polarization over the electromotive force requisite to separate water into its component gases. In other words, the work done by the current against this residual electromotive force is accounted for by the local heat $H' - H$ developed in the cell (see Joule's statement above, p. 89). A glance at the column L in the above table shows that this residual electromotive force depends greatly on the nature of the electrodes. Thus when the positive and negative electrodes are plates of platinum and zinc respectively the residual electromotive force is 1.39, whereas with platinized silver plates it is only .34. Local heat and the corresponding electromotive force play a very important part in the working of batteries. Owing to this cause there is an evolution of heat in the cell itself which varies with the strength of the current, and uses up a certain definite fraction of the energy furnished by the solution of the zinc. By sufficiently increasing the external resistance, the amount of heat developed in the cell according to the law $JH = RI^2$ can be made as small a fraction as we please of the whole heat thus developed; but the amount of local heat generated in the cell during the solution of 65 gm. zinc is not greatly altered in this way, at least not in a cell of Daniell, or in any other of the so-called constant batteries. Did our space permit we might quote a great variety of experimental results to illustrate the principles we have been discussing. Most of these investigations are due to the French physicists Favre and Silbermann, whose researches have greatly advanced this department of the science of energy.

Very copious extracts from the memoirs of these and other physicists who have worked in this department will be found in Wiedemann, Bd. ii. 2, §§ 1121 *sqq.* The reader who desires to follow this interesting subject to the sources will find his labour much lightened by referring to M. J. Violle's excellent bibliography of the mechanical theory of heat, appended to the second volume of Verdet's *Théorie Mécanique de la Chaleur*. Much has been done for the theory of the subject by a series of papers on the mechanical theory of electrolysis by Bosscha,² in which the somewhat complicated phenomena involved are analysed in a remarkably clear and able way. Any reader who desires to know what has been done in this department will do well to consult these papers. We quote the following as an example of Favre and Silbermann's results and of the calculations of Bosscha.

The heat evolved in a cell of Smee³ and in platinum wires of different lengths through which it was circulated was measured with the following result:—

Heat in cell.	Heat in wire.	Length of wire.	Heat in cell calc.
13127	4965	25 mm.	13523
11690	6557	50 "	11788
10439	7746	100 "	10188
8992	9030	200 "	9048

The heat in each case is that evolved during the liberation of 1 grm. of hydrogen in the cell. If we assume that the whole heat in the cell and in the wire is generated according to Joule's law, and calculate on this hypothesis the resistance of the cell in mm. of the wire, we should get

¹ The amount of oxygen that finally escapes in the active state as ozone is very small.

² *Pogg. Ann.*, cl., ciii., cv., cviii., 1857, &c.

³ Amalgamated zinc and platinized copper.

Local heat and residual electromotive force.

Favre and Silbermann. Bosscha.

values varying from 66 to 200 mm. If, however, we assume, in accordance with the principles explained above, that a constant fraction of the whole energy per grm. of liberated hydrogen appears as local heat in the cell, then, Q denoting the whole heat which appears in the cell, L the local heat, H the heat in the wire, R the resistance of the cell, S that of the wire, we have

$$\frac{Q-L}{H} = \frac{R}{S};$$

and it is found that on making $R=32.3$ and $L=7589$, this formula will represent the results of experiment very fairly. The last column in the above table gives the value of Q thus calculated. In general so good an agreement is not to be expected, because L may and does vary with the strength of the current.

Theories of residual electromotive force.

Thus far we have been dealing with the direct results of experiment, but when we inquire into the reason for the existence of this residual electromotive force and of the local development of heat corresponding to it, and, in particular, when we ask why the effect is so much greater with some metals than with others, the answers become less satisfactory. We meet, in fact, with considerable divergence of opinion.

Joule's view was that the effect is due to the affinity of the metal of the electrode for oxygen. This is endorsed to a certain extent by Sir William Thomson, who puts the matter thus:—"In a pair consisting of zinc and tin the electromotive force has been found by Poggendorff to be only about half that of a pair consisting of zinc and copper, and consequently less than half that of a single cell of Smee's. There is therefore an immense loss of mechanical effect in the external working of a battery composed of such elements, which must be compensated by heat produced within the cells. I believe, with Joule, that this compensating heat is produced at the surface of the tin in consequence of hydrogen being forced to bubble up from it, instead of the metal itself being allowed to combine with the oxygen of the water in contact with it. A most curious result of the theory of chemical resistance is that, in experiments (such as those of Faraday, *Exp. Res.*, 1027, 1028) in which an electric current passing through a trough containing sulphuric acid is made to traverse a diaphragm of an oxidizable metal (zinc or tin) dissolving it on one side and evolving bubbles of hydrogen on the other, part (if not all) of the heat of combination will be evolved, not on the side on which the metal is being eaten away, but on the side at which the bubbles of hydrogen appear. It will be interesting to verify this conclusion by comparing the quantities of heat evolved in two equal and similar electrolytic cells in the same circuit, each with zinc for negative electrode, and one with zinc the other with platinum or platinized silver for the positive electrode."²

Bosscha dissents from the theory of "chemical resistance" thus expounded, and advances a different explanation. According to him, the local heat arises from the energy lost by the liberated ions in passing from the active to the ordinary state. We know that the hydrogen which is being liberated at the surface of an electrode can effect reductions which hydrogen in the ordinary state cannot accomplish; it is liberated in fact in a state of greater intrinsic energy than usual. It is this excess of intrinsic energy which appears as local heat, and gives rise to the residual electromotive force in electrolysis. Different metals possess in very different degrees the power of reducing active hydrogen to the ordinary state; and therefore

the proportion of hydrogen which gets away from the electrode in the active state differs according to circumstances. Bosscha's theory is that it is the intrinsic energy thus carried away from the electrode that appears as local heat. Similar remarks apply of course to oxygen, the active form of the gas being probably ozone. As a verification of the theory, the fact is cited that at the surface of a plate of carbon, which possesses in an eminent degree the power of reducing ozone to the ordinary state, the residual electromotive force and local heat are very small. At all events the theory of "chemical resistance" is held to be inadequate to explain the facts; for calculating from some results of his own, combined with those of Lenz and Saweljew, he finds for the residual electromotive force at electrodes of

Pt	Fe	Cu	Sn	Hg	Zn
.45	.49	.64	.86	1.20	1.20;

from which it appears that the order of magnitude of the electromotive forces is not that of the affinities of the metals for oxygen.

Electrical Measure of Chemical Affinity.—In a paper³ sent to the French Academy to compete for a prize offered for the best essay on the heat of chemical combination, Joule elaborates an ingenious method for measuring chemical affinity. By direct observation it is ascertained how much heat is developed in a given time in a certain standard coil of wire, when it is traversed by a current whose strength is measured by means of a tangent galvanometer. Then three readings of the tangent galvanometer are taken—first, when the galvanometer alone is in circuit with the battery, secondly, when the standard coil is also inserted, thirdly, when an electrolytic cell is inserted instead of the coil. The amount of the ions liberated and the heat evolved in the cell during a given time is also observed in the last case. If A , B , C be the readings of the galvanometer in the three cases, and if x be the resistance of a metallic wire capable of retarding the current equally with the electrolytic cell,⁴ then we easily get, taking the resistance of the standard coil as unity,

$$x = \frac{(A-C)B}{(A-B)C}.$$

Now if the resistance x were put in the place of the electrolytic cell, the current would be the same; hence by Faraday's law the amount of chemical energy absorbed in the battery would be the same. Also the heat evolved in the rest of the circuit, excluding x , would be the same. It follows, therefore, that the heat H which would be evolved in x is the equivalent of the whole energy given out in the electrolytic cell. If therefore we subtract from H the heat K which actually appears in the cell, the remainder $H-K$ is the heat equivalent of the intrinsic energy of the liberated ions; and, provided they appear finally in the "ordinary" condition, $H-K$ is the heat which would be developed when they are allowed to combine.

In this way Joule found for the heat evolved in the combustion of 1 grm. of copper, zinc, and hydrogen respectively 594, 1185, 33553.

Galvanic Batteries.—It would be inconsistent with our general plan to attempt an exhaustive discussion of all the different electromotors which depend for their energy on chemical action. Wiedemann's *Galvanismus*, or books on telegraphy and other arts in which electricity is applied to technical purposes, may be consulted by the reader who wishes for fuller information. A brief discussion of some typical batteries will, however, be useful, were it only to illustrate the principles we have just been laying down.

All the earlier batteries were one-fluid batteries. The

² Noticed in the *Comptes Rendus*, Feb. 1846, and published at length in *Phil. Mag.*, 1852.

⁴ Notice that it is not said that x is equal to the resistance of the electrolyte. Bosscha in the papers we have quoted, either from not having seen the paper we are now analysing, or through a misunderstanding, accuses Joule of error here. The reasoning (*Pogg. Ann.*, c. p. 540) which he seems to quote from Joule is not to be found in this or in any other of Joule's papers that we know of. Polarization is taken into account by Joule (see *Phil. Mag.*, 1852 (1), p. 485). The criticisms of Verdet, who seems to follow Bosscha, are equally groundless (*Théorie Mécanique de la Chaleur*, t. ii. p. 204).

⁵ This word is left purposely a little vague, and is used to avoid a long discussion of points which need not be raised here.

¹ *Phil. Mag.*, 1851 (2), p. 556.

² The effect here predicted was afterwards observed by Thomson himself, *Rep. Brit. Assoc.*, 1852, and later still by Bosscha, *Pogg. Ann.*, ciii. p. 517.

aid plates usually consisted of zinc and copper, and the exciting fluid was in general sulphuric acid. Various improvements were made by Cruickshank, Wollaston, Hare, and others, in the way of rendering the battery more compact, and of reducing its internal resistance by enlarging the plates. Hare carried the last-mentioned improvement to great lengths; by winding up together in a spiral form sheets of copper and zinc, insulated from each other by pieces of wood, plates of 40 or more square feet surface were obtained. In this way the internal resistance was very much reduced, and powerful heating effects could be obtained. When small internal resistance is no object, the cells of the battery may be filled with sand or sawdust, moistened with the dilute acid. In this form the battery is more portable.

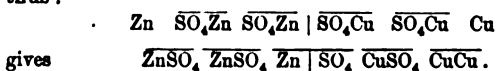
There are two capital defects to which all one-fluid batteries are more or less subject. In the first place, whether there is or is not external metallic connection between the plates, a certain amount of chemical action goes on at the surface of the zinc, which consumes the metal without aiding in the production of the current. To this is given the name of local action. It is supposed to arise from inequalities in the zinc, on account of which one portion of the metal is electropositive to a neighbouring portion; hence local currents arise causing an evolution of hydrogen at some places and solution of the zinc at others. In the second place, when the battery is in action, there is always an evolution of hydrogen at the copper or electronegative plate of the cell, a certain amount of which adheres to the plate and causes a strong electromotive force of polarization. The first of these evils is remedied to a great extent by amalgamating the zinc plate. We thus reduce the surface metal to a fluid condition everywhere, and thereby eliminate differences of hardness and softness, crystalline structure, and so on. The oldest method was to use zinc amalgam for the electronegative metal; but it is now universally the custom to amalgamate the surface of the zinc plates simply, which may be done by rubbing them with mercury under dilute sulphuric acid. No such effective cure has been found for the hydrogen polarization. Smee introduced the plan of using instead of the copper plates thin leaves of platinum or silver foil, which are platinized by the process already described (p. 87). This, in accordance with what we have already seen, diminishes the polarization.¹ A similar result is obtained by using for the electronegative plate cast iron or graphite; the action of the latter is much improved by steeping it in nitric acid.

This last fact introduces us to a new principle for improving the action of batteries, viz., the use of an oxidizing agent to get rid of the hydrogen polarization. When the plates of a Smee's battery have been exposed to the air for some time, it is always found that the current is much more energetic than usual just after the first immersion. The improvement is of course only temporary, for the stock of oxygen is soon exhausted, and on raising the plates and dipping them again immediately, the phenomenon does not appear. Davy found that dilute nitric acid acted better than dilute sulphuric acid as an exciting fluid, and the cause of this is the action of the HNO_3 on the hydrogen evolved at the copper plate. Good instances of this kind of action are furnished by the bichromate battery of Bunsen and the Léclanché cell, which occupy a sort of middle position between one and two fluid batteries.

The bichromate cell consists of an amalgamated zinc plate, usually suspended between two parallel carbon plates, so that it can be raised or depressed at pleasure. The bichromate solution is made,

according to Bunsen, by mixing 605 parts of water with 61.8 of potassium bichromate and 116 of sulphuric acid. The electromotive force of the bichromate cell is very great to start with (more than twice that of a Daniell's cell), but it falls very quickly when the external resistance is small. The cell recovers pretty quickly however, and may be used with advantage where powerful currents of short duration are often wanted. In the cell of Léclanché the electronegative metal is replaced by a porous vessel filled with carbon and pounded peroxide of manganese. The exciting liquid used is chloride of ammonium. Chloride of zinc is formed at the zinc plate, and ammonia and hydrogen appear at the negative plate; the latter reduces the MnO_2 , so that H_2O and Mn_2O_3 are formed, while the ammonia is partly dissolved and partly escapes. This element is tolerably constant if it be not used to produce very strong currents, but its great merit consists in being very permanent; for it will keep in condition for months with very little attention, furnishing a current now and then when wanted; hence its extensive use in working electric bells, railway signals, and so on.

It cannot be said that any of the batteries we have described, or in fact any battery on the one-fluid system, satisfies to any great extent the requirements of a constant electromotor, which are to give the same electromotive force whatever the external resistance, and to preserve that electromotive force unaltered for a considerable time. The best solution of the problem to construct a constant battery is the two-fluid principle invented by Daniell; and on the whole, the best application of that principle is the cell originally given by him. This consists essentially of a plate of copper immersed in a saturated solution of copper sulphate, and a plate of zinc immersed in dilute sulphuric acid or zinc sulphate, the copper solution being separated from the other by some kind of diaphragm, usually a porous vessel of unglazed earthenware. The chemical action consists of the solution of the zinc plate to form zinc sulphate, the formation of zinc sulphate at the diaphragm, and the deposition of copper at the copper plate; thus:—



The evolution of hydrogen and the polarization arising therefrom are thus avoided.

A very common arrangement of this cell is a porous vessel, containing the copper plate and the sulphate of copper, with a small store of large crystals to keep the solution saturated. This vessel is dipped into another nearly filled with a semi-saturated solution of zinc sulphate, in which is placed the zinc plate. With a little care to keep the cell clean by occasionally removing some of the zinc solution and diluting to prevent incrustation, to feed the copper solution, so that it may not get weak and deposit hydrogen instead of copper on the copper plate, to keep down the level of the copper solution, which is constantly rising by osmose (see art. ELECTROLYSIS), and a few other obvious precautions, a battery of Daniell's cells will furnish a very nearly constant current, and keep in order for a long time. It is necessary to keep the current going, otherwise the solutions diffuse through the porous vessel, the result of which is a deposit of copper on the zinc, and other mischiefs, which stop the action of the cell altogether.

Daniell's element has been constructed in a great variety of forms, to suit various purposes. The sawdust Daniell, invented by Sir Wm. Thomson¹ (1858), is very convenient when portability is desired. In this form the copper plate, soldered to a gutta-percha covered wire, is placed at the bottom of a glass vessel and covered with crystals of copper sulphate; over these wet sawdust is sprinkled, and then more sawdust, moistened with a solution of sulphate of zinc, upon which is laid the zinc plate. The cell of Minotto is very similar to this.

In these batteries the sawdust takes the place of the porous diaphragm, and retards the interdiffusion of the fluids. In another class of batteries, of which the element of Meidinger may be taken as the type, the diaphragm is dispensed with altogether, and the action of gravity alone retards the diffusion. In Meidinger's cell the zinc is formed into a ring, which fits the upper part of a glass beaker filled with zinc sulphate. At the bottom of this beaker is placed a smaller beaker, in which stands a ring of copper, with a properly insulated connecting wire. The mouth of the beaker is closed by a lid, with a hole in the centre, through which passes the long tapering neck of a glass balloon, which is filled with

¹ Fleeming Jenkin gives .47 volt as the available electromotive force of Smee's cell. The electromotive force when the circuit is broken is much greater. See above, p. 90.

² Jenkin, *Electricity and Magnetism*, p. 224.

crystals of copper sulphate; the narrow end of this neck dips into the small beaker, the copper sulphate runs slowly out, and being specifically heavier than the zinc sulphate, it collects at the bottom about the copper ring.

Yet another form of Daniell's element is the tray cell of Sir William Thomson, which consists of a large wooden tray lined with lead, the bottom of which is covered with copper by electrotyping. The zinc is made like a grating, to allow the gas to escape, and is enveloped in a piece of parchment paper bent into a tray-shape, the whole resting on little pieces of wood placed on the leaden bottom of the outer tray. Sulphate of copper is fed in at the edge of the tray, and sulphate of zinc is poured into the parchment. The zincs in these elements are some 40 centimetres square, so that the internal resistance is as low as 0.2 ohm.

Grove's element. One of the best known in this country, and perhaps the most used of all the two-fluid cells, is the element of Grove. This differs from Daniell's element in having nitric acid with a platinum electrode in the porous cell, instead of the copper solution and the copper electrode of Daniell's element. The hydrogen evolved at the platinum is oxidized by the nitric acid, and the polarization thus avoided. The nitrous fumes given off by the chemical action are very disagreeable, and also very poisonous, so that it is advisable to place the battery outside the experimenting room or in a suitable draught chamber. The electromotive force of Grove's cell is a good deal higher than that of Daniell's, and its internal resistance is very much less, 25 ohm being easily attained with a cell of moderate dimensions. On this account the cell is much used for working induction coils, generating the electric light, and so on, notwithstanding that it is troublesome to fit up, and must be renewed every day.

Cells of Bunsen, &c. In Bunsen's element the platinum foils of Grove are replaced by carbon. The prime cost of the battery is thus considerably reduced, the more so now that carbons for the purpose have become articles of commerce. The electromotive force of the element thus altered is as great as, or with good carbons even greater than, in Grove's original form; but the internal resistance is greater. There is a difficulty sometimes in obtaining good connection with the carbons, and trouble arises from their fouling; but the fact that this cell is a universal favourite in Germany proves its practical utility. It is comparatively little used in this country.

In the cell of Marié Davy, which is, or was, much used for telegraphic purposes in France, the copper solution and copper plate of Daniell are replaced by a watery paste of protosulphate of mercury, into which is inserted a carbon electrode. The electromotive force of this cell is said to be about 1.5 volts,¹ and its internal resistance to be greater than that of Daniell's cell.

Besides these, various bichromate elements of merit might be described; but we have dwelt long enough on this subject already.

The following table of Latimer Clark's, quoted by Maxwell, will give the reader an idea of the relations as to electromotive force of the commoner elements:—

Daniell.....	H ₂ SO ₄ + 4Aq	CuSO ₄	1.079
Do.	H ₂ SO ₄ + 12Aq	CuSO ₄	0.978
Do.	H ₂ SO ₄ + 12Aq	Cu(NO ₃) ₂	1.000
Bunsen....	H ₂ SO ₄ + 12Aq	HNO ₃	1.964
Do.	H ₂ SO ₄ + 12Aq	HNO ₃ (sp.g. 1.38)	1.888
Grove.....	H ₂ SO ₄ + 4Aq	HNO ₃	1.956

The electromotive force is stated in volts, and the solutions in the third column are concentrated, unless it is otherwise stated.

Thermoelectricity.—We have already alluded to the law of Volta, according to which there can be no resultant electromotive force in a circuit composed solely of different metals; and it will be remembered that we added the condition that all the junctions must be at the same temperature. Seebeck was the first to discover² that this law is subject to exception when the junctions are not all at the same temperature. If we form a circuit with an iron wire and a copper wire, and raise the temperature of one of the junctions a little above that of the other, a current flows round the circuit, passing from copper to iron over the hotter junction; similarly, if we solder together a piece of bismuth and a piece of antimony, and connect the free ends with the copper wires of a galvanometer, then when the junction of the bismuth and antimony is heated the galvanometer indicates a current passing from bismuth to antimony over the hot junction. It will be perceived that the second of

our two illustrative instances is more complicated than the first, inasmuch as three metals enter into the circuit instead of two. Nevertheless the experimental result is not altered by the intervention of the copper wire (abstraction being made of its resistance), provided the temperatures of the points where it joins the bismuth and antimony respectively be the same. It is easy to give a direct experimental proof of this assertion by inserting between the pieces of bismuth and antimony a piece of copper wire so that the circuit now is Bi.Cu.Sb.Cu.Bi; if the junctions of the inserted wire with the bismuth and antimony be raised to the same temperature as the BiSb junction in our second experiment, and the junctions with the copper wire of the galvanometer be at the same lower temperature as before, the total electromotive force in the circuit will be the same; and, provided the resistance of the circuit has not been sensibly increased by the interpolation of the copper wire, the galvanometer indication will also be the same as before. The same result is obtained however many different metals we insert between the bismuth and antimony, provided the temperatures of all the junctions be the same and equal to that of the BiSb junction in the original experiment.

The law of Volta therefore still holds if stated thus: *A series of metals whose junctions are all at the same temperature may be replaced by the two end metals of the series without altering the electromotive force in any circuit of which the series forms a part.*

It is not unlikely that the above statement of the fundamental facts concerning thermoelectromotive force has suggested to the reader two notions:—1st, that the phenomena may be completely explained by a *contact force* at the junctions of the metals which is a function of the temperature of the junction; and 2d, that this contact force is the true contact force of Volta. It is perhaps as well to mention even at this early stage that the first of these notions is certainly not correct, and that the second is not admitted by some of the greatest authorities on the subject.

Seebeck examined the thermoelectric properties of a large number of metals, and formed a thermoelectric series, any metal in which is thermoelectrically related to any following one as bismuth (see above) is to antimony, the electromotive force in a circuit formed of the metals being *ceteris paribus* greater the farther apart they are in the series. The following is a selection from Seebeck's series:—

Bi. Ni. Co. Pd. Pt. Cu. Mn. Hg. Pb. Sn. Au. Ag. Zn. Cd. Fe. Sb. Te.

This series has only a general interest, and is not to be regarded as in any way absolute. Seebeck himself showed the great effect that slight impurities and variations of physical condition may have on the position of a metal in the series. Some specimens of platinum for instance come between zinc and cadmium. Another instance of the same kind is afforded by iron: Joule³ found the following order to hold—cast iron, copper, steel, smithy iron.

Thermoelectric series have been given by Hankel, Thomson, and others, but we need not reproduce them here. It may be well, however, to direct the attention of the reader to the properties of metallic sulphides and of alloys which in many cases occupy extreme positions in the thermoelectric series. Alloys present anomalies in their thermoelectric properties somewhat similar to those already noticed in our discussion of their conductivity. These properties have been much studied with a view to practical applications in the construction of thermopiles. Considerable progress has been made in this direction (see above p. 11), notwithstanding the fact that many of the alloys most distinguished for their thermoelectric power are very brittle and have a tendency to instability under the continued action of heat.⁴

Seebeck's discovery.

¹ Jenkin, *Electricity and Magnetism*, p. 225.

² *Pogg. Ann.*, vi. 1826. The discovery was made about 1821 or 1822.

³ *Phil. Mag.*, 1857.

⁴ For further information consult Wiedemann, *Galv.*, Bd. I. § 593. &c.

Many measurements of the electromotive force of thermoelectric couples have been made by Matthiessen,¹ Wiedemann,² E. Becquerel,³ and others, but the results are of no great value owing to the effect of impurities and the want of sufficient data to determine all the thermoelectric constants of any one couple (see below, p. 99). Numerical data, such as they are, will be found in Wiedemann, Fleming Jenkin's *Electricity and Magnetism*, or Everett's *Illustrations of the Centremetre-gramme-second System of Units*. It will give the reader an idea of the order of the magnitudes involved to state that the electromotive force at ordinary temperatures of a BiSb couple is somewhere about 11700 C. G. S. absolute units when the difference between the temperatures of the junctions is 1° C. The corresponding number for a CuFe couple is 1600 or 1700.

Thermoelectric currents, or at least what may very likely be such, have been obtained in circuits other than purely metallic, *e. g.*, in circuits containing junctions of metals and fluids,⁴ metals and melted salts,⁵ fluids and fluids.⁷ The phenomena in all these cases are complicated, and the results more or less doubtful; so that no useful purpose could be served by discussing the matter here. The same remark applies to the curious electrical phenomena of flames,⁸ of which no proper explanation, so far as we know, has as yet been given.

The experiments of Magnus⁹ have shown that in a circuit composed entirely of one metal, every part of which is in the same state as to hardness and strain, no thermoelectromotive force can exist, no matter what the variations of the section or form of the conductor or what the distribution of temperature in it may be (so long as there is neither discontinuity of form nor abrupt variation of temperature).

This statement is of great importance, as we shall see, in the theory of thermoelectricity. Its purport will be all the better understood if we dwell for a little on the three limitations which accompany it.

The great effect of the hardness or softness and crystalline or amorphous structure of a metal on its electric properties was observed by Seebeck soon after the discovery of thermoelectricity.¹⁰ The effect of temper in wires may be shown very neatly by the following experiment due to Magnus. On a reel formed by crossing two pieces of wood are wound several turns of hard-drawn brass wire softened in a number of places adjacent to each other on the reel. The free ends of the wire being connected with a galvanometer, and the parts of the wire lying between neighbouring hard and soft portions being heated, a thermoelectric current of considerable strength is obtained, whose direction is from soft parts to hard across the heated boundaries. Effects of a similar kind were obtained with silver, steel, cadmium, copper, gold, and platinum. In German silver, zinc, tin, and iron, the current went from hard to soft across the hotter boundary.

Sir William Thomson made a number of experiments on the effect of strain on the electric properties of metals. The results, some of them very surprising, are contained in his Bakerian Lecture,¹¹ along with many other things of great importance for the student of thermoelectricity.

Two of his experiments may be described as specimens.

They afford convenient lecture-room illustrations of the subject under discussion. (1.) A series of copper wires A, B, C, D, E, F, G, &c., are suspended from a horizontal peg. A and B, C and D, E and F, &c., are connected by short horizontal pieces of copper wire, all lying in the same horizontal line, and B and C, D and E, F and G, &c., are connected by a series of pieces lying in another horizontal line below the former. An arrangement is made by means of which the alternate wires A, C, E, G, can be more or less powerfully stretched, while B, D, F, &c., are comparatively free. A piece of hot glass is applied to heat either the upper or lower line of junctions. A thermoelectric current is then observed passing from the stretched to the unstretched copper across the hot junctions. This thermoelectric current increases with the traction up to the breaking point. But the most remarkable point that comes out in such experiments is that when we free the wire after powerful traction, leaving it with a permanent set, there is still a thermoelectric current; but the direction is now from the soft or unstrained towards the permanently strained parts across the hot region. (2.) Iron gives similar results, only the direction of the current is in each case opposite to that in the corresponding case for copper. The following experiment exhibits this in a very elegant manner. One end of a piece of carefully annealed iron wire is wound several times round a horizontal peg, the free end being slightly stretched by a small weight, and connected with one terminal of a galvanometer. The other end of the wire is wound a few times round one side of a rectangular wooden frame, the free end being stretched by a small weight and connected with the other terminal of the galvanometer. The parts of the wire on the peg or the part on the frame is then heated, and weights are hung to the frame. As the weight increases, the deflection of the galvanometer goes on increasing. If we stop a little short of rupture, and gradually decrease the weight, the deflection of the galvanometer gradually decreases to zero, changes sign before the weight is entirely removed, and finally remains at a considerable negative value when the wire is again free.

These experiments of Sir Wm. Thomson's were repeated Le Roux, by Le Roux. The results of the two experimenters are &c. not very concordant. This may be due to differences in the qualities of the materials with which they worked, or to the fact that Le Roux¹² worked at higher mean temperatures than Thomson.¹³

Le Roux also repeated the experiments of Magnus, confirming his general result, but adding the two last qualifications given above. He found, contrary to the result of Magnus, that when a lateral notch is filed in a wire and one side heated, there is in general a thermoelectric current, which is greater, up to a certain limit, the deeper the notch. He also found that when two wires of the same metal, with flat ends, are pressed together, so that one forms the continuation of the other, and the wire on one side of the junction is heated, no current is obtained; but he observed a current in all cases where there was dissymmetry,—*e. g.*, where an edge of one end was pressed on the flat surface of the other, where the wires overlapped or crossed, or where the chisel-shaped end of one wire fitted into a notch in the end of the other, and the axes of the wires were inclined, and so on.

Whether a very abrupt variation in temperature in a continuous part of a metallic wire would produce a thermoelectromotive force is a question which possesses little physical interest, since it is impossible to realize the

¹ *Pogg. Ann.*, 1853.

² *Galv.*, Bd. i. § 590.

³ *Ann. de Chim. et de Phys.*, 1864.

⁴ That is, roughly, '000117, if we take for our unit the electromotive force of a Daniell's cell.

⁵ By Walker, Faraday, Henrici, Gore, and others; see Wiedemann, Bd. i. § 639, &c.

⁶ Andrews, *Phil. Mag.*, 1837; Hankel, Wiedemann (*l. c.*), Gore, *Phil. Mag.*, 1864.

⁷ Nobili, Wiedemann, Becquerel; see Wiedemann, *l. c.*

⁸ See Wiedemann, *l. c.*

⁹ *Pogg. Ann.*, 1851.

¹⁰ *Pogg. Ann.*, 1856.

¹¹ *Phil. Trans.*, 1856.

¹² *Ann. de Chim. et de Phys.*, 1867.

¹³ Wiedemann, Bd. i. § 610. It appears from a note at the end of Le Roux's paper (*l. c.*) that Sir Wm. Thomson has lately repeated some of his experiments and confirmed his former results.

imagined conditions. There can be no doubt, however, that, when the two unequally heated ends of a wire composed of the same metal throughout are brought together, a thermoelectric current is in general the consequence. Such currents were, it appears, observed by Ritter¹ in 1801, when cold and hot pieces of zinc wire were brought into contact. Becquerel, Matteucci, Magnus, and others have experimented on this subject. The results obtained are, no doubt, greatly influenced by the state as to oxidation, &c., of the surfaces of the metals experimented on, as has been pointed out by Franz and Gauguin. The experimental conditions are, in truth, very complicated, and a discussion of the matter would be out of place here.² We may mention, however, that, at the instance of Professor Tait, Mr Durham³ made experiments on the transient current which arises when the unequally heated ends of a platinum wire are brought into contact. It was found that the first swing of a galvanometer of moderately long period was proportional to the temperature difference and independent of the mean temperature through a considerable range.

Thermoelectric inversion. Cumming.

Cumming, who experimented on thermoelectricity about the same time as Seebeck, and apparently independently, discovered the remarkable fact that the thermoelectric order of the metals is not the same for high temperatures as for low. He found that, when the temperature of the hot junction in a circuit of iron and copper, or iron and gold, is gradually raised, the electromotive force increases more and more slowly, reaches a maximum at a certain temperature T , then decreases to zero, and finally changes its direction. The higher the temperature of the colder junction, so long as it is less than T , the sooner the reversal of the electromotive force is obtained. If the temperature of the hot junction be $T + \tau$, where τ is small, then the reversal of the electromotive force takes place when the temperature of the colder junction is $T - \tau$. If both junctions, A and B, be at the temperature T , then either heating or cooling A will cause a current in the same direction round the circuit, and either heating or cooling B will cause a current in the opposite direction.

The reversal of the current may be shown very conveniently in the manner recommended by Sir Wm. Thomson.⁴

A circuit is formed by soldering an iron wire to the copper terminal wires of a galvanometer. If one junction be at the temperature of the room and the other at 300°C . or thereby, a current flows from copper to iron across the hotter junction; but, if we raise the temperature of both junctions over 300°C ., one being still a little hotter than the other (which can be managed by keeping both in a lamp flame, one in a slightly hotter place than the other), then the current will flow from iron to copper across the hot junction. If both junctions be allowed to cool, the difference between their temperatures remaining the same, the current will decrease, becoming zero when the mean temperature of the two junctions is about 280°C .; and, on still further lowering the mean temperature, it will set again in the opposite direction, *i.e.*, from copper to iron across the hot junction. The fundamental facts of thermoelectric inversion were confirmed by Becquerel,⁵ Hankel,⁶ Svanberg,⁷ &c.; but the matter rested there till it was taken up⁸ by Sir Wm. Thomson⁹ in the course of his classical researches on the applications of the laws of thermodynamics to physical problems.

¹ Wiedemann, Bd. i. § 627.

² Consult Wiedemann, Bd. i. § 627, &c., and Mascart, t. ii. § 932, &c.

³ Proc. R. S. E., 1871-2.

⁴ Bakerian Lecture, Phil. Trans., 1856, p. 699.

⁵ Ann. de Chim. et de Phys., 1826. ⁶ Pogg. Ann., 1844.

⁷ Pogg. Ann., 1853; cf. Wiedemann, Bd. i. § 623.

⁸ In consequence, it appears, of a remark of Joule's, cf. Proc. R. S. E., 1874-5, p. 417. ⁹ Trans. R. S. E., 1851.

The application of the first law of thermodynamics leads to no difficulty; and it indicates that the heat absorbed according to Peltier's law, in the ordinary case when a current passes from copper to iron across the hotter of the junctions, minus the heat evolved at the colder junction where the current passes from iron to copper, is to be looked on as a source of part at least of the energy of the thermoelectric current. If absorption or evolution of heat occur anywhere else than at the junctions, this must be taken account of in a similar manner.

The application of the second law is of a more hypothetical character. It is true that the Peltier effects, as we may for shortness call the heat absorption and evolution at the junctions, are reversible in this sense that we might suppose the thermoelectric current, whose energy arises wholly or partly from the excess of the heat absorbed at the junction A over that evolved at the junction B, used to drive an electromagnetic engine and raise a weight; and that we might suppose the potential energy thus obtained again expended in sending, by means of an electromagnetic machine, a current in the opposite direction round the circuit, absorbing heat at B, evolving heat at A, and thus restoring the inequality of temperature. This process, however, must always be accompanied by dissipation of energy, (1) by the evolution of heat in the circuit according to Joule's law, and (2) by conduction from the hotter towards the colder parts of the wires. The first of these effects varies as the square of the current strength, while Peltier's effect varies as the current strength simply; so that the former might be made as small a fraction of the latter as we please by sufficiently reducing the current, and thus, theoretically speaking, eliminated. The second form of dissipation could not be thus got rid of, and could only be eliminated in a circuit of infinitely small thermal but finite electric conductivity, a kind of circuit not to be realized, as we know (see above p. 51). Still it seems a reasonable hypothesis to assume that the Peltier effects, and other heat effects if any, which vary as the first power of the strength of the current, taken by themselves are subject to the second law of thermodynamics. Let us now further assume that all the reversible heat effects occur solely at the junctions. Let Π , Π' denote the heat (measured in dynamical equivalents) absorbed and evolved, at the hot and cold junctions respectively in a unit of time by a unit current. Let E be the electromotive force of an electromotor maintaining a current I , in such a direction as to cause absorption of heat at the hot junction. Then, if R be the whole resistance of the circuit, we have, by Joule's law and the first law of thermodynamics,

$$EI + \Pi I - \Pi' I = RI^2, \dots (1),$$

supposing the whole of the energy of the current wasted in heat. Hence we get

$$I = \frac{E + \Pi - \Pi'}{R} \dots (2).$$

It appears then that, owing to the excess of the absorption of heat at the hot junction over the evolution at the cold junction, there arises an electromotive force $\Pi - \Pi'$ helping to drive the current in the direction giving heat absorption at the hot junction. We may suppose (and shall henceforth suppose) that $E = 0$, and then the current will be maintained entirely by the thermoelectromotive force.

If we now apply the second law, we get

$$\frac{\Pi I}{\theta} - \frac{\Pi' I}{\theta'} = 0,$$

θ and θ' being the absolute temperatures of the hot and cold junctions. Hence

$$\frac{\Pi}{\theta} - \frac{\Pi'}{\theta'} = 0 \dots (3);$$

or, in other words, $\Pi = C\theta$, where C is a constant depend-

Application of laws of thermodynamics. Sir W. Thomson.

ing only on the nature of the metals. In accordance with this, the thermoelectromotive force in the circuit would be $C(\theta - \theta')$; that is, it would be proportional to the difference between the temperatures of the junctions. Now this conclusion is wholly inconsistent with the existence of thermoelectric inversions. We must therefore either deny the applicability of the second law, or else seek for reversible heat effects other than those of Peltier. This line of reasoning, taken in connection with another somewhat more difficult, satisfied Sir Wm. Thomson that reversible heating effects do exist in the circuit elsewhere than at the junctions. These can only exist where the current passes from hotter to colder parts of the same wire or the reverse. Thomson was thus led to one of the most astonishing of all his brilliant discoveries; for he found, after a series of researches distinguished alike for patience and experimental skill, that an electric current absorbs heat in a copper conductor when it passes from cold to hot, and evolves heat in iron under similar circumstances. This phenomenon was called by its discoverer the electric convection of heat. He expressed the facts above stated by saying that positive electricity carries heat with it in an unequally heated copper conductor, and negative electricity carries heat with it in an unequally heated iron conductor. The first statement is perhaps clearer; the value of the one given by Thomson consists in the suggestion which it conveys of a valuable physical analogy with the transport of heat by a current of water in an unequally heated pipe.¹

If two points AB of a uniform linear conductor, in which a current I is flowing from A to B, and evolving heat, be kept at the same constant temperature, but for the electric transport of heat the temperature distribution would be symmetrical about a point of maximum temperature half way between A and B. Owing to the electric transport of heat, the maximum will be shifted towards A in iron, towards B in copper.² This remark contains the principle of the experiments made by Thomson to detect the new effect.

The first experiment in which the effect was satisfactorily established was made with a conductor ABCDEFG, formed of a number of strips of iron bound together at A, C, E, and G, but opened out widely at B, D, and F, to allow these parts to be thoroughly heated or cooled. At C and E small cylindrical openings allowed the bulbs of two delicate mercurial thermometers to be inserted in the heart of the bundle of strips. The part D of the conductor was kept at 100° C. by means of boiling water, and the parts B and F were kept cool by a constant stream of cold water. The current from a few cells of large surface was sent for a certain time from A to G, then for the same length of time from G to A, and so on. In this way the effects of want of symmetry were eliminated, and the result was that the excess of the temperature at E over that at C was always greatest when the current passed from G to A; whence it follows, as stated above, that a current of positive electricity evolves heat in an iron conductor when it passes from cold to hot.

Le Roux³ has made a series of interesting experiments on the Thomson effect in different metals. He found that the effect varies as the strength of the current, and gives the following numbers representing its relative magnitudes in different metals. In lead the effect is insensible.

+		-	
Sb	64	Fe	31
Cd	31	Bi	31
Zn	11	Arg	25
Ag	6	Pt	18
Cu	2	Al	0.1
		Sn	0.1

We may now apply the mathematical reasoning given above, taking into account Thomson's effect.

Suppose for simplicity we have a circuit of two metals only. Let the current go from A to B over the hot junction, and let the heat absorbed in passing from a point at temperature θ to a neighbouring point at temperature $\theta + d\theta$ in A be $\sigma_1 d\theta$ per unit of current per unit of time; let $\sigma_2 d\theta$ be the corresponding expression for B. Then it is obvious, from the result of Magnus (see above, p. 95), that σ_1 and σ_2 can be functions of the temperature merely; they depend, of course, on the nature of the metal, but are independent of the form or magnitude of the section of the conductor. The first and second laws now give respectively

$$E = \Pi - \Pi' + \int_{\theta'}^{\theta} (\sigma_1 - \sigma_2) d\theta \quad \dots (4),$$

$$0 = \frac{\Pi}{\theta} - \frac{\Pi'}{\theta'} + \int_{\theta'}^{\theta} \frac{\sigma_1 - \sigma_2}{\theta} d\theta \quad \dots (5),$$

where E is the whole thermoelectromotive force, and Π and Π' are the same functions of θ and θ' respectively. By differentiation we get from (5)

$$\frac{d}{d\theta} \left(\frac{\Pi}{\theta} \right) + \frac{\sigma_1 - \sigma_2}{\theta} = 0 \quad \dots (6);$$

whence we easily get

$$E = \int_{\theta'}^{\theta} \frac{\Pi}{\theta} d\theta \quad \dots (7).$$

or

$$\frac{\Pi}{\theta} = \frac{dE}{d\theta}$$

This last equation enables us to determine E in terms of Π , and conversely.

When the difference between the temperatures of the junctions is very small, equal to $d\theta$ say, the thermoelectromotive force is

$$\frac{\Pi}{\theta} d\theta \quad \dots (8).$$

The coefficient $\frac{\Pi}{\theta}$ by which we must multiply the small temperature difference to get the electromotive force is called by Thomson the thermoelectric power of the circuit. If we have a circuit of three metals, A, B, C, all at the same temperature θ , then we know that

$$\Pi_{BC} + \Pi_{CA} + \Pi_{AB} = 0, \quad \dots (9);$$

whence $\frac{\Pi_{AB}}{\theta} = \frac{\Pi_{AC}}{\theta} - \frac{\Pi_{BC}}{\theta}$, or, in other words, the thermoelectric power of B with respect to A is equal to the difference between the thermoelectric powers of a third metal C with respect to A and B respectively.

Thus far we have been following Thomson. But as yet Tait's we have no indication how σ , the coefficient of the Thomson effect, depends on the temperature. Thomson himself seems (see his Bakerian Lecture, *l.c.*, p. 706) to have expected that σ would turn out to be constant. Certain considerations concerning the dissipation of energy led Tait, however, to conjecture that σ is proportional to the absolute temperature. If we adopt this conjecture, Thomson's equations give us at once the values of the Peltier effect and the electromotive force in the circuit. If $\sigma_1 = k_1\theta$, $\sigma_2 = k_2\theta$, we get from (6) and (7) successively⁴

$$\Pi = (k_1 - k_2)(\theta_1 - \theta)\theta \quad \dots (10),$$

$$E = (k_1 - k_2)(\theta - \theta') \left\{ \theta_1 - \frac{1}{2}(\theta + \theta') \right\} \quad \dots (11),$$

where θ_1 is the neutral temperature. Also, since in a circuit of uniform temperature there are no Thomson effects, and the sum of the Peltier effects is zero, we get for any three metals

$$(k_2 - k_3)\theta_{21} + (k_3 - k_1)\theta_{31} + (k_1 - k_2)\theta_{13} = 0 \quad \dots (12).$$

Taking up the idea of a thermoelectric diagram originally suggested by Thomson, Tait has shown how to represent the above results in a very elegant and simple manner. Suppose we construct a curve whose abscissa is the absolute temperature θ , and whose ordinate is the thermoelectric power of some standard metal with respect to the

¹ *Trans. R. S. E.*, 1851.

² See Verdet, *Théorie Mécanique de la Chaleur*, t. ii. § 250.

³ *Ann. de Chim. et de Phys.*, 1867

⁴ Tait, *Proc. R. S. E.*, 1870-1-2.

metal we are considering, then, from what has been shown (10), Tait's conjecture leads to the result that this curve is a straight line; and if the standard metal be lead, for which, according to Le Roux's results, the Thomson effect is zero, then the coefficient k of the Thomson effect is the tangent of the inclination of the representative line to the axis of abscissæ. And not only so, but it follows from formulæ (9) and (7) that, if $A'AN$, $B'BN$ (fig. 54) be the

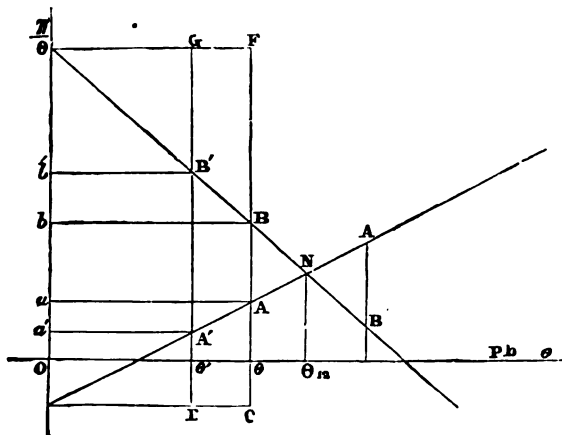


Fig. 54.

lines corresponding to two metals, say Cu and Fe (of which the former is above the latter in the thermoelectric series at ordinary temperatures), and if AB , $A'B'$ be the ordinates corresponding to θ and θ' , then the electromotive force in a circuit of the two metals whose junctions are at the temperatures θ and θ' , tending to send a current from Cu to Fe across the hotter junction, is represented by the area $ABB'A'$. The Peltier effects at the two junctions are represented by the rectangles $ABba$ and $A'B'b'a'$, and the Thomson effects, in the Cu and Fe respectively, by $AA'DC$ and $BB'GF$, or by $AA'a'a$ and $BB'b'b$, which are equal to these. At N , where the lines intersect, the Peltier effect vanishes. N therefore is the neutral point; and, if the higher temperature lie beyond it, the electromotive force must be found by taking the difference of the areas $NA'B'$ and NAB , and so on. All the phenomena of inversion may be studied by means of this diagram, and the reader will find it by far the best means for fixing the facts in his memory.

For several years back Tait¹ and his pupils have been engaged in verifying the consequences of this conjecture; and it has been shown, first, for temperatures within the range of mercury thermometers, and latterly for temperatures considerably beyond this range, that the hypothesis accords with experience. The methods employed by Tait in his experiments at high temperatures are of great interest and importance. One of these was to construct a curve whose ordinate and abscissa are the simultaneous readings of two thermoelectric circuits whose hot and whose cold junctions are kept at common temperatures. It is a consequence of the foregoing assumption that the curve thus obtained ought to be a parabola. Very good parabolas were in many cases obtained. In some cases, however, the curves, so far from being parabolas, were actually curves having points of contrary flexure. This anomaly led Tait to the discovery of the astonishing fact that the Thomson effect in iron changes its sign certainly once at a temperature near low red heat, if not a second time near the melting point. It was found that the inflected curves could be represented by piecing together different parabolas. Hence the line for iron in the

thermoelectric diagram is a broken line made up of two if not three straight pieces. This peculiarity of the iron line was very strikingly shown by forming circuits of iron with the alloys PtIr or PtCu. Such circuits exhibit *two or even three neutral points* (see fig. 55). Another very elegant

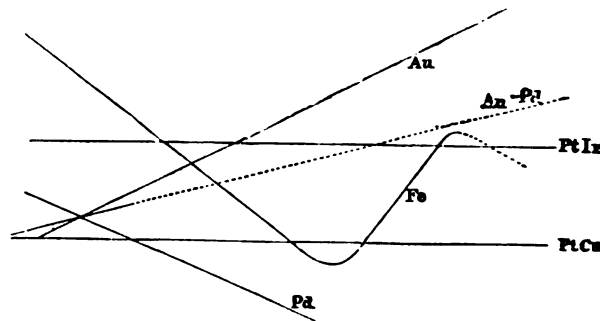


Fig. 55.

method of verification consisted in using along with an iron wire a multiple wire of Au and Pd, the resistances of whose branches could be modified at will. It is easy enough to show that the line for the Au-Pd wire is a straight line, passing through the neutral point of Au and Pd, and such that it divides the part of an ordinate lying between the Au and Pd lines in the ratio of the respective conductivities of the Au and Pd branches. Thus, by increasing ratios of the conductivities of the Pd and Au branches from 0 up to ∞ , we can make the Au-Pd line sweep through the whole of the space between Au and Pd (fig. 55), and thus explore the part of the Fe line lying in the space. We get in this way first one neutral point, then two, then one, and then none in our Fe, Au-Pd circuit.

Tait has pointed out that, by using PtIr and Fe, and keeping the hot and cold junctions at the two neutral temperatures, we get a current maintained solely by the excess of the heat absorbed in the hotter iron over that developed in the colder. The electromotive force is represented by the area inclosed by the part of the zigzag on the Fe line cut off by the PtIr line (fig. 55). A similar case of thermoelectromotive force without Peltier effects may be obtained with three metals, such as Fe, Cd, Cu, whose neutral points lie within reasonable limits. The electromotive force in this case is represented by the triangle between the three lines.

We subjoin a table, calculated by Professor Everett from Tait's diagram. The thermoelectric power is given in electromagnetic (C.G.S.) units, in terms of the temperature (t) in centigrade degrees, by means of the formula $\alpha + \beta t$, where α and β have the tabulated values:—

	α	β		α	β
Fe	-1734	+4.87	Cd	-266	-4.29
Steel	-1139	+3.28	Zn	-234	-2.40
Pt Ir	-839	+0.00	Ag	-214	-1.50
Pt Ir (5 p.c. Ir)	-662	+0.55	Au	-283	-1.02
Do. (10 do.)	-593	+1.34	Cu	-136	-0.95
Do. (15 do.)	-709	+0.63	Pb	+0	+0.00
Do. (15 do.)	-577	+0.00	Sn	+43	-0.55
Pt soft	+61	+1.10	Al	+77	-0.39
Pt hard	-260	+0.75	Pd	+625	+3.59
Pt Ni	-544	+1.10	Ni to 175° C.	+2204	+5.12
Mg	-224	+0.95	Do. 250° to 310° C.	+8449	-24.10
German silver	+1207	+5.12	Do. from 340° C.	+307	+5.12

We need scarcely warn the reader that the results in this table must not be rashly applied to any specimens of the metals taken at random. The temperature limits lie between 18° C. and 420° C.

It would be extremely interesting to compare the results

¹ Trans. R. S. E., 1873.

of absolute measurements of the Peltier effect with Tait's theory; but, unfortunately, no data that we know of are available for the purpose. It is absolutely necessary for this purpose to have heat measurements and determinations of the lines of the metals in the same specimens. The data of Edlund¹ and Le Roux are quite useless for such a purpose. One result of Le Roux's is, however, interesting. He finds for the amount of heat developed at the junction BiCu, the values 3.09 and 3.95 at 25° C. and 100° C. respectively. Since the neutral temperature of BiCu is very high, the Peltier effect ought, according to Tait's theory, to vary as the absolute temperature. The absolute temperatures corresponding to 25° C and 100° C. are 298° and 373°, and we have $3.95 \div 3.09 = 1.278$, while $373 \div 298 = 1.252$; the agreement between these numbers bears out the theory so far.²

General Considerations regarding the Seat of Electromotive Force.—Before proceeding to notice the remaining cases of the origin of electromotive force, in which the phenomena are more complicated, and the experimental conditions less understood, it may be well to call attention to a principle that appears to hold in most of the cases already examined. In most of these cases the seat of the electromotive force appears to be at the places where energy is either taken in or given out in the circuit.³

It is very natural to ask ourselves what the consequences would be if we applied this principle to the voltaic circuit. It would probably be admitted by most that the energy in the voltaic circuit is taken in mainly at the surface of the electropositive metal. This admission, taken in conjunction with the general principle above stated, leads us to the conclusion that the electromotive force resides mainly at the surface of the electropositive metal. The absorption or evolution of energy at the junction of the dissimilar metals is quite insignificant, and we should, on the same view, deny that any considerable part of the electromotive force resides there.

This view appears to be at variance with the theory of metallic contact, as now held by Sir William Thomson and others; and the burden of explaining the experiments made by him and others on the contact force of Volta is doubtless thrown on those who adopt this view. The position of such would very likely be that there is an uneliminated source of uncertainty in all these experiments⁴ (see above, p. 85). On the other hand, those who adopt the contact force of Volta at the junction of copper and zinc as the main part of the electromotive force of Daniell's element are under the necessity of distinguishing this from the electromotive force corresponding to the Peltier effect, which must be a distinct effect, since it is but a very small fraction of that of a Daniell's cell.

We are, however, so very ignorant of the nature of the motion which is the essence of the electric current that the very form in which we have put the question may be misleading. If this motion be in the surrounding medium, as there is great reason to believe it to be, it would not be surprising to find that speculations as to the exact locality of the electromotive force in the circuit were utterly wide of the mark. The very language which we use implies a certain mode of analysing the problem which may be altogether wrong. The only thing of which we can as yet be sure is that the mathematical equations deduced

from Ohm's law and other proximate principles are in exact accordance with experiment.

Pyroelectricity.—Some account of this interesting subject has already been given in the Historical Sketch at the beginning of this article. It will be well, however, to state here some of the conclusions of those who have recently investigated the matter. It seems now to be settled that it is not merely high or low temperature, but change of temperature, which gives rise to the electrical phenomena of pyroelectric crystals. The properties exhibited by tourmaline may be described thus. One end A of the crystal is distinguishable from the other end B by the dissymmetry of the crystalline form. A is called the *analogous* pole of the crystal, and B the *antilogous* pole. When the temperature of the crystal is increasing uniformly throughout, the analogous pole is positively electrified and the antilogous pole negatively electrified. When the temperature is decreasing uniformly throughout, the analogous pole is negative and the antilogous pole positive. This law was originally discovered by Canton,⁵ but it seems to have been lost sight of again and rediscovered both by Bergman and by Wilcke in 1786. When the temperature is uniform, the positive and negative regions are symmetrically distributed about the central zone of the crystal, which is neutral. If the ends be unequally heated, this symmetry no longer obtains. It must not be forgotten that complications may arise from the crystal becoming electrical as a whole by friction, usually positive, like most other vitreous bodies.

Gauguin⁶ made a series of interesting experiments on the electrical properties of tourmaline, and concluded that a tourmaline whose temperature is varying may be compared to a voltaic battery of great internal resistance, consisting of an infinite number of cells, each of infinitely small electromotive force; so that the electromotive force is proportional to the length of the tourmaline, and its internal resistance is proportional to the section inversely and to the length directly. He also concluded that the amount of electricity furnished by a tourmaline, while its temperature varies either way between two given temperatures, is always the same.

In order to explain the properties of the tourmaline, it has been supposed⁷ that the crystal is naturally in a state of electrical polarization, like that assumed by Maxwell in a medium, under the influence of electromotive force, or more nearly (since no sustaining force having an external origin is supposed) like that of a permanent magnet. The intensity of this polarization is supposed to be a function of the temperature. Supposing the tourmaline to remain for some time at the same temperature, a surface layer of electricity would be formed, which would completely mask the electrical polarization of the crystal, inasmuch as it would destroy all external electrical action. This neutralization would be instantly effected by running the crystal through the flame of a lamp. If, however, the temperature increase, then the polarization will, let us say, increase, so that the surface electrification no longer balances it. We shall thus get polar electrical properties of a certain kind. If the temperature decrease, the polarization will decrease, and we shall thus get polar properties of the opposite kind.

In many pyroelectric crystals there are more than one electric axis, so that we have several analogous and corresponding antilogous poles. An enumeration of the various crystals in which pyroelectric properties have been found, and a discussion of the peculiarities in their crystalline form, belongs more properly to the science of Mineralogy. Much has been done in this department by Köhler,⁸ Gustav Rose and Riess,⁹ and Hankel.¹⁰ For some very interesting researches by Friedel see *Annales de Chimie et de Physique*, 1869.

Frictional Electricity.—In accordance with the general principle laid down at the beginning of this section, we should expect to find of non-an electromotive force at the surface which separates two different non-conducting media, just as we have found it at the boundary of two different conducting media. The effect of such a contact force would be very different however in the former of these cases, from what we have seen it to be in the latter. In the case of non-conductors the electricity cannot leave the surface of separation, but will simply accumulate on the two sides of it, till the force arising from electrical separation is equal to the contact force. On separating the bodies, in certain cases, we may carry away with us these surface layers of electricity, and it is an obvious consequence of our principles that the electrifications of parts of the two bodies that have been in contact must be equal and opposite. While the bodies are in contact the difference of potential between the layers of electricity corresponding to very considerable surface density may be very small, just as in Volta's condensing electroscope (see above, p. 34); but when we separate the bodies work is done against the electrical attractions, and the potential increases enormously.

¹ Wied. Galv., Bd. i. § 694.

² Since the above was written further experimental evidence in support of the theory has appeared. See Naccari and Bellati, *Atti del R. Ist. Veneto di Sc. Litt. ed Arti*, November 1877.

³ Maxwell, vol. i. § 249. By "being taken in," in the case of heat for instance, is meant "disappearing as heat and appearing as electrokinetic energy." In a thermoelectric circuit this transformation occurs wherever there is Peltier or Thomson effect.

⁴ Maxwell, l.c.

⁵ Phil. Trans., 1759.

⁶ Mascart, t. ii.

⁷ Thomson, *Phil. Mag.*, 1878, p. 26; or Nichol's *Cyclopædia of the Physical Sciences*, 1860.

⁸ Pogg. Ann., xvii., 1829.

⁹ Abh. der Berl. Akad., 1836 and 1843.

¹⁰ Pogg. Ann., xlix., l., lvi., 1840-2; also cxxxi., cxxxii., 1867, &c.

These hypothetical results tally very well with the electrical phenomena observed when non-conducting bodies are lightly rubbed together; and the above is nearly the explanation that most physicists of the present day would probably give (if they gave any) of what is called the "frictional generation of electricity."

All experimenters are agreed that equal quantities of positive and negative electricity appear in this case as in every other case of electrical separation; an experiment to prove the contrary would have to be very demonstrative indeed before it would now be accepted as conclusive. A single case of exception would revolutionize our fundamental ideas completely. The reader should consult on this point Faraday's *Experimental Researches*, series xi. ¶ ii.

The other consequences of our hypothesis are by no means so firmly established. One of these is that we ought to be able to arrange non-conducting bodies in a series such that any body rubbed with one below it in the series becomes positive, and rubbed by one above it negative.

Many electricians have attempted to establish such electromotive series, but the experimental conditions (see the admirable remarks of Riess, *Reibungselektricität*, § 907) are so complicated that nothing absolute has been attained. Yet it would appear that, if we could make sure that we were always dealing with definite materials under definite surface conditions, electromotive series could be constructed in which every different body would have a fixed position. As it is, the body bearing the same name in the lists of different experimenters was in all probability not exactly of the same material in all cases, and (we might say certainly) was not under the same surface conditions. We refer the reader to Riess (*l.c.*) for an admirable résumé of the work of different electricians in this department. Mascart has given a very interesting account of the matter (t. ii. § 834, &c.) from a more modern point of view. From these sources, together with indications in Young's *Lectures on Natural Philosophy*, the reader will be able to follow up the literature of this somewhat uninviting department of electricity.

Frictional series.
Wilcke.

We give two instances of frictional electromotive series which may be useful in giving the reader a general idea how different bodies stand.

The following is Wilcke's series¹ (1758):—Glass, woollen cloth, feathers, wood, paper, shellac, white wax, ground glass, lead, sulphur, metals.

Faraday.

Faraday² gives—cat and bear skin, flannel, ivory, feathers, rock crystal, flint glass, cotton, linen, white silk, the hand, wood, shellac, metals (iron, copper, brass, tin, silver, and platinum), sulphur.

Peclet's experiments.

To which Riess adds (in order) the highly negative bodies—gutta-percha, electrical paper,³ collodion, gun cotton.

Considered as evidence for the contact hypothesis, the experiments of Peclet seem to be important. He used an apparatus which was virtually a Nairne's machine (see below, p. 101), in which the rubber could be varied at will. His general conclusions are quite in accordance with the contact theory. He found, for instance, that for the great majority of materials the quantity of electricity generated was independent of the pressure and of the breadth⁴ of the rubber, and varied as the angular velocity of the cylinder, and it even appeared to be the same for rolling friction as for sliding friction, so long as the material of the rubber was unchanged.

Contact of conductor with non-conductor.

Besides the case of two non-conductors, we might consider the case of a conductor and a non-conductor in contact. Much of what has just been said would apply to this case also, an excellent example of which is furnished by a frictional electrical machine of the ordinary construction when the cushions are well furnished with amalgam. This is the place to give a short account of these time-honoured pieces of electrical apparatus. For a history of them we cannot do better than refer to Mascart⁵ (*l.c.*), who has devoted much attention to the theory as well as the history of electrical machines in general.

Frictional machines.

A very common form of machine, called Ramsden's, is pictured in fig. 56. It consists, like all other frictional machines, essentially of three parts—(1) the rubbed or moving body, (2) the rubbers, and (3) the collectors and prime conductors. In the present instance the rubbed body is a disc of glass, which can be turned about a horizontal axis by means of a suitable handle. The efficiency of the machine depends very much on the quality of the glass of which the disc is made. According to Mascart, glass of old manufacture is superior to the more modern specimens, owing to the smaller proportion of alkali in the former; it appears, however, that the disc improves in most cases with age and use. Many

other materials have been proposed to replace glass, which is somewhat costly when large discs are required. Ebonite has been tried a good deal of late, and has great advantages so far as its electrical properties are concerned; but it has the disadvantage that it warps very readily if heated incautiously, and its surface will not keep good for any length of time. Owing to decomposition under the action of light, a layer of sulphuric acid forms on the surface, after which it is very difficult to restore the electrical virtue so remarkable in the new material, although washing with hot water or immersion in a blast of steam are said to be effective in some degree.

The rubbers consist of two rectangular pieces of wood, hinged to supports attached to the framework of the machine, and fitted with springs and screws, so that they can be made to clip the plate with any required pressure. The rubbing surfaces are usually formed of leather, stretched as smooth and flat as possible (oiled silk is sometimes used, but it is not so durable). Before the leather cushions are fit for use, they must be carefully coated with amalgam. The amalgam most commonly used is Kienmayer's, which is a composition of two parts of mercury with one of zinc and one of tin. A great variety of different compounds of this kind have been used by different electricians, bisulphide of tin being a general favourite. The amalgam must be powdered as finely as possible, all grit being carefully removed. The cushions are then to be lightly smeared with lard, and worked together till the surface is very smooth and the greasiness almost gone; then the amalgam is to be carefully spread over them, and the surfaces again worked together till a uniform metallic surface is attained;⁶ they are then ready for use. The amalgam aids the action of the machine in two ways,—first, by presenting a surface which is highly negative to glass; secondly, by allowing the negative electricity evolved by friction to flow away without hindrance from the points of contact. In order to secure the second of these advantages still more perfectly, the cushions should be carefully connected by strips of tinfoil, or otherwise, with knobs, which can be put to earth during the action of the machine.

The collectors are two stout metal forks bestriding the glass disc at the ends of a horizontal diameter. They are armed, on the sides next the glass, with rows of sharp points, which extend across the rubbed part of the disc. The prime conductor in the specimen we are describing forms a metal arch rising over the framework of the machine, and insulated from the sole by two glass pillars. Various forms are given to this part of the machine, according to the fancy or convenience of the experimenter. One important thing to be seen to is, that there be no salient points on it which might facilitate the dissipation of electricity by brush, convective, or spark discharge.

After what has been said, the action of the machine requires little explanation. The disc, electrified positively by contact with the amalgam, carries away a positive charge, whose potential rises rapidly as it leaves the cushion,—so high, in fact, that there is a tendency to discharge to the air, which is prevented by covering the excited parts of the disc by pieces of oiled silk. When the highly charged glass comes opposite the points of the collector, owing to the inductive action, negative electricity issues from the points and neutralizes the charged plate, which at this point is virtually inside a closed conductor. The result of this is that the prime conductor becomes positively charged. The neutralized parts of the disc now pass on to be rubbed by the other cushion, and so on. The electricity goes on accumulating in the prime conductor until the potential is so great that discharge by surface conduction, or by spark, takes place between the collectors and the cushion, or between the collectors and the axis.

If it is desired to obtain negative electricity from a machine with a glass disc, we have simply to connect the prime conductor to earth, insulate the cushions, and collect the electricity from them.

We have said that there is a limit to the potential to which the



FIG. 56.—Ramsden's electrical machine.

¹ According to Riess, the earliest. ² *Exp. Res.*, 2141.

³ The parchment-like paper obtained by treating ordinary paper with concentrated sulphuric acid.

⁴ That is, the dimension of the rubber perpendicular to the axis of rotation.

⁵ A few notices of the earlier machines will be found in the *Historical Sketch*.

⁶ Mascart, *l.c.*

charge on the prime conductor can be raised. We can never get a longer spark from the machine than the length of the interval between the collector and the cushion or the axis, as the case may be. The limiting potential can, however, be increased by insulating the axis of the machine, or making the axis itself wholly or partially of insulating material, and by using only one rubber and one collector, and placing them at the extremities of a diameter. The machine of Le Roy, often called Winter's machine (fig. 57), is

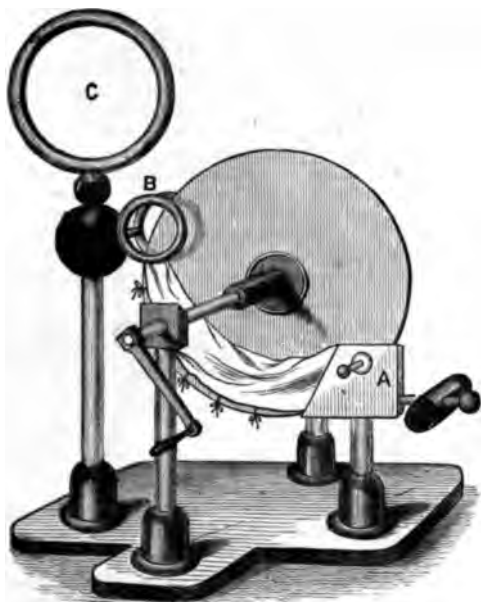


FIG. 57.—Le Roy's machine.

constructed on this pattern. We get, of course, *ceteris paribus*, only half as much electricity per revolution with a machine of this kind as with Ramsden's; but the spark is longer, in consequence of the greater insulation between the cushion (A) and the collector (B).

The cylinder machine, also called Nairne's machine, was one of the first machines in which all the essential parts of the modern frictional machine appeared. It consists of a glass cylinder, which can be turned about a horizontal axis by a multiplying gear, or (as is now more usual) by means of a winch handle simply. The cushion is affixed to one horizontal metal cylinder, and the collector to another. It is necessary to insulate the axis in this machine, owing to its proximity to the ends of the conductors. Positive or negative electricity can be obtained with equal readiness by insulating either of the conductors, and connecting the other with the earth.

Those who desire more minute information concerning the functions of the different organs of the frictional machine, are referred to Mascart, tom. ii. § 834, &c. In the same place will be found a description of the famous machine with double plates constructed by Cuthbertson for Van Marum, and still to be seen in Teyler's Museum at Haarlem. A description of another of Van Marum's machines will be found in the article "Electricity" in the *Encyclopædia Metropolitana*. We take this opportunity of calling the scientific reader's attention to that article, which contains a great quantity of very valuable matter. Much of the work of the earlier electricians that we have been obliged to pass over in silence is fully described there.

Electric machines have also been constructed of less costly materials than glass or even vulcanite—of cloth and paper, for instance—for an account of these, see Riess, Bd. ii. §§ 936, 937.

Many experiments have been made on the electrification of sifted powders. We have already, in describing Lichtenberg's figures, alluded to some cases of this kind. As a rule, either the results are very uncertain, or the conditions of the experiment very complicated, so that the experiments are, in most cases, more curious than valuable, from a scientific point of view. Such as desire it will find abundant indications of the sources of information in Riess, Bd. i. §§ 938 *sqq.*, and *Ency. Metrop.*, art. "Electricity," §§ 193 *sqq.* One case of this kind, however, was so famous in its day, that we ought to mention it. In the year 1840 a workman at Newcastle, having accidentally put one hand in the steam which was blowing off at the safety valve of a high-pressure engine boiler, while his other hand was on the lever of the valve, experienced a powerful electric shock in his arms. Armstrong investigated the matter, and was led to construct his famous hydroelectric machine. This apparatus consists simply of an insulated boiler for generating high-pressure steam, fitted with a series of nozzles,

kept cool by a stream of water. The steam issues from these nozzles and impinges on a conductor armed with points for collecting the electricity. The boiler gets electrified to a high potential, and a torrent of dense sparks may be drawn from it. The machine far surpassed any ordinary electrical machine in the quantity of electricity furnished in a given time. By means of it water was decomposed, and the gases collected separately. It was reserved for Faraday to trace the exact source of the electromotive force. He demonstrated, by a series of ingenious experiments, that the electrical action arose from the friction of the particles of water in the condensed steam against the wood of the nozzles.¹

Remaining Cases.—Of these the most important are atmospheric electricity,² which belongs properly to meteorology, animal electricity, comprehending the study of the properties of the electrical fishes, and the electric phenomena of nerve and muscle. We have already indicated the literature of the former subject, and the latter belongs, for the present at least, to physiology. Evaporation, combustion, and in fact chemical action generally, have been brought forward by some experimenters as sources of electromotive force. About the last of all there is, of course, in one well-known case no doubt. As to the experiments generally alluded to under the other two heads—in particular, those of Laplace and Lavoisier, Volta, Pouillet, and others—there has been considerable difference of opinion, and we need not occupy space here with fruitless discussion of the matter.³ Similar remarks apply to the electrification caused by pressure, cleavage, and rupture.

*Machines founded on Induction and Convection.*⁴—The oldest electric machine on this principle is the electrophorus of Volta. This consists of a plate of resinous matter (now usually vulcanite) backed by a plate of metal, and a loose metal plate, which we may call the collector, fitted with an insulating handle. The vulcanite is electrified by flapping it with a cat-skin, the collector is placed upon it, uninsulated for a moment by touching it with the finger,⁵ and then lifted by the insulating handle. The collector plate is then found to be charged (positively) to a high potential, and sparks of some length may be drawn from it. The explanation of the action of the electrophorus is simple enough, if we keep clearly in view the experimental fact that the surface electrification of a non-conductor, like vulcanite, will not pass to a metal plate in contact with it under ordinary circumstances. If the surface density of the electrification be very great, discharge to the metal may no doubt take place; and if the collector be kept for a very long time in contact with the vulcanite, it is said that it may become negatively electrified. In the normal state, however, the negative electricity of the vulcanite remains upon it, and the thin layer of air intervening between it and the collector forms the dielectric in a condenser of very great capacity, so that a quantity of electricity collects on the lower surface of the condenser very nearly equal to that on the vulcanite. The difference of potential between the plates is very small (just as in Volta's condensing electroscope, see above, p. 34). When the collector is raised it carries away the positive charge—the potential of which, owing to the decrease in the capacity of the collector, rises enormously. It is to be noticed that the potential of the charge on the vulcanite rises to a corresponding extent. This remark partly explains the remarkable fact that, when the collector is kept on the excited vulcanite, its electrification may be kept for a long time (for weeks under favourable circumstances), whereas it speedily dissipates if the vulcanite be left uncovered. According to Riess, the fact that a plate of metal laid on an excited piece of glass tends to preserve its electrification was discovered by Wilcke in 1762.

If each time we charged the collector it were discharged by contact with the interior surface of a hollow conductor A, it is obvious that we could raise A by a sufficient number of such contacts to as high a potential as we please, provided it were sufficiently well insulated. This remark brings Volta's electrophorus into the present category of electrical machines.

In the rest of the induction machines to be described the excited dielectric is dispensed with, and an electrified conductor substituted in its place.

The earliest apparatus that involved the principle of such machines appears to have been Bennet's doubler.⁶ The principle of this apparatus may be explained thus. Let A and C be two fixed discs, and B a disc which can be brought at will within a very short distance of either A or C. Let us suppose all the plates to be equal, and

¹ *Exp. Res.*, ser. xviii. 2075.

² See Riess, § 1028 *sqq.*, and Thomson's papers in *Reprint* already alluded to; also *Ency. Metrop.*, art. "Electricity," § 219, for bibliography of older investigators.

³ See Riess, §§ 943 *sqq.*

⁴ This highly-descriptive title is Sir William Thomson's.

⁵ In most modern specimens this is rendered unnecessary by a brass pin, which is in metallic connection with the metal backing of the vulcanite, and comes up flush with the surface of the vulcanite, so as to touch the collector when it is *in situ*.

⁶ *Phil. Trans.*, 1787.

Miscellaneous results.

Electrophorus.

Bennet's doubler.

let the capacities of A and C in presence of B be each equal to p , and the coefficient of induction between A and B, or C and B, be q . Let us also suppose that the plates A and C are so distant from each other that there is no mutual influence, and that p' is the capacity of one of the discs when it stands alone. A small charge Q is communicated to A, and A is insulated, and B, uninsulated, is brought up to it; the charge on B will be $-\frac{q}{p}Q$. B is now uninsulated and brought to face C, which is uninsulated; the charge on C will be $\frac{q^2}{p^2}Q$. C is now insulated and connected with A, which is always insulated. B is then brought to face A and uninsulated, so that the charge on A becomes rQ , where

$$r = \frac{p}{p+q} \left(1 + \frac{q^2}{p^2}\right).$$

A is now disconnected from C, and here the first operation ends. It is obvious that at the end of n such operations the charge on A will be $r^n Q$, so that the charge goes on increasing in geometrical progression. If the distance between the discs could be made infinitely small each time, then the multiplier r would be 2, and the charge would be doubled each time. Hence the name of the apparatus.

Darwin.
Cavallo.
Nicholson.

Darwin, Cavallo, and Nicholson¹ devised mechanism for effecting the movements which in Bennet's instrument were made by hand. Cavallo's was a reciprocating movement, but in the machines of Darwin and Nicholson the motion was continuous and rotatory. Nicholson's doubler is a very elegant instrument. A drawing of it is given by Mascart (t. ii. § 845); the specimen there represented is very like one which was found among the late Professor Willis's apparatus, and is now in the Cavendish Laboratory at Cambridge. A still more elegant machine is "Nicholson's spinning condenser," which bears a remarkable resemblance to the induction machine of Töpler.² A description, with a figure, will be found in the *Encyclopædia Metropolitana*, art. "Electricity," § 112.

It is obvious that if any conductor be connected with the part of any of these machines corresponding to the conductor A in the above description, and the potential of A be raised to any small positive or negative value,³ we can by means of the machine increase the charge, and therefore the potential, up to any required amount. We have, in fact, an electric machine which may be used for all the ordinary purposes. It was not with this view, however, that these pieces of apparatus were first invented, but rather for the purpose of demonstrating small electric differences. In this they were but too successful, for it was found that it was impossible to prevent them from indicating electric differences unavoidably arising within the apparatus itself. It was this difficulty no doubt that led to their being ultimately abandoned, and for a time forgotten, although they were once in high favour. Of late, however, they have been taken up as electrometers with great success.

Typical inductive machine. The type of all these machines is an arrangement of the following description. A conductor or carrier C, or a series of carriers, is inductive fastened upon the circumference of an insulating disc. At the ends of a diameter are two hollow conductors, A and B, embracing the disc on both sides, so that twice in the course of a revolution the carrier is virtually in the interior of a hollow conductor. Inside each conductor are two springs: one of these is in metallic connection with the conductor, and may be called the receiving spring; the other, called the inductor spring, is insulated from the conductor, and is connected either to earth or with the corresponding spring belonging to the other conductor. Suppose A to be at a small positive potential, and B at zero potential; starting with C in connection with the inductor spring inside A, it becomes negatively electrified and carries away its charge; it next comes in contact with the receiving spring in B, and, being now part of the interior of a hollow conductor, it parts with the whole of its charge to B; then it passes on and is charged positively at B's inductor spring; then discharges to A at A's receiving spring; and so on. The positive and negative charges are each a little increased every revolution, and the difference of potentials accordingly augmented. This is the principle of Varley's machine⁴ (1860), and of Thomson's mouse mill and replenisher⁵ (1867); it is virtually that of Bennet's doubler.

Water-dropping machine. Closely allied to these machines is Thomson's water-dropping potential equalizer. This consists of an insulated reservoir of water, with a long pipe, from the nozzle of which water is allowed to break in drops. It is obvious that if the potential of the reservoir be above that of the air surrounding the spot where the water breaks into drops, each drop will carry away with it a positive charge, and this will go on till the potentials are equalized. This device was introduced by Thomson in observations on atmospheric

electricity. The burning match which he uses in conjunction with the portable electrometer acts in the same way. He has also constructed a water-dropping electric machine on a similar principle. Two streams of water break into drops inside two inductors connected with the internal armatures of two Leyden jars, A and B; the drops from each inductor fall into a receiver connected with the other inductor. A very small difference of potential between the jars starts or reverses the action of the apparatus; in fact, it will in general start of itself, and very soon sparks are seen passing between the different parts, and the drops are scattered in all directions by the strong electrical forces developed.

The most remarkable, as well as the most useful, of all these machines is that of Holtz.⁶ Here the convection is effected by means of a disc of glass, which is mounted on a horizontal axis F (fig. 58), and can be made to rotate with considerable angular velocity by means of a multiplying gear, part of which is seen at X. Close behind this glass disc is fixed another vertical disc of glass, in which are cut two windows, B, B. On the side of the fixed disc next the rotating disc are pasted two sectors of paper, A, A, with short blunt points attached to them, which run out into the windows towards the rotating disc, without quite touching it. Two metal combs C are placed on the other side of the rotating

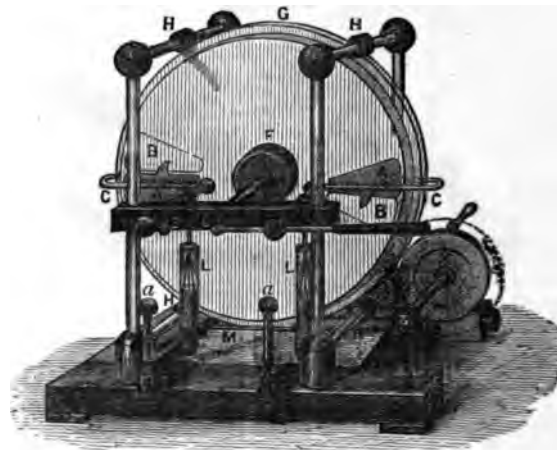


FIG. 58.—Holtz's machine.

disc (that nearest the reader), the teeth being put opposite the parts of A, A which lie towards the windows. The combs are fixed to metal shanks, which pass through a stout horizontal bar of ebonite. One of these shanks terminates in a couple of balls at E, and the other carries a sliding electrode D with a long ebonite handle. The framework which carries the horizontal ebonite bar and supports the fixed plates, &c., will be understood from the figure.

The machine, as originally constructed by Holtz, contained only the parts we have described. Poggendorff doubled all the parts (except, of course, the electrodes D and E). The figure represents Ruhmkorff's modification of this construction. Behind the fixed disc there is another fixed disc, with windows and armatures like the first, and, beyond that, another movable disc mounted on the axis F. The combs are double, as will be seen from the figure. To start the machine, D and E are brought together, and one of the armatures (or one pair), say the right hand one, is electrified in any manner, let us say positively, and the disc set in rotation. After a little time a hissing noise is heard, and the machine becomes sensibly harder to turn, as if the disc were moving through a resisting medium. If the room be dark, long curved pencils of blue light will now be seen issuing from the points of the left hand comb, and running along the surface of the disc in a direction opposite to its motion, while little stars shine upon the points of the right-hand comb. After this state has been reached, the balls D, E may be separated, and a continuous series of brush discharges will take place between them, even when the distance is very considerable. If two Leyden jars, L, L, be hung upon the conductors which support the combs, the outer coatings being connected by a conductor M, then a succession of brilliant and sonorous sparks will take the place of the brushes. Instead of using the two jars L, L, we may connect D and E with the internal and external armatures of a condenser; it will then be found that, as we augment the capacity of the condenser (the angular velocity of the disc being constant), the frequency of the sparks diminishes, while their brilliancy increases. If we insert a high resistance galvanometer between D and E, it will indicate a current flowing from D

¹ Phil. Trans., 1788.

² Pogg. Ann., 1865.

³ By connecting the conductor with the positive or negative pole of a small galvanic battery, for instance.

⁴ Jenkin, *Elect. and Mag.*, cap. xix.

⁵ Described in the art. ELECTROMETER.

⁶ Pogg. Ann., 1865.

to E, the intensity of which, under given atmospheric conditions and given state of the machine, will vary as the angular velocity, being independent, within very wide limits, of the resistance¹ between D and E.

It is not difficult to give a general account of the action of this machine, although it is very hard to assign the precise importance of the individual parts, very slight modifications of which greatly affect the efficiency. Suppose D and E in contact; the right-hand armature, charged +, acts by induction on the right-hand comb, causing - electricity to issue from the points upon the disc. At the same time the positive electricity of the right comb passes through DE to the left comb, and issues from its teeth upon the parts of the disc at the other end of the horizontal diameter. This + electricity electrifies the left armature - by induction, + electricity issuing from the blunt point upon the further side of the rotating disc. The charges thus deposited on the disc are carried along, so that the upper half is electrified - on both sides, and the lower half + on both sides, the sign of the electrification being reversed as the disc passes between the combs and the armature by the electricity issuing from the combs and from the armatures. If it were not for dissipation in various ways, the electrification everywhere would obviously go on increasing; but in practice a stationary condition is soon attained, in which the loss from the armatures is just balanced by the gain owing to the action of the blunt points. After this, both sides of the disc are similarly electrified, the upper half² always -, the lower always +; + electricity continually issuing from the points of the right comb, - electricity from the points of the left. This is, of course, accompanied by a current of + electricity from right to left through DE.

The machine of Holtz, as we have described it, is somewhat uncertain in its action in our moist climate; but a slight modification of it gives excellent results. Upon the axis X is fixed a disc of ebonite, large enough just to reach between the paper armatures. This disc is fitted with a small rubber attached to the frame of the apparatus, and forms a small electric machine, which keeps the armatures continually electrified.³ The whole is inclosed in a glass case, with a beaker of sulphuric acid to dry the air. There is a machine of this kind at present in the Cavendish laboratory at Cambridge, which never fails when the auxiliary apparatus is at all in good order.

A very remarkable phenomenon often occurs when the electrodes of Holtz's machine are in connection with the armatures of a condenser of considerable capacity, and are so far separated that a spark does not pass. The machine charges the condenser up to a certain point, and then the condenser discharges along the surface of the disc. If the experiment be conducted in a dark room, a flash of light will be seen to pass along the surface of the disc, and thereafter it will be observed that the long positive brushes have shifted from one comb to the other; after a little the condenser discharges again, and the brushes will now be seen in their old place, and so on. This phenomenon, though interesting to study, is often inconvenient in practice. To prevent it, Holtz introduced the diagonal conductor which is seen on many machines. For an account of this, and for other details concerning these machines, we refer the reader to Mascart, t. ii. § 847 *seq.*, whose account of the more obvious principles of this apparatus is among the most lucid we have seen. His account of the experiment of causing one Holtz's machine in action to turn the disc of another by the electrical reaction is of peculiar interest.

Electromagnetic Induction Machines.—The type of these is the induction coil or inductorium, sometimes called Ruhmkorff's coil, after the great Parisian instrument-maker who first brought the instrument to perfection. The object of such machines is to obtain great electromotive force from sources which furnish large quantities of electricity, but have only small electromotive force.

The principles on which the action is founded has been sufficiently indicated above in our section on the induction of electric currents. We have also given in the Historical Sketch (p. 12) some notices of the literature of the subject; a brief enumeration of the essential parts of the machine is all that is necessary here.

We have first the primary coil—of thick wire and few windings, so as to have a small resistance and a small coefficient of self induction; the secondary coil surrounding the primary is of thin wire ($\frac{1}{4}$ mm. or so), with many windings, the length in large machines being often 100,000 metres. In order to avoid the danger of disruptive discharge between parts of the insulated wire, the coil is divided up by insulating septa, so that parts at very different potentials are separated. In the centre of the primary is placed a bundle of iron wires; this greatly strengthens the action, and a good deal depends on the quality of the iron, which should be very soft. The interruptor is simply a lever, worked by the coil itself or by an electromagnet separate from the coil, by means of which the circuit

of the primary is made and broken automatically. A variety of forms have been given to the part of the apparatus; the interruptor of Foucault is a very common one.⁴ For some purposes a break driven by clock-work is used. The condenser, a very important part of the apparatus, is made of a number of sheets of tinfoil, interleaved with sheets of oiled silk or varnished paper. One set of leaves of the condenser is connected with one side of the break, and the alternate set with the other side. The function of the condenser is to provide a way for the electricity when the circuit is broken, and thus to prevent the intense spark of the extra current in the primary, which destroys the contact surfaces of the break, and, what is worse, prolongs the fall of the primary current, and thereby reduces the average electromotive force of the induction current.

Other devices have been tried for effecting the same object as the condenser, such as inserting a fine metallic wire or an electrolyte as an alternative circuit to the break; and these answer the purpose to a considerable extent. An important improvement affecting this part of the apparatus has recently been introduced by breaking the primary circuit between the poles of a magnet, the effect of which is that the spark is suddenly drawn aside (blown out as it were). A considerable increase of striking distance between the poles of the secondary results from this arrangement.

ABSOLUTE MEASUREMENTS.

We have already indicated the considerations which determine the fundamental units in the two systems that have come into practical use. We ought now to explain how practical standards can be constructed to represent these fundamental units, or at least known multiples of them. It is necessary to have such standards in order that we may be able to measure electrical quantities in absolute measure by simple and expeditious methods of comparison, it being obviously impossible in practice to make absolute measurements directly on all occasions.

Electrostatical System.—By means of Thomson's absolute electro-measure meter we can determine any electromotive force in absolute measure. In this way Thomson found the electromotive force of Daniell's E. M. F. battery to be '00374 C.G.S. electrostatical units.⁵

By using the absolute electrometer (see art. **ELECTROMETER**), or Resistor, another that had been compared with it, we could by the method above, p. 46, find a resistance (which was large enough to suit the method) in electrostatical measure.

Then, having standards of electromotive force and resistance, we could easily measure a current in electrostatic measure by applying Ohm's law. The same thing might be done by constructing the standard of quantity, which is the charge on an isolated sphere of unit radius charged to unit potential. By comparing the throw of a galvanometer when unit quantity is discharged through it with the deflection produced by any current, we could determine the latter in absolute measure by observing the time of oscillation of the galvanometer and the logarithmic decrement of its oscillation (see Maxwell, vol. ii. § 749).

Among the absolute measurements in the present system of units, we must not omit to mention Sir Wm. Thomson's determinations of the dielectric strength of different thicknesses of air. From these, and from the measurement of the electromotive force of Daniell's cell just mentioned, he concluded that a Daniell's battery of 5510 elements would be competent to produce a spark between two slightly curved metallic surfaces at $\frac{1}{4}$ of a centimetre asunder in ordinary atmospheric air.⁶

Electromagnetic System.—The great majority of the absolute determinations hitherto made have reference to this system. We make no attempt here to instruct the reader concerning the details of this subject; such an attempt would lead us into technical particulars intelligible only to a few scientific men. We are fortunate, however, in being able to refer the English reader to two books which contain in a collected form all, or nearly all, the requisite information, viz. Maxwell's *Electricity and Magnetism*, and the collected Reports of the Committee of the British Association on Electrical Standards.⁷

As a specimen of the theoretical considerations involved, the reader may take Maxwell's method for determining the coefficient of self-induction of a coil (given above, p. 80). If we know the value of L (in centimetres) from calculation, then equation (33) might be used to find α in absolute measure. This would not be a practicable method, inasmuch as the calculation of L would be difficult if not impossible; we might, however, determine L by comparison⁸ with a coefficient of mutual induction which could be calculated.

The earliest absolute measurement of the resistance of a wire (by Kirchhoff).

⁴ See Wiedemann's *Galv.*, or Du Moncel, *Notice sur l'Appareil de Ruhmkorff*.

⁵ *Reprint of Papers*, § 305, &c.

⁶ *Reprint of Papers*, § 340.

⁷ Such as wish to go deeply into the matter must read the *Maassbestimmungen* of Weber.

⁸ Maxwell, vol. ii. § 756.

¹ We speak of resistances of 1 to 10,000 or 100,000 ohms.

² The line of division is not horizontal, however, if, indeed, it be exactly a diameter. See Mascart.

³ Compare Carré's machine, Mascart, t. ii. § 856.

Kirchhoff in 1849) was of the kind just alluded to; that is to say, it involved the comparison of a resistance with a coefficient of mutual induction, the time measurement being that of the period of oscillation of a galvanometer.

Weber. Weber used two methods,—(1) the method of transient currents, in which he measured the throw of a galvanometer caused by the current from an earth inductor of known area when it was turned about a vertical axis, so that the number of the earth's lines of force through it increased from zero to a maximum; and (2) the method of logarithmic decrements, in which he observed the time of oscillation and the logarithmic decrement of a magnet in a galvanometer of known constant. In the last of these two methods the horizontal component of the earth's horizontal force comes in directly, and the magnetic moment of the galvanometer magnet must be determined, which is a matter of great difficulty.

B. A. committee. The determination of the British Association committee was carried out by Messrs Maxwell, Balfour Stewart, and Fleeming Jenkin, and the result of it was the construction of a standard called the ohm, which professes to represent a velocity of an earth quadrant per second ($10^9 \frac{\text{cm.}}{\text{sec.}}$).—The method they used is due to Sir Wm. Thomson. It consists essentially in causing a coil of wire of known dimensions to rotate about a vertical axis, and observing the deflection of a magnet of very small moment suspended at its centre.

Kohlrausch. In a recent determination, F. Kohlrausch¹ has combined the two methods of Weber, and thereby avoided some of the difficulties which arise in either method used by itself. His value for the resistance of Siemens's mercury unit is $0.9717 \frac{\text{Earth quadrant}}{\text{Second}}$.

According to Dehms and Hermann Siemens, the resistance of the coil called the ohm is equal to 1.0493 mercury units. According to Kohlrausch, therefore, the actual British Association standard is $1.0196 \frac{\text{Earth quadrant}}{\text{Second}}$ in absolute measure; or, in other words, the determination of the British Association Committee is out by nearly 2 per cent.

Lorenz. Lorenz² has, still more recently, made a determination of the value of the mercury unit in absolute measure. He causes a copper disc to rotate inside a coil of known dimensions. The two ends of a circuit C are kept in contact with the axis and circumference respectively of this disc. At two points A and B of C, the resistance between which is R, are attached the two terminals of the coil of wire, in circuit with which is also a battery. A sensitive galvanometer is placed in the circuit C, and the angular velocity of the disc is adjusted till this galvanometer indicates no current. If n be the number of revolutions per second, and E the electromotive force of induction per unit of inducing current, calculated from the dimensions of the coil, then the resistance R is equal to nE in electromagnetic measure.

The result obtained by Lorenz for the value of the mercury unit is $.9337 \frac{\text{Earth quadrant}}{\text{Second}}$; this would make the value of the B. A. standard $.9797 \frac{\text{Earth quadrant}}{\text{Second}}$.

There is thus considerable discordance between the different results. It is a curious fact that the mean of the result of Kohlrausch and Lorenz gives for the value of the B. A. standard $.9996 \frac{\text{Earth quadrant}}{\text{Second}}$. Fresh determinations are, however, in progress, and it is to be hoped that the doubt which hangs over the matter will be dispelled.³

Calorimetric method. Besides these methods, there is yet another of a totally different character, originally suggested by Thomson in 1851, in his paper on the "Mechanical Theory of Electrolysis." This method consists in measuring the amount of heat developed in a wire by a current the square of whose strength is known in electromagnetic measure. If we know the mechanical equivalent of heat with sufficient accuracy, we can calculate from these results the resistance of the wire in absolute measure by means of Joule's law. Measurements of this nature have been made by Von Quintus Icilius,⁴ Joule,⁵ and H. Weber.⁶

Current. We can, by means of a tangent galvanometer, find the value of any current in electromagnetic measure (see art. GALVANOMETER). If the resistance of the circuit be found, by comparison with the

ohm or other absolute standard, we can determine the value of the electromotive force in the circuit by Ohm's law. Measurements of this kind have been made by Boescha,⁷ by Von Waltenhofen, F. Kohlrausch, and Latimer Clark. The results of Kohlrausch⁸ for the cells of Daniell and Grove, when no current is passing, are 1138×10^8 and 1942×10^8 C.G.S. units respectively. Latimer Clark⁹ gives 1110×10^8 and 1970×10^8 for the same constants. The results, of course, depend on the constitution of the cells.

Taking the number of electromagnetic units in an electrostatic unit to be 3×10^{10} , we get from Thomson's electrostatic measurements for the electromotive force of Daniell's element 1120×10^8 in C.G.S. units.¹⁰ The agreement among the different results is so far good.

The determination of the electrochemical equivalent of some elementary substance in this system of units is of great importance. Determinations exist by Weber, Bunsen, Casselmann, Joule, equivalent, and F. Kohlrausch. The result of the last is no doubt the best, lent as he combined with his voltametric experiments a determination of the horizontal component of earth's magnetic force, which is the most uncertain factor in the result. According to his result, one C.G.S. unit of electricity deposits $.011363 (\pm .000002)$ gm. of silver. From this we get for the electrochemical equivalent of water $.0009476$.

Ratio of Electrostatic to Electromagnetic Unit.—If we measure the same quantity of electricity first in electrostatic and then in electromagnetic measure, the fundamental units of mass, length, time, and time being the same in both cases, the ratio of the two fundamental measures will vary directly as the magnitude of the unit of length, and inversely as the magnitude of the unit of time adopted. This velocity ratio may therefore be regarded as a velocity which will remain the same whatever three fundamental units we adopt.¹¹

This velocity was found by Weber and Kohlrausch by the direct process of measuring the same quantity of electricity, first in terms of the one unit and then in terms of the other. This result was $31 \times 10^9 \frac{\text{cm.}}{\text{sec.}}$.

Five other methods will be found described by Maxwell, vol. ii. § 768 *seq.* Two of these have actually been carried into execution,—one by himself, the other by Sir Wm. Thomson. The results for the fundamental velocity are $28.8 \times 10^9 \frac{\text{cm.}}{\text{sec.}}$ and

$28.2 \times 10^9 \frac{\text{cm.}}{\text{sec.}}$ respectively.

THEORIES OF ELECTRICAL PHENOMENA.

Throughout this article we have limited ourselves as much as possible to an exposition of the experimental facts of electricity. Where mathematical developments have occurred, they have been in most cases been simply deductions from some principle or principles well established by experience. To have made our survey of the present state of electrical science complete, we ought to have added a section on the different attempts which have been made by the doctors of the science to penetrate a little farther into the secrets of the hidden mechanism by which electrical phenomena are brought about. But any attempt at a review of this kind must be relinquished. We refer the reader to our indications of the literature (Historical Sketch, p. 10). The most important work in this department lies at hand for the English reader in Professor Clerk Maxwell's *Treatise on Electricity and Magnetism*.¹² Particularly important are his theory of electric displacement and its application to statical as well as to current electricity; his investigation of the stresses in the medium, by which the electrostatic forces on the one hand, and the electromagnetic forces on the other, may be produced; the application of the theory of displacement to the case of electrical equilibrium when the dielectric medium is not everywhere the same; the dynamical theory of the electromagnetic field; and the electromagnetic theory of light. Maxwell gives, at the end of his work, a most instructive summary of the different speculative theories. The student who desires to pursue this department farther will do well to master this summary at the outset. (G. CH.)

⁷ Whose result has already been quoted. It is too low, on account of polarization.

⁸ *Pogg. Ann.*, 1870, and *Ergbd.*, 1874.

⁹ Everett, *Illustrations of C. G. S. System of Units*, § 125, or *Journ. Soc. Tel. Eng.*, 1873.

¹⁰ Everett, *l.c.*

¹¹ Maxwell, *Elect. and Mag.*, vol. ii. § 768.

¹² We have followed throughout the views expounded in this work; and we are also under great obligations to its author for his advice on many points. For aid in collecting facts we are indebted mainly to the works of Riess, Wiedemann, and Mascart. Without their aid many sections of this article could not have been written. Wiedemann's treatise, in particular, lightened our task by the extent of its information and the profusion and accuracy of its references to original authorities for the facts in electrical science.

¹ *Pogg. Ann.*, *Ergbd.*, 1873.

² *Pogg. Ann.*, 1873.

³ Since the above was written, an account has appeared of a new determination by H. Weber of Zurich. His results, from three distinct methods, differ by less than $\frac{1}{10000}$, and give $.9550 \times 10^9 \frac{\text{cm.}}{\text{sec.}}$ for the

Siemens unit. This would make the B. A. unit $1.0014 \times 10^9 \frac{\text{cm.}}{\text{sec.}}$

⁴ *Pogg. Ann.*, 1857.

⁵ *Brit. Assoc. Rep.*, 1867.

⁶ *Dissertation*, Leipsic, 1863, quoted in Wiedemann, *Bd. ii.* § 1109.

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ELECTROLYSIS. A very slight acquaintance with the phenomena of conduction of electricity by different bodies shows us that conductors may be arranged in two very distinct classes. In one the passage of electricity produces no change in the chemical composition of the substance, unless indeed the electromotive force be so great that disruptive discharge occurs, or so large an amount of heat is generated that chemical effects ensue; the conductivity diminishes slowly as the temperature rises, and if the resistance of the rest of the circuit be small compared with that of the substance under consideration, an amount of heat is produced in the latter equivalent to the energy expended by the sources of electricity. To this class of conductors probably belong all solids, with the exception of hot glass, which conducts with decomposition at a temperature below the fusing point. The conductivity differs enormously in the different cases; those which conduct most readily are the metals, alloys, the chemical elements generally, and some few metallic oxides and sulphides (Faraday, *Exp. Res.*, 440, ser. iv.; Skey, *Chem. News*, xxiii.). Besides fused metals Faraday added one liquid, fused periodide of mercury, to the list, but subsequently gave reasons for considering that it was misplaced (*Exp. Res.*, 691, ser. vii.). The other class of conductors presents a remarkable contrast to the one just described. In these the passage of electricity results in the chemical decomposition¹ of the substance of the conductor at the points where the electric current² enters and leaves the body; a rise of temperature produces in such bodies a very considerable increase in the conductivity, but the specific resistance of even the best conducting among them is always very great compared with that of the metals. (For details see article ELECTRICITY, p. 46 *seq.*) Only part of the energy of the circuit is spent in heating the conductor, as a transformation of energy takes place in the chemical and molecular actions at the points where the current enters and leaves the conductor.

It is the behaviour of the second class of bodies under the influence of the electric current that we have now to discuss. The physical side of the subject has already been considered in the article ELECTRICITY; so we shall principally confine our attention to the phenomena of electrolysis which bear on the laws and principles of chemistry. Before going further it will be necessary to introduce the technical terms which have now become familiar, and, in order to be definite, we will consider somewhat closely a particular instance of electrochemical decomposition of the simplest type.

The cell in which the action takes place consists of a wide tube of hard glass, bent into a V-shape; into this is introduced some silver chloride, which is kept fused during the experiment; into the liquid in one leg of the tube is dipped a platinum wire connected with the negative pole (zinc) of a battery³ of 3 or 4 Grove's cells, and into that in

the other a piece of graphite or gas carbon connected with the positive pole of the same battery. We will suppose a galvanometer introduced into the circuit, and that the current strength as indicated thereby is, roughly speaking, constant, so that the quantity of electricity which passes can be measured roughly by the time occupied in passing. After the circuit has been closed a short time, bubbles of chlorine will begin to come off from the carbon, while pure silver is deposited upon the platinum wire, but *except at these points no alteration will take place at any part of the fluid*. If the platinum wire with the attached silver be weighed at intervals, it will be found that the amount deposited after the current has become constant is proportional to the time, i.e., to the amount of electricity which has passed through the liquid. The same will be true of the chlorine if collected in the other leg of the tube, due allowance being made for the small bubbles retained by the carbon, &c. And the amount of chlorine will be chemically equivalent to the amount of silver; thus for every 108 grammes of silver on the platinum there will be 35.5 grammes of chlorine set free in the other leg of the tube. Moreover if the current be varied by varying the number of battery cells, it will be found that the amount of decomposition in a given time is proportional to the current, that is, again, to the quantity of electricity which traverses the substance.

Faraday, who was the first to define the laws which hold in electrochemical decomposition, introduced, for the sake of precision, a system of nomenclature which has since been generally employed. Wishing to regard the terminals corresponding, in any similar case, to the carbon and platinum in the above experiment merely as the "doors" by which the electricity enters and leaves the liquid, he denominated them *electrodes*, and, comparing the "path" of the current to those of the currents which may produce terrestrial magnetism, and hence to the course of the sun, he called the homologue of the carbon (where the current, so to speak, "rose," or entered) the *anode*, that of the platinum (where the current "set," or left) the *cathode*. The component parts, no matter how complex, into which the liquid was decomposed, corresponding to the Ag and Cl of the above, received the name of "ions"—that component which went *down* with the current to the cathode, and there either was set free or combined with the cathode or the surrounding liquid, being the *cation*, and that which went *up* against the current, and appeared or promoted some chemical action at the anode, the *anion*. Moreover, the substance decomposed was called an *electrolyte*, and the process itself *electrolysis*. (Faraday, *Exp. Res.*, 662 *seq.*)

The phenomena which occur at the electrodes when the ions there set free react upon the electrode or the surrounding fluid, so that the resulting products of electrolysis are not the ions themselves, are called *secondary actions*.

The anion and the cation are frequently called the negative and positive ion respectively. Similarly the cathode and anode are termed the negative and positive electrodes; Daniell denoted them the platinode and the zincode, but these terms have fallen into disuse.

Of the bodies which are capable of electrolytic conduction nearly all, if not all, are liquids. Faraday (*Exp. Res.*, 433, 1340) apparently obtained some chemical decomposition in sulphuret of silver and a few other salts when solid, but this did not alter his opinion that the mobility secured in the fluid state, either by fusion or by solution, was necessary to the phenomena of electrolysis; and his view, which he supported by experiments on ice and other solids that conduct when fused (*Exp. Res.*, 380–397, 419–428), still obtains. Electrolytic action doubtless sometimes takes place in gases, but accurate investigation of the subject is difficult on account of the extreme mobility of the particles

Typical electrolytic action.

¹ We have not space here to discuss whether or not conduction in electrolytes is always attended with decomposition, although the question has engaged the attention of many writers on the subject. The reader who wishes for information upon the point may consult Faraday, *Exp. Res.* 986–987½, ser. viii.; Despretz, *Compt. Rend.*, t. xlii. p. 707; De la Rive, *Archives*, t. xxxii. p. 38; Logeman and Van Breda, *Phil. Mag.* [4], viii. 465; Buff, *Ann. d. Chem. u. Pharm.*, Bd. xciv. s. 15; Foucault, *Compt. Rend.*, t. xxxvii. p. 580; De la Rive, *Ann. de Chimie*, [3], t. xlvii. p. 41; Favre, *Compt. Rend.*, lxxiii. p. 1463; Helmholtz, *Berlin Monatsbericht*, 1873, Nachtrag zum Juliheft; and, for a summary of results, Wiedemann, *Galv.*, Bd. I. § 314–316, and Nachtrag, 36, § 334.

² The standard direction of the current is taken, as usual, to be from the copper through the wire to the zinc of an ordinary zinc-copper cell.

³ It is not necessary to use a voltaic battery,—any source of electricity serves,—but either a voltaic or a thermoelectric battery is usually employed, since these so conveniently supply a large quantity of electricity, with an electromotive force sufficient for the purpose.

and the danger of confusing electrolytic effects with effects due to disruptive discharge by convection. Gases have, however, been decomposed by the silent discharge, as CO_2 into $\text{CO} + \text{O}$.

From Faraday's time attempts have continually been made to classify strictly, according to their chemical composition or constitution, the liquids capable of electrolytic conduction, but hitherto without very much success. It must be remembered that, as the resistance of a liquid increases, the tests of electrolytic conduction become less and less sensitive. We can consider a body an electrolyte if we can (1) collect the products of decomposition, or (2) demonstrate their presence on the electrodes by means of the return current due to polarization. If the resistance be very great the former method becomes evidently very difficult, and in the latter complications are introduced which cannot here be discussed (see ELECTRICITY). On the other hand, we might easily be misled into considering a body an electrolyte from the presence of mere traces of a foreign substance. Thus at one time water was regarded as the only electrolyte, but it is found that the purer the water is the less does it conduct electricity, and now Kohlrausch and Nippoldt have shown that the presence of one 10-millionth of H_2SO_4 would be sufficient to account for its observed conducting power, so that the weight of evidence goes to show that water itself is not an electrolyte at all.

It is not, then, surprising that views on the question of what constitutes an electrolyte have changed considerably. Davy and the older chemists, as mentioned above, considered water to be the only electrolyte; Faraday, by electrolyzing fused chlorides, &c., dissipated these notions, but still regarded water as the electrolyte which was decomposed when acids were subjected to the electric current, and his general conclusion was that an electrolyte must be a compound consisting of an equal number of chemical equivalents of its elements, that is, in modern notation, must be of the type $\text{M}_x^{(y)}\text{R}_y^{(x)}$ where x and y are the atomicities or valencies of the elements whose atomic weights are represented by M and R , and thus that two elements would by uniting form only one electrolyte (*Exp. Res.*, 679-701, 830). The oxygen salts for which Faraday assigned no law were included by Daniell in the same formula as binary compounds, of which the part R acting as anion was no longer an element but a compound; thus ZnSO_4 was shown to be split up by electrolysis into Zn and SO_4 ; in that case y would represent the basicity of the acid forming the salt.

This hypothesis lacks definiteness, on account of the variation of the atomicity of the elements, and falls through altogether in the case of copper and iron, which form each two chlorides, $(\text{CuCl}, \text{Cu}_2\text{Cl}_2)$, $(\text{FeCl}, \text{Fe}_2\text{Cl}_2)$, both electrolytes, and in consequence Wiedemann (*Galv.*, Bd. i. §§ 295, 346a, 418 (5)) modifies the statement of the hypothesis, and considers that for a body to be an electrolyte it must be capable of formation by double decomposition from one of the simple binary electrolytes, the exchanging atoms or groups of atoms forming the ions of the new compound. Thus silver acetate gives, by double decomposition with sodium chloride, silver chloride and sodium acetate. Sodium acetate and silver chloride are therefore electrolytes of which Ag , Cl , Na , $\text{C}_2\text{H}_3\text{O}_2$ are the respective ions. This hypothesis may be illustrated by a great number of instances:—the case of the decomposition of uranium compounds, as UOCl into UO and Cl , is a very good example. But Wiedemann's view would indicate that a body, in order to be an electrolyte, need not be one of a "series of salts," and we then see no reason for excluding the hydrogen salts from the class; thus H_2O and HCl can be easily formed by double decomposition, yet the former is, when pure, one of the worst liquid conductors, while the latter as liquefied

gas is apparently not decomposed even by 5640 cells of De la Rue's chloride of silver battery, but gives vibrations indicating very high resistance.¹ Bleekrode has also shown that, of all the pure liquefied hydrogen acids, only HCN is an electrolyte. On the other hand, liquefied NH_3 , which is not formed, so far as we are aware, by double decomposition, is electrolysable by only a moderate battery of Bunsen's cells, giving a blue liquid at the cathode. Moreover, Buff (*Ann. d. Chem. und Pharm.*, Bd. cx.) has electrolysed molybdic and vanadic anhydrides after the manner $\text{MoO}_3 = \text{MoO}_2 + \text{O}$, but these bodies are not obtainable by double decomposition with a simple electrolyte.

Miller (*Elements Chem.*, i. § 282 (v)) considers that an electrolyte must be a combination of a conductor and a non-conductor, and so the majority of electrolytes are. But alloys behave to a certain extent as electrolytes when fused (see Wied., *Galv.*, Bd. i. § 328), and SnCl_4 , though consisting of a conductor and a non-conductor, is not an electrolyte; so that this classification is not exclusive.

It would therefore appear that the condition does not lie in the chemical constitution of the body, but rather in its molecular state, and to this points the fact that two non-conductors, as H_2O and HCl , on being mixed form a very good conductor. In addition to this, quantitative measurements of the resistance of electrolytes show that, in the case of many salt and acid solutions, there is a point of concentration below saturation, for which the conductivity is a maximum. This would scarcely be the case if one alone of the bodies were the conductor.

The liquids which do not conduct are very various, including, besides oils and resins and other organic bodies, benzene, iodide of sulphur, carbon disulphide, glacial acetic acid, fused boracic anhydride, antimonious oxide and oxychloride, the higher halogen salts of tin, liquid sulphurous anhydride, pure water, and pure halogen acids. For others see article ELECTRICITY, p. 51.

In the description of the phenomena, in the typical case Faraday of electrolysis given above, it was stated that the amount of chemical decomposition in any time is proportional to the whole quantity of electricity which passes through the liquid in that time; this is true in all cases of electrolysis, and was established by Faraday (*Exp. Res.*, v. 505, and ser. vii.). It forms part of the general law to which his name is attached, but we prefer to consider it separately for reasons that will appear when we discuss the statement of that law. We may put it thus:—If W be the mass of an electrolyte,² decomposed by the passage of a quantity E of electricity, then, as long as the ions remain of the same nature,

$$W = KE \dots \dots (1),$$

where K is a constant dependent only on the nature of the electrolyte, and therefore independent of the nature or size of the electrodes and of any secondary actions which may take place.

It is evident that if we can prove the truth of this law Volta for one electrolyte, with ions which do not vary with variations of electromotive force, we shall have a very convenient means of measuring the total amount of electricity which passes through any circuit in a given time by introducing such an electrolyte into the circuit, and measuring the amount of decomposition in the given time. Faraday

¹ Bleekrode and De la Rue, *Proc. Roy. Soc.*, xxv. p. 323. In fact, disruptive discharge occurs by convection currents, or, if the electrodes be sufficiently near, by spark. Similar phenomena may be observed by immersing the poles of a Holtz machine in paraffin oil.

² In what follows, the term electrolyte is used in its most general sense, to signify any liquid or mixture of liquids through which the current passes, and not necessarily one definite chemical compound. Hence the necessity for the condition that the ions shall not vary, as in mixed electrolytes ions for high electromotive forces are different from those for low (*vid. inf.*).

day demonstrated the truth of the law in the case of dilute sulphuric acid by experiments with vessels in which the products of decomposition of the dilute acid between platinum electrodes could be conveniently collected, either separately or together, and measured (*Exp. Res.*, 714-728.) Such an instrument he called a volta-electrometer, and subsequently a voltameter. After demonstrating that the amount of decomposition was independent of the size of the electrodes, he connected up two voltameters A and B, in multiple arc, as in the accompanying diagram, and then passed the whole current through a third C, and found that the amount of decomposition in C was equal to the sum of the amounts in A and B. He therefore applied the voltameter¹ to measure quantities of electricity in other cases.

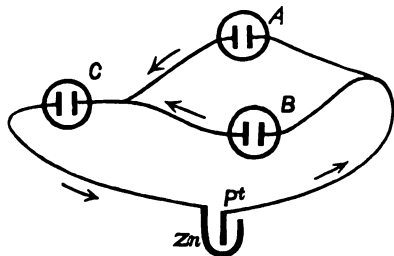


Diagram showing connection of voltameters.

Various forms of voltameter have been employed (see Wiedemann, *Galvanismus*, Bd. i. § 317-319). The most accurate is the silver voltameter of Poggendorff, which consists of a vertical rod of silver with the lower end immersed in a solution of silver nitrate contained in a platinum vessel; the silver is connected with the positive, the platinum vessel with the negative pole of the battery, and the amount of decomposition is ascertained by weighing the platinum vessel with the attached silver before and after the experiment. Buff directly proved the truth of equation (1) for such an instrument by electrolysing silver nitrate solutions of different strengths between silver electrodes. The currents employed were varied for different experiments, and were measured by a tangent galvanometer, and the quantity E of electricity was deduced by observing the time of passage of the current. (*Ann. d. Chem. u. Pharm.*, xciv. 15.)

We have, then, in order to demonstrate generally the law expressed in equation (1), to measure the amount of the ions set free in any case of electrolysis, while the amount of electricity is measured at the same time by means of a voltameter included in the circuit. But the measurement of the amount of ions liberated is not always an easy task; in the great majority of cases secondary actions (see above, p. 106) occur, the primary results of electrolysis are obscured, and in order to determine the nature and amount of the ions special apparatus and further investigation are necessary.

Since the ions are liberated at the electrodes the products of secondary action will remain in the immediate neighbourhood if the action be not too long continued. We may therefore determine the ions by collecting any gaseous products, ascertaining the loss or gain in weight of the electrodes, and analysing the electrolyte in the immediate neighbourhood of the electrodes, taking care that the products at the two do not mix by gravitation, by diffusion, or otherwise.

For instance, if a fused chloride (e.g., PbCl_2) be electrolysed with platinum electrodes, no chlorine will be

evolved at the anode, although Pb will be deposited at the cathode; but if the liquid round the anode be analysed, for every 414 grammes of lead at the cathode will be found 339 grammes of PtCl_4 round the anode. Now the platinum must have been derived from the anode, which will be found to have lost 197 grammes in weight, consequently the 142 grammes of Cl were derived by the electrolysis from the PbCl_2 , and hence PbCl_2 is electrolysed as $\text{Pb} + \text{Cl}_2$.

In order to separate the fluids at the two electrodes, various forms of apparatus have been employed. For fused electrolytes a W-shaped tube, which can be divided after the fluid has solidified, is sufficient; with solutions, however, where the solvent introduces new complications the separation is more difficult, owing to the "migration of the ions" and other causes which will be considered below. Daniell and Miller (*Phil. Trans.*, 1844) used a cylindrical glass vessel separated into three compartments by porous clay diaphragms, the two end compartments containing the electrodes, and having tubes for conducting away gaseous products; while Hittorf, in a classical series of experiments (*Pogg. Ann.*, lxxxix. xcvi. ciii. cvi.), used a number of bell-shaped glass vessels fitted to each other with india-rubber washers, the electrodes being inserted in the bottom and top vessels respectively. The lower end of each bell was covered with membrane to prevent mixing of the products; the whole apparatus was filled with the electrolyte to be decomposed; and the products at the two electrodes were known to be separated if the composition of the fluid in one of the intermediate bells remained the same throughout the experiment.

Great numbers of experiments have been made by different experimenters in one or other of the ways mentioned, and they have thus proved that, whatever the electrodes, and whatever the electromotive force, the secondary action at the electrodes has no effect upon the amount of chemical decomposition,² and therefore the law of equation (1) always holds.

We can give here but a few examples of secondary action. A very good account will be found in Wiedemann, *Galvanismus*, Bd. i. § 326-385, with, however, the drawback of the use of an obsolete chemical notation.

(1) *The ions themselves are set free, but separate into component parts.* That this is the case with oxygen salts, which are separated into the metal and a complex anion which is resolved into oxygen and an anhydride, was pointed out by Daniell (*Phil. Trans.*, 1839), who gave to the SO_4 , derived as electro-negative ion from sulphates, the name of oxysulphion, and so on. Many similar cases occur in electrolyses of organic compounds. Thus potassium acetate is electrolysed originally as $\text{KC}_2\text{H}_3\text{O}_2 = \text{K} + \text{C}_2\text{H}_3\text{O}_2$; but the anion splits up (partly at least) thus: $2\text{C}_2\text{H}_3\text{O}_2 = \text{C}_2\text{H}_4 + 2\text{CO}_2$. All the potassium salts of the fatty acids behave similarly, so that this becomes a general method of preparing the normal paraffins.

(2) *The ions appear in an abnormal molecular state.* The deposit of copper in Gladstone and Tribe's ZnCu couple is a black crystalline powder (see p. 114). The most important instance, however, is the formation of ozone in the oxygen liberated at the anode by the electrolysis of acid solutions, which was recognized by Schönbein in 1840, although the smell and powerful oxidizing properties of the gas evolved had previously been noticed by Franklin and Van Marum. The amount of ozone, though very small, may be recognized by all the ordinary tests (KI, indigo, &c.); it diminishes with rise of the temperature at which the electrolysis takes place, and is above 3 per cent. when the electrolyte dilute H_2SO_4 is cooled by ice and salt, and the electrodes are platinum-iridium wires (Soret). With dilute H_2SO_4 at 6°C ., 100 c.c. of oxygen contained '00009 gramme ozone, and '00027 gramme at a mean temperature of -9°C .; dilute H_2CrO_4 gave at 0°C ., '00052 gramme per 100 c.c. of oxygen (Soret). The amount varies with the different acids, solutions of chromic and permanganic acids giving the largest percentage.

These points are of importance in correcting observations by the water voltameter.

² Of course, if the products of decomposition be allowed to accumulate until the electrode is surrounded with an envelope of liquid differing from the original electrolyte, the whole character of the decomposition changes.

Determina-
tion of
the ions.

¹ Many corrections have to be applied to the observations with a water voltameter in consequence of—(1) the formation of ozone in the collected oxygen; (2) the formation of H_2O_2 ; (3) the solution of the evolved gases in the water, varying with different strengths of acid, and greater for oxygen than hydrogen; (4) the re-combination of the oxygen and hydrogen if in contact with platinum (see Wied. *Galv.*, l.c.). A diagram and description of the water voltameter will be found in any of the numerous works on the subject.

The molecular state of the deposit varies very much with the density of the current, i.e., the current strength per unit area of electrode (Bunsen, *Pogg. Ann.*, xci. 619). With small current density the metals are deposited as well-shaped crystals; on increasing the density, reguline metal (similar to the metal when smelted) is obtained, but with great density the deposit is amorphous, botryoidal, or pulverulent. With some metals, the molecular state differs with the solutions from which they are deposited. Thus silver from dilute solution of the nitrate, with great current density, appears as a black powder, becoming grey-white and crystalline when the current ceases (Wied., *Galv.*, Bd. 1. § 336a) but from solution of potassium silver cyanide it is electrolysed as reguline metal. Gold and platinum exhibit a similar behaviour. For a good instance of amorphous deposit, see the account of Gore's explosive antimony in his *Electrometallurgy*, p. 103.

(3.) *The ions very frequently react upon the electrodes* and produce in some cases very interesting chemical actions. If the cation and cathode are both metals, an alloy of the two is the usual if not universal result. This is well known in the case of the electrolysis of many metals and salts with mercury electrodes, and the combination of the hydrogen set free by electrolysis with electrodes of palladium, nickel, and iron may be similarly regarded; and perhaps the compounds derived when ammonium salts are decomposed with a mercury cathode. Copper, when deposited on platinum, alloys with it to a certain extent, the alloy penetrating to a considerable depth (Gore, *Electro-metallurgy*, p. 47). Faraday noticed the combination of tin and lead with platinum electrodes in the electrolysis of the fused salts of those metals.

The action of the anion upon the anode furnished Faraday with an accurate and convenient means of estimating the amount of chemical decomposition produced by a definite quantity of electricity, and thereby of confirming the law given by equation (1) (*Exp. Res.*, 807-822). Thus by varying the anodes, while the cathode remained the same, in the decomposition of acidulated water he found the amount of hydrogen liberated at the cathode, and therefore the chemical decomposition, independent of the nature of the electrodes; and by electrolysing various chlorides, as of silver, tin, lead, with an anode of the same metals respectively, he was enabled to determine very accurately the amount of chlorine separated. We shall have more to say on the bearing of this hereafter. The oxygen liberated by the electrolysis of acidulated water frequently unites with the anode; even if this is of carbon it becomes oxidized to CO and CO₂; this was noticed by Faraday (*Exp. Res.*, 744), and is interesting as showing the active state of the oxygen when separated.

But perhaps the most interesting examples of the action of the ions on the electrodes are furnished by the capillary phenomena exhibited by mercury in contact with dilute acid, on the passage of the current. If we have a drop of water upon a surface of Hg, and the water be connected with the positive, while the Hg is connected with the negative pole of a battery, the water will gather itself up into a spherical drop, and on reversing the current will spread itself over the metal. This phenomenon is supposed by Wiedemann to be due, in the former case, to the reduction of a film of oxide on the surface of the Hg by the liberated H, thereby giving a cleaner surface with a higher capillary constant, and, in the latter, to the oxidation of the surface by the liberated oxygen, and this view is borne out by numerous experiments. Thus a reducing agent, such as crystal of sodium thio-sulphate (Na₂S₂O₃), introduced into the drop of water produces similar contraction of the drop, while an oxidizing agent, as K₂Cr₂O₇, produces on the contrary a similar dispersion. A drop of Hg in dilute sulphuric acid, connected with the positive pole of a battery, while the negative electrode is near it, extends toward that electrode on the passage of the current, becoming covered with a film of suboxide, which then dissolves in the H₂SO₄, and leaves again a bright surface, when the drop returns to its original position, and a series of oscillations are thus set up (see Wied. *Galv.*, i. 368 *seq.*). With solutions of alkaline cyanides containing mercury Gore obtained oscillations producing sounds (*Elec.-Metall.*, p. 197; *Proc. Roy. Soc.*, 1862). It was observed by Erman that a drop of mercury in a horizontal tube, with dilute acid on both sides, moved at the passage of the electric current through the tube towards the negative electrode. These phenomena have been investigated further by Lippmann (*Pogg. Ann.*, cxlix. 547, trans. in *Phil. Mag.* [4] xlvii. 281). One of the forms of his apparatus is as follows. A glass tube A, drawn out to a short capillary point of about $\frac{1}{16}$ mm. radius, contains mercury which penetrates into the fine point and partly fills it, the remainder being filled with dilute H₂SO₄, into which the capillary opening dips; below the electrolyte is a surface of mercury, serving as the positive electrode, sufficiently broad for the capillary effects there to be neglected. The negative electrode is the mercury in the tube A. Lippmann showed by this apparatus that, in order to compensate the change in the capillary constant of the mercury produced by a definite electromotive force of polarization, a definite increase of pressure on the mercury in A is required. As for an electromotive force of polarization equivalent to a Daniell cell the

compensating pressure was 260 mm., and as the quantity of electricity required to polarize the electrodes is very small, this apparatus, when once it has been graduated by observing the compensating pressure for known electromotive forces, may evidently be employed as a sensitive and convenient electrometer for electromotive forces less than the maximum of polarization of the electrodes.

We may mention one other example of the action of the ions *Passiv-* upon the electrodes. An iron wire is usually attacked by dilute HNO₃ (sp. gr. 1.3); but if previously to its being immersed in that liquid it is employed as the anode in the electrolysis of diluted oxygen acids, the nitric acid has no longer any effect upon it, not even tarnishing the surface, and the wire differs from ordinary iron in being strongly electro-negative to it, and indeed to copper, in dilute acids (Martens, *Pogg. Ann.*, lxi. 121). It is then said to be in the *passive state*, and is considered to be covered with a film of oxide which is strongly electro-negative, and insoluble in dilute nitric acid (Faraday, *Phil. Mag.*, ix. p. 60, 1836, x. p. 175, 1837; Beetz, *Pogg. Ann.*, lxii. 234, lxiii. 415). De Regnon, however (*Comptes Rendus*, lxxix. 299), attributes the phenomena to polarization. This peculiar state may be induced by various processes; Keir (*Phil. Trans.*, 1790) observed it when an iron wire was dipped into strong nitric acid (sp. gr. 1.5), by which its surface is not attacked. A more dilute solution has the same effect (Schonbein, *Pogg. Ann.*, xxxviii. 444), if the wire be immersed several times, or if the solution contain chromic or sulphuric and permanganic acids (Boutmy and Chateau, *Cosmos*, xix. 117). Iron when dipped in very strong solution of AgNO₃ does not precipitate the silver, and is electro-negative even to that metal. Another method of rendering iron passive, evidently the same in principle as the one first mentioned, is to touch the iron wire immersed in dilute nitric acid, by carbon, platinum, or other electro-negative element itself in contact with the liquid; and on the contrary, passive iron becomes active if it be touched by a body electro-positive to it, as copper or zinc. If a passive wire be partly immersed in the dilute acid, and an active wire in contact with it be *slowly* introduced into the liquid, the latter becomes *passive* too; but if they touch under the surface, both are rendered active. Iron is rendered passive also by heating in a current of oxygen or an oxidizing flame until it is tarnished. On the other hand, the passive metal becomes active under the influence of any reducing action upon its surface, whether by deposition of H upon it by electrolysis, by heating the metal in a reducing flame, or by abrading the surface. One modification of the electrolytic method is to touch the metal in dilute nitric acid, for a moment, with a copper wire. The point touched becomes immediately active, and therefore electro-positive to the rest, and so currents are set up from active to passive metal through the acid, which accordingly reverse the state of both parts, and a curious series of oscillations result, ending in the whole becoming active. (Schonbein, *l.c.* Compare these with the phenomena of alternation of passive and active states of iron, and of the oxidized and bright surfaces of amalgamated zinc described by Joule, *Phil. Mag.*, 1844, i. 106).

Iron is not the only metal which behaves thus. Nickel, cobalt, tin, bismuth, and even copper, all exhibit similar phenomena in strong HNO₃ and as positive electrodes; and aluminium thus treated is electro-negative even to passive iron (see Wiedemann, *Galv.*, Bd. i. § 539-542).

(4.) *The ions act upon the fluid surrounding the electrodes.* Secondary Actions of this kind in both fused and dissolved electrolytes nearly always occur unless the ions combine with the electrodes; thus per-chlorides, if such exist, are formed from the chlorides, and perchlorates from chlorates at the anode (Kolbe). At the cathode the secondary actions are cases of reduction; thus if solution of potassium iodide be electrolysed, corresponding to 1 equivalent of iodine at the anode, there will appear not only 1 equivalent of H, at the cathode, but an equivalent of KHO as well, so that the potassium liberated from the iodine must have acted upon the water and formed KHO. If ammonium chloride be electrolysed, the chlorine at the anode reacts upon the NH₄Cl, giving free nitrogen and nitrogen-chloride. The electrolysis of ammonium nitrate is still more interesting, as NH₃ and H are separated at the cathode, where the hydrogen reduces the nitric acid of the nitrate, and nitrogen is evolved, while at the anode NO₂ is deposited, which forms with the water nitric acid and oxygen, the latter reacting upon the ammonia of the nitrate, again evolving nitrogen, so that that element appears at both poles,—at one mixed with ammonia, at the other with oxygen (Miller). Some of the reactions investigated by Kolbe and Burgoin with organic salts are very interesting, but more exclusively to the chemist. The oxidizing and reducing actions are very powerful, as the bodies probably act in the "nascent state."

Solutions of acetate and nitrate of lead, when electrolysed by currents of small density, deposit at the positive electrode hydrated peroxide of lead as a black powder. If a polished iron plate be used as the anode, the deposit shows prismatic colours depending on the thickness, and the process has been applied in the arts to colour metallic toys, under the name of metallochromy. If a fine wire as cathode be placed vertically above the anode plate.

the colours are arranged in circles long known as Nobili's rings. Similar phenomena are exhibited by salts of bismuth, nickel, cobalt, and manganese, all of which are precipitated as peroxides, usually hydrated (Wernicke, *Pogg. Ann.*, cxli. 109), upon the anode by the action of the oxygen liberated by the passage of electricity. Silver is also thrown down as a black peroxide, together with some oxygen from a solution of sulphate and nitrate, and iron behaves somewhat similarly in an ammoniacal solution of the protoxide in vacuo.

Such secondary actions vary very conspicuously with the density of the current and the temperature. Bunsen (*Pogg. Ann.*, xci.) electrolysed solution of chromic chloride, and by increasing the current density obtained in succession H , Cr_2O_3 , CrO_3 , and metallic Cr at the cathode; the reason for this is evidently that with high current densities the supply of ions in any time is greater than can take part in secondary action, and hence some of the original ion is deposited. A rise of temperature favours chemical action, and promotes rapid mixture of the ions with the solution at the same time; so the higher the temperature the greater is the current density required to isolate the ions. From concentrated sulphuric acid, for instance, below 80° only H and O are obtained; between 80° and 90° oxygen is given off at the anode, while at the cathode H and S , due to reduction of H_2SO_4 by hydrogen, appear; above 90° sulphur alone is deposited at the cathode (Warburg, *Pogg. Ann.*, cxxxv. 114).

Mixed
electro-
lytes.

Instructive and important cases of secondary action occur when the electric current is made to traverse a mixture of several solutions. Magnus (*Pogg. Ann.*, cii. 23) determined by experiments on dilute $CuSO_4$ solution, in an apparatus with a porous diaphragm of clay, colloid paper, or animal membrane, specially arranged that the lines of flow should be parallel, and the current density therefore uniform, that there was a limiting value of the density above which both copper and hydrogen appeared at the cathode, but below only copper. His results show that this density is independent of length of the electrolyte and material of the electrodes, but varies directly as the size of the electrodes. The specific resistance of the constituents, as well as the relative position of the two ions in the "electro-chemical series" (*vid. inf.*), are of great importance, the electro-negative metal always appearing first.

In order to determine whether the current traversed both electrolytes or only one, Hittorf (*Pogg. Ann.*, ciii. 48), with the apparatus above described (p. 108), electrolysed mixed solutions of potassium chloride and iodide in different proportions, and arrived at the important conclusion that for all densities the current traversed both electrolytes, as it were in multiple arc (though the resistance of the mixture apparently bears no definite relation to the resistances of its constituents except for some of the haloid salts); but the products liberated depend on the secondary action at the electrode, and hence on the current density. The formation of an envelope of liquid of altered composition would also introduce complications (Smee, *Phil. Mag.*, xlv. 437). Buff, by experimenting on solution of HCl , with a small amount of H_2SO_4 , substantially confirms Hittorf's results (*Ann. d. Chem. u. Pharm.*, cv. 166).

These considerations are, of course, especially useful in effecting the deposition of alloys by electrolysis. The possibility of so doing appears to depend upon the composition of the solution employed. An acid solution of Cu and Zn deposits only copper, but the addition of potassium cyanide determines the deposition of brass. Gore (*Electro-metallurgy*, p. 51) points out that, in order to deposit an alloy of two metals, there must be no electric separation when the two metals are in contact with the liquid; if indeed such were the case, a deposit of the two metals, say of Cu and Zn , would immediately act as a $CuZn$ couple (see p. 114), and the electro-negative metal alone would be deposited at the expense of the electro-positive.

Migra-
tion of
the ions.

Although the amount of a salt decomposed by the passage of a given quantity of electricity is the same whether the salt be fused or dissolved in alcohol, water, or other solvent, yet the presence of the solvent produces an important effect upon the electrolyte, which should not be lost sight of in quantitative experiments. The phenomenon is known as the "migration of the ions" (Hittorf), or the "unequal transfer of the ions" (Miller). Suppose, for example, we electrolyse a solution of $CuSO_4$ containing .16 gramme of salt per cubic centimetre, in a vessel separated by a porous diaphragm into two portions A and B. Let electricity be passed through the solution between platinum electrodes from B to A, until 1.59 grammes of $CuSO_4$ have been decomposed. Then—

- (1.) 1.59 g. of $CuSO_4$ has been removed from the solution;
- (2.) .63 g. of Cu has been deposited on the platinum cathode;
- (3.) .16 g. of O has been evolved at the anode, and
.80 g. of SO_4 absorbed there by the water of the solution.

Now, had the electrolyte been a single fused compound, no complication could have arisen; the liquid remaining must still have been homogeneous (except for the presence of the ions near one or other electrode). But when the salt is dissolved, it is important to

consider from what part of the solution the salt has been removed. Suppose that of the $CuSO_4$ decomposed $\frac{1}{n}$ th was taken from the vessel B, and therefore $\frac{n-1}{n}$ ths from A. The result of electrolysis may then be exhibited thus (assuming that no diffusion takes place through the diaphragm):—

	In A.	In B.
Before Electrolysis	x g. $CuSO_4$.	y g. $CuSO_4$.
After "	$\left(x - \frac{n-1}{n} \cdot 1.59\right)$ g. $CuSO_4$ + .63 g. Cu , including Cu deposited.	$\left(y - \frac{1}{n} \cdot 1.59\right)$ g. $CuSO_4$ + .96 g. SO_4 , including oxygen collected.

If the volumes of the two vessels are equal, x and y are of course equal, since the fluid is originally homogeneous.

Hence A will gain $\frac{1}{n}$.63 g. Cu , and lose $\frac{n-1}{n}$.96 g. SO_4 .

B will lose $\frac{1}{n}$.63 g. Cu , and gain $\frac{n-1}{n}$.96 g. SO_4 .

We may therefore state the result thus:—For every equivalent of copper deposited upon the cathode the entire gain of copper in the vessel A is $\frac{1}{n}$ th equivalent, and the entire gain of SO_4 in B is $\frac{n-1}{n}$ equiv. The experiment shows that the entire gain of copper in A is $.276 \times .63$ g., and the gain of SO_4 in B is $.724 \times .96$ g.; and hence, for solutions of $CuSO_4$ of that strength, $\frac{1}{n} = .276$, and consequently $\frac{n-1}{n} = .724$, so that, of the $CuSO_4$ decomposed, 72.4 per

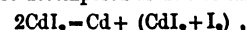
cent. is taken from A, and 27.6 per cent. from B, and the solution round the cathode is weakened much faster than that round the anode. This will be observable by the depth of the blue colour of the solution. If the anode be of copper and be vertically above the cathode, the effect is well seen; for although the total amount of $CuSO_4$ in solution remains constant, the difference of colour at the two electrodes is very apparent, and, if the action be continued, strong dark-blue solution drops down in thin streams from the anode through the more dilute (Magnus).

The value of n differs for different salts, and usually for solutions of the same salt of different strengths, though in some cases, as K_2SO_4 , KNO_3 , $NaCl$, and KCl , the variations for great difference of concentration are very slight. The following table shows a few of the results obtained by Hittorf, with the apparatus described above, by which errors due to diffusion were avoided. The numbers

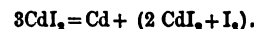
in the third column indicate what is called above $\frac{n-1}{n}$, i.e., the total excess in equivalents of the anion in the vessel containing the anodes corresponding to a decomposition of one equivalent of salt; or, except in the last few cases, that part of the salt decomposed which is taken from the vessel containing the cathode.

Salt.	No. of cc. of solvent containing one gramme of salt.	$\frac{n-1}{n}$
HCl	2.9	.319
HCl	36.2	.168
HCl	140.9	.171
HCl	2125.9	.210
HBr	8.6	.178
HIO_3	18.3	.102
K_2SO_4	11.8	.600
K_2SO_4	412.8	.498
$NaCl$	20.7	.634
Fe_2Cl_6	25.25	.800
CdI_2	4.2	1.14
.....	116.7	.618
CdI_2 in alcohol	1.1	2.102
.....	37.2	1.318
ZnI_2 in alcohol	0.5	2.16

The iodides of zinc and cadmium are anomalous, but it may be supposed that they are decomposed as double salts thus:—



or



The total increase in the amount of an ion in one part of a vessel divided by a porous partition is also affected by a mechanical transference of the electrolyte through the pores of the diaphragm, or endosmosis, generally in the positive direction of the current, which is very noticeable in cases of electrolytes of high resistance. This was discovered by Reuss in 1807, and observed by Forret soon after.

wards; it has been investigated by Wiedemann (*Pogg. Ann.*, lxxxvii. 321), and Quincke (*Pogg. Ann.*, cxiii. 513). The former worked with a porous cell, and estimated the effect either by the quantity of the electrolyte which passed through the wall of the cell, the pressure remaining constant, or by the rise of pressure in the porous cell measured by a mercury manometer. A current of moderate intensity through distilled water caused 17.77 g. of the electrolyte to pass through the diaphragm towards the cathode in a quarter of an hour, and with a 19 per cent. solution of CuSO_4 , a pressure of 176.5 mm. was observed in the cell containing the cathode, due to the current of a battery of Daniell's cells. Quincke, however, employed, instead of a porous cell, a capillary tube without diaphragm, open at one end, and connected with a reservoir at the other containing one electrode, while the other electrode consisted of one of several pieces of platinum wire, sealed into the tube in various positions. His current was obtained from either a Leyden battery or 40 to 80 Grove's cells. The two ways of experimenting gave concordant results, and showed that the pressure on the cathode vessel varies as the electromotive force between the electrodes, and so diminishes with the resistance if the current be kept constant. It is also, in Quincke's apparatus, inversely proportional to the square of the diameter of the tube, and, for tubes of the same sectional area, is greatly increased by increasing the perimeter. The direction of motion is, as stated above, usually towards the cathode, and is immediately reversed on a reversal of the current, and stops when the circuit is broken. The rate of transfer is increased by coating the tube with shellac; it is different for different fluids, and with certain specimens of absolute alcohol, and with turpentine oil, the direction is reversed, unless in the latter case the tube is coated with sulphur, when the direction is as before.

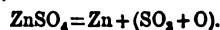
Intimately connected with these phenomena is the motion of solid particles contained in fluids of high resistance. Faraday observed the motion of silk threads in water, and Jürgensen made many experiments on the subject with a capillary tube in the form of three sides of a rectangle with bulbs at the two corners which contained the electrodes; in one was a porous diaphragm as well. Quincke (*l.c.*) used a similar apparatus to this, as well as the one described above, and observed by means of a microscope a double motion of particles of starch contained in water subject to the action of an electric machine. Near the sides of the tube the particles moved towards the negative electrode, but in the middle in the opposite direction; on turning the machine more quickly the particles near the sides gradually lost their velocity, and then began to move towards the positive electrode in common with those in the middle. So that it is highly probable that near the sides the particles are in the first instance carried along by the motion of the fluid there, but on increasing the current the friction of the liquid in contact with the tube prevents its velocity increasing so fast as that of the particles in the opposite direction, and ultimately the motion of the particles in that direction becomes apparent. Similar phenomena are observed with many finely divided bodies suspended in water, as gold, copper, graphite, silica, feldspar, sulphur, lycopodium, &c., as well as minute drops of liquid, as CS_2 , and oil of turpentine, and bubbles of oxygen, marsh gas, &c. All these are urged in water towards the positive electrode, but in oil of turpentine the direction is reversed except in the case of particles of sulphur; the direction is also reversed for silica in carbon disulphide.

Considering now our first equation $W = KE$ established, K being, as stated, dependent only on the nature of the electrolyte, we proceed to examine the constant K and its value for different electrolytes. The primary investigation is due to Faraday, who found that if A and B be two electrolytes, and if a quantity E of electricity decomposes a mass X of A and Y of B, then X and Y are *chemically equivalent*, that is, are the amounts of A and B which would take part in a double decomposition between them. According to this view we have for any electrolyte $W = \mu \epsilon E$, where μ is the amount of the electrolyte chemically equivalent to 1 gramme of water, and ϵ is the number of grammes of water decomposed by a unit of electricity, and is called the *electrochemical equivalent* of water. This appears to be always true, but the law as usually stated refers to the amounts of the ions separated. The most general statement which the facts allow is the following, known as Faraday's law:—*In any electrolytic decomposition whatever, the mass w of one at least (usually of each) of the ions, simple or complex, separated by the passage of a quantity of electricity E, is chemically equivalent to the amount of hydrogen separated by the same quantity of electricity in a water voltameter, and hence $w = m\epsilon E$, where m is the chemi-*

cal equivalent of the ion, and h the electrochemical equivalent of hydrogen.

Since water contains $\frac{1}{8}$ th its weight of hydrogen $h = \frac{1}{8}\epsilon$.

Faraday admitted as electrolytes only bodies containing an equal number of equivalents of their components, and accordingly found that the amount of either ion was equivalent to the hydrogen evolved in a voltameter included in the circuit. The seventh series of *Experimental Researches* was devoted to proving this most important law. Two methods were adopted—(1) by collecting and measuring the products of decomposition, a voltameter being included in the circuit, and (2) by introducing an anode with which the anion could combine (as for instance a Pb anode in fused PbCl_2 , a silver one in fused AgCl), and determining the loss in weight of the anode. By these means the law was proved for simple fused electrolytes, such as the chlorides, &c. Daniell extended it to oxygen salt solutions, and showed that they were decomposed into a metal and a complex ion, this last splitting up into oxygen and an anhydride which united with water to form the corresponding acid, e.g.,



Matteucci and E. Becquerel added a large amount of evidence in defence of the law, which was demonstrated with great accuracy (to $\frac{1}{2}$ per cent.) by Soret (*Ann. de Chim. et de Phys.*, [3] xlii. 257) for a series of copper salts; and by Buff for great variations of current strength with silver compounds.

So long as we confine ourselves to normal salts there is little difficulty about the statement of the relation; even with such compounds as the series of phosphates, the double cyanides, &c., which are decomposed as in the following table, the amount of either ion may be considered equivalent to the H of the voltameter.

Electrolyte.	Anion corresponding to H in voltameter.	Cation.	Observer.
Na_3PO_4	$\frac{\text{P}_2\text{O}_5}{3} + \frac{\text{O}}{2}$	Na.	Hittorf.
NaPO_3	$\frac{\text{P}_2\text{O}_5}{2} + \frac{\text{O}}{2}$	Na.	
$\text{Na}_4\text{P}_2\text{O}_7$	$\frac{\text{P}_2\text{O}_5}{4} + \frac{\text{O}}{2}$	Na.	
Na_2HPO_4	$\left(\frac{\text{P}_2\text{O}_5}{4} + \frac{\text{H}_2\text{O}}{4}\right) + \frac{\text{O}}{2}$	Na.	Daniell and Miller, <i>Phil. Trans.</i> , 1844, p. 1.
$\text{NaH}_2\text{NH}_4\text{PO}_4$...	$\frac{\text{H}_2(\text{NH}_4)_2\text{P}_2\text{O}_7}{2} + \frac{\text{O}}{2}$	Na.	
K_4FeC_7	$\frac{\text{FeC}_7}{4} + \text{Cy.}$	K.	
KAgC_7	$\frac{\text{AgC}_7 + \text{Cy.}}{4}$	K.	
$\text{K}_2\text{Al}_2(\text{SO}_4)_3$	$\frac{1}{2}(\text{Al}_2\text{SO}_4) + \frac{\text{SO}_3}{2} + \frac{\text{O}}{2}$	K.	Hittorf.

Faraday's law is nearly always true for both ions, but there are, as Becquerel's before stated, examples of elements forming two series of electrolytically-modifiable salts, especially when dissolved in water. In these cases the *electronegative ion* is usually equivalent to the H of the voltameter, or we may consider that the chemical equivalent of the positive ion varies, while that of the negative ion remains unchanged in the different combinations; so that ferric chloride may be regarded as a dichloride with formula FeCl_2 , where $\text{Fe} = \frac{1}{2}\text{Fe}$; cuprous chloride CuCl , where $\text{Cu} = \frac{1}{2}\text{Cu}$, and so on. Considerable confusion, too, arose from the arbitrary numbers for chemical equivalents which formerly obtained, and which caused such compounds as Al_2Cl_6 , SbCl_3 , AuCl_3 , to appear anomalous, and warranted E. Becquerel (*Ann. de Chim. et de Phys.*, [3] t. xi. p. 178) in considering that generally the amount of *electro-negative ion* alone was equivalent to the H of the voltameter.¹ This was borne out by his electrolysis of $2\text{N}_2\text{O}_5$, 7PbO , $3\text{H}_2\text{O}$, and N_2O_4 , 2PbO , H_2O , which gave $\frac{1}{2}$ and $\frac{1}{2}$ an equivalent of Pb at the cathode respectively; but the law as thus modified fails in the case of $\text{K}_2\text{Cr}_2\text{O}_7$, which gives $\text{K} + (\text{CrO}_3 + \frac{1}{2}\text{O})$ both in the melted and dissolved state, and in that of Na_2S_8 , which gives $\text{Na} + (\text{S}_8 + \frac{1}{8}\text{S})$, and also for basic acetate of lead.

¹ This oxygen is set free.

² The well-known deposit of silver in electro-plating is due to secondary action of the K.

³ The chemical equivalents of Al, Sb, Au were taken to be 18.5, 61.98 respectively, instead of 9.1, 40.6, 65.5 as now.

Electro-
lysis of
solutions
in con-
tact.

Faraday's law receives striking confirmation from the electrolysis of several solutions arranged in series in contact with each other by means either of porous septa, asbestos wicks, or siphon tubes. Each liquid then acts as an electrode to the adjacent ones, and so at the junction we have separated the anion of one electrolyte and the cation of the next. These in general unite, and if the resulting compound be insoluble, a precipitate is thrown down. Faraday thus precipitated magnesia from its sulphate by electrolysis a solution of that salt in contact with water, the current passing from the salt solution to the water. Now, in all cases in which the ions unite at the junction, and do not appear free at all, the amount of the cation of one liquid must be chemically equivalent to that of the anion of the succeeding one, and hence obey Faraday's law. Many of the decompositions and combinations thus effected are very interesting, a list showing in a tabulated form the results of experiments by Hisinger and Berzelius, Davy, Daniell, Miller, and others will be found in Wiedemann (*Galv.*, Bd. i. § 368). We can only mention one example which is of theoretical importance. If the positive electrode be in solution of iodic acid which is in contact with dilute sulphuric acid containing the cathode, then at the surface of separation there will be formed I and SO₄, or H and SO₄, according as the I observed at the negative electrode in the electrolysis of HIO₃ solution is an ion or due to secondary action. By the union of the two ions at the junction the latter is shown to be the case; therefore iodic acid is electrolysed as H₂ + (I₂O₅ + O).

Electro-
chemical
equiva-
lents.

We gather at once from the truth of Faraday's law that we can assign to each ion an electrochemical equivalent (which may be referred to as E.C.E.), which will enable us to determine at once the amount of the ion which will be separated by a given quantity of electricity. With the notation already used the E.C.E. of an ion = $\frac{1}{m}$ me. The value of ϵ —the amount of water decomposed by one C.G.S. electromotive unit of electricity—from experiments of Weber, Joule, Bunsen, Casselmann, and Kohlrausch is .00093 gramme (Wied. *Galv.*, Bd. iii. § 1077-1079). The quantity m is one of the chemical equivalents of the ion, usually that deduced from its most stable salts; some metals, indeed, with two series of salts have two E.C.E.s. The following table of the elements gives the values of m and the E.C.E.s in absolute units, as far as they have been experimentally determined. Since m bears a simple ratio to the atomic weight, its value can be corrected by the results of chemical analysis.

Table of Electrochemical Equivalents.

Element.	Atomic Weight.	Electrochemical Equivalent in Hydrogen units (m).	E.C.E. in grammes per C.G.S. unit of Elec. = $\frac{1}{m}$ me.	Element.	Atomic Weight.	E.C.E. in Hydrogen units (m).	E.C.E. in absolute units = $\frac{1}{m}$ me.
Al ...	27.5	9.1(2)	.00094	Mo ...	96	(2)	...
Sb ...	123	40.6(2)	.00420	Ni ...	59	29.5(1)	.00305
As ...	75	25(2)	.00258	Nb ...	97.5	(2)	...
Ba ...	137	68.5	.00708	N ...	14	(2)	...
Bi ...	210	71(1)(2)	.00733	Os ...	199	(2)	...
B ...	10.9	(2)	...	O ...	16	8(1)	.00083
Br ...	80	80(1)	.00826	Pd ...	106	(2)	...
Cd ...	112	56(1)	.00578	P ...	31	(2)	...
Cs ...	133	133(4)	.01374	Pt ...	197	98.5(1)	.01018
Ca ...	40	20	.00206	K ...	39.1	39.1(1)	.00404
C ...	12	(2)	...	Rb ...	104.3	(2)	...
Cl ...	35.5	35.5(1)	.00366	Ra ...	85	85(4)	.00878
Co ...	59	29.5(1)	.00305	Se ...	79.5	(2)	...
Cu ...	63	31.6(1)	.00326	Si ...	28	(2)	...
D ...	96	(2)	...	Ag ...	108	108(1)	.01116
F ...	19	19(1)	.00196	Na ...	23	23(1)	.00237
G ...	9.3	(2)	...	Sr ...	87.5	43.7	.00452
Au ...	196.6	65.5(2)	.00677	S ...	32	16(2)	.00165
H ...	1	1	.000103	Ta ...	138	(2)	...
In ...	113.4	(2)	...	Te ...	129	64.5(2)	.00666
I ...	127	127(1)	.01312	Tl ...	204	204(2)	.02108
Ir ...	197	(2)	...	Th ...	119	(2)	...
Fe ...	56	28(1)(2)	.00289	Sn ...	118	29.5(2)	.00305
La ...	92	(2)	...	Ti ...	50	50(1)(2)	.00610
Pb ...	207	103.5(1)	.01069	W ...	184	(2)	...
Li ...	7	7	.00072	U ...	120	(2)	...
Mg ...	24.3	12(1)(2)	.00124	V ...	137	(2)	...
Mn ...	55	27.5(1)	.00284	Zn ...	65	32.5(1)(2)	.00336
Hg ...	200	200(1)(2)	.02066	Zr ...	89.5	(2)	...
		100(2)	.01033				

Every complex ion has also a definite electrochemical equivalent, usually coinciding with its chemical equivalent. The E.C.E. of an electrolyte is the sum of the E.C.E.s of its component ions.

¹ Faraday, *Exp. Res.*, ser. vii.

² Renault (*l.c.* § 478).

³ Either these elements have not been obtained as ions by electrolytic action, or quantitative experiments are wanting.

⁴ Bunsen.

Renault² determined the E.C.E.s by an inverse method. He observed the amount of the metal which, forming the negative pole of a battery with various electrolytes, gave a current equivalent to that produced by the dissolution of a definite amount of zinc in a ZnPt cell, the two currents passing through a differential galvanometer, and thus compared the amounts of elements which generate the same quantity of electricity in combining. It is perhaps necessary to observe that the electrolytic reactions taking place in a galvanic cell which generates a current are in every way identical with those due to a current from an external source sent through the electrolyte. In the former case, the energy of chemical affinity at the electrodes is transformed into the energy of electrical separation, and in the latter the converse is the case.

The Electrochemical Series.

It is evident from all the examples we have given that it is Elect not an accident whether an ion will appear at the anode or cathode; the cations have been all more or less similar in character, and series were either metals or more allied to the metals than the corresponding anions, which were bodies like Cl, Br, I, CN, O, &c. Faraday (*Exp. Res.*, 847) was accordingly led to consider that an element or radicle was unalterably either an anion or a cation; this, however, was contradicted by the fact that the same element may act as an anion in one solution and a cation in another, as is the case with iodine, which in KI is an anion, but from a solution of iodine bromide (IBr) appears at the cathode. The electrolysis of alloys³ points in the same direction, so that the conclusion is suggested to us that "anion" and "cation" have only relative meanings, and that we might arrange the elements in a series such that, in a compound of an element A with any one of those above it, A would appear as a cation, but in a compound with any of those below, as an anion. To do this by purely electrolytic means is out of the question, as binary electrolytes do not exist for each pair of elements. As far, however, as the series can be thus made out, it is found that, as a rule, if two elements A and B, such that A is above B in the series, be immersed in a simple electrolyte, as dilute H₂SO₄, and connected by means of a wire, the current flows from B to A through the liquid. Hence in unknown cases we may observe the direction of the current when the two elements are immersed in an electrolyte, say H₂SO₄, and determine the relative position in the series.⁴ With the series thus roughly formed, it is observed that the wider two elements are apart the greater is the chemical affinity between the two, and thus that if we have a compound MR, where M is the electro-positive element, a more electro-positive element M' having a greater affinity for R than M tends to replace M from the compound, and a more electro-negative element R' tends to replace R as iron replaces copper from CuSO₄, and chlorine iodine from KI. This further assists us in forming an electrochemical series of the elements, but it is still not very strictly arranged, and many of the members of the series are placed by their analogy to elements whose positions are known. Moreover, it is supposed that the relative position of two elements may vary with the temperature. Thus carbon which is used in batteries as the negative element, is at a full red heat electro-positive even to potassium, or at least reduces the carbonate of that element. Jablockhoff (*Comptes Rendus*, Dec. 3, 1877) describes a cell of which the positive element is coke. The electrolyte is fused sodium or potassium nitrate, and the negative element is a cast-iron vessel containing the fused salt. The current is from coke to cast-iron through the nitrate, and the electromotive force 2 to 3 volts.

Berzelius's final series stands thus:—

Electro-negative.

Oxygen.	Boron.	Mercury.	Thorium.
Sulphur.	Carbon.	Silver.	Zirconium.
Selenium.	Antimony.	Copper.	Aluminium.
Nitrogen.	Tellurium.	Bismuth.	Didymium.
Fluorine.	Tantalum.	Tin.	Lanthanum.
Chlorine.	Titanium.	Lead.	Yttrium.
Bromine.	Silicon.	Cadmium.	Glucinum.
Iodine.	Hydrogen.	Cobalt.	Magnesium.
Phosphorus.	Gold.	Nickel.	Calcium.
Arsenic.	Osmium.	Iron.	Strontium.
Chromium.	Indium.	Zinc.	Barium.
Vanadium.	Platinum.	Manganese.	Lithium.
Molybdenum.	Rhodium.	Uranium.	Sodium.
Tungsten.	Palladium.	Cerium.	Potassium.

Electro-positive.

⁵ "Vérification expérimentale de la réciproque de la loi de Faraday, sur la décomposition des électrolytes." Paris, 1867; *Ann. de Chim.* [4] xi. 137.

⁶ Alloys of tin and lead, potassium and sodium, sodium amalgam, gold amalgam, and fused cast-iron have all been shown to suffer chemical decomposition on the passage of the electric current (Wied. *Galv.*, i. § 528).

⁷ This is not always conclusive evidence, as the direction of the current for the same two elements sometimes varies with the electrolyte employed, as will be seen by referring to the list of chemico-electric series in Gore, *Electro-metalurgy*, p. 66. The boracic acid series is peculiarly anomalous.

no matter how small it is, it causes the atoms, when liberated as usual, to tend in one direction, viz., along the lines of force. Hence the collection of the ions at the electrodes, where they will separate if the electromotive force be sufficient to prevent them reacting and again recombining,—in other words, sufficient to bear the polarization. This, though by no means a complete theory, is indeed applicable to ultimate atoms, and is the only one which admits decomposition for all electromotive forces. Clausius shows that the finite electromotive force is necessary to maintain the ions in the free state at the electrodes.

Quincke's theory.

One theory, which we must mention because it accounts at once for conduction, the migration of the ions, and "electric endosmose," is that due to Quincke (*Pogg. Ann.*, cxiii., extended in cxliv.), who considers the ions of each molecule charged with quantities of electricity ϵ and ϵ' ; then the force K tending to separate the ions from each other $= -\frac{dv}{dx}(B\epsilon - B'\epsilon')$, where B and B' are constants, and $\frac{dv}{dx}$ is the electromotive force per unit of length of the electrolyte, and is consequently $= -\frac{i}{qk}$, where i is the current intensity, q the sectional area of the electrolyte, and k its specific conductivity; so that $K = -\frac{i}{qk}(B\epsilon - B'\epsilon')$, and electrolysis takes place when this is greater than the force of chemical affinity. This is a weak point of the theory, as a finite electromotive force would be required to produce any decomposition or polarization.

The forces on the ions when separated, and hence their respective velocities, will be proportional to ϵ and ϵ' . This will account for the migration of the ions, for which ϵ and ϵ' are supposed unequal and of different signs in all cases except ZnI and CdI , &c., for which $(1 - \frac{1}{n})$ is greater than unity; for these ϵ and ϵ' may be of the same sign. If, on the other hand, ϵ be the amount of free electricity on a molecule of the electrolyte (supposed of high resistance) in contact with the glass, then $-B\frac{dv}{dx}\epsilon$ will represent the force urging the fluid in the positive direction of the current, and perhaps producing endosmose, since ϵ will be positive except for turpentine oil. So the motion of particles may be similarly explained by supposing ϵ to be the charge on them due to contact with the fluid; this is negative with particles in water, and positive for all particles except sulphur in turpentine oil. The results thus obtained will be found to agree closely with the experiments mentioned above (p. 111); and the quantitative results also agree, since the force on a particle equals $B\frac{i}{qk}\epsilon$, and therefore varies as the current density i , and inversely as the conductivity k .

Zinc copper couple.

An application of electrolysis, which has already proved to be of great value in chemistry, has been introduced of late years by Gladstone and Tribe. In a paper read before the British Association in 1872 (*Trans. of Sections*, p. 75, see *Proc. Roy. Soc.*, vol. xx. p. 218) they showed that although zinc alone does not decompose distilled water, yet if zinc foil be immersed in dilute solution of cupric sulphate, and be thereby coated with metallic copper, which is thrown down as a black crystalline powder, containing traces of zinc only if the time of immersion be very long (*Journal Chem. Soc.*, 1873, p. 452), and if the zinc copper couple thus produced be immersed in distilled water at ordinary temperature, about 4 cc. of H can be collected per hour. The hydrogen is seen by the microscope to collect upon the copper crystals, while the zinc is oxidized, and forms a hydrate. The rate of evolution of hydrogen varies with the temperature; the relation may be exhibited by a curve very similar to the curve of tension of water vapour. Gladstone and Tribe have found this a powerful method of acting upon many organic bodies, particularly the halogen compounds of the alcohol radicles. In all cases either new reactions were set up, or the temperature at which reaction takes place was very much lower than with ordinary zinc (see the series of papers by Gladstone and Tribe in the *Jour. Chem. Soc.*, 1873-6). To the chemist the $ZnCu$ couple affords an exceedingly convenient way of arranging electrolysis, since the whole may be contained in one vessel. For the copper in the arrangement, gold or platinum may with great advantage be substituted by immersing zinc foil in solutions of the chlorides.

This easily explains the well-known custom of generating hydrogen from zinc and sulphuric acid, to which a little $CuSO_4$ is added; and the "local action" in batteries, when currents pass from one part to the other of the same mass of metal and consequently energy is expended for which no external equivalent is obtained, may be similarly referred to the difference of composition of the metals in the two places. It should be remembered that Davy suggested the preservation of the copper sheathing of ships by attaching plates of Zn ; the same object is now achieved by using an alloy of the two metals.

The application of the principle of the conservation of energy to electrolysis has already produced valuable results; research, how-

ever, in this direction is rendered difficult on account of the great number of circumstances which have to be taken into account, in computing the balance of energy expended and work done; the chemical composition and physical state of the electrolyte, the molecular condition of the ions, and the secondary actions at the electrodes have all to be taken into account. For a notice of the present state of this branch of the subject the reader is referred to the article ELECTRICITY.

(W. N. S.)

ELECTRO-METALLURGY, a term introduced by the late Mr Alfred Smee to include all processes in which electricity is applied to the working of metals. It is far more appropriate than the French equivalent *galvanoplastie*, or the German *Galvanoplastik*, since the metals are certainly not rendered plastic under galvanic action, though it is true that in electrolysis, which forms one branch of electro-metallurgy, the metal is deposited in moulds, and can thus be used to reproduce works of plastic art.

It was observed as far back as the beginning of the present century that certain metals could be "revived" from solutions of their salts on the passage of a current of electricity. The germ of the art of electro-metallurgy may undoubtedly be traced to the early experiments of Wollaston, Cruickshank, Brugnatelli, and Davy; but it remained undeveloped until the late Professor Daniell devised that particular form of battery which bears his name, and which he described in the *Philosophical Transactions* for 1836. A Daniell's cell consists, in its usual form, of a copper vessel containing a saturated solution of blue vitriol or sulphate of copper, in which is placed a porous cylinder containing dilute sulphuric acid; a rod of amalgamated zinc is immersed in the acid, and on the two metals being connected by means of a conductor, electrical action is immediately set up. The zinc, which forms the positive or generating element, is dissolved, with formation of sulphate of zinc; whilst the blue vitriol is reduced, and its copper deposited, in metallic form, upon the surface of the copper containing vessel, which forms the negative or conducting element of the combination. Any one using this form of battery can hardly fail to observe that the copper which is thus deposited takes the exact shape of the surface on which it is thrown down, and indeed presents a faithful counterpart of even the slightest scratch or indentation. Mr De la Rue incidentally called attention to this fact in a paper published in the *Philosophical Magazine* in 1836, but it does not appear that any practical application was at the time suggested by this observation. Indeed, the earliest notice of electro-metallurgy as an art came from abroad two or three years later.

Sturgeon's *Annals of Electricity* for March 1839 contained a letter from Mr Guggsworth, announcing that Professor Jacobi, of St Petersburg, had recently discovered a means of producing copies of engraved copper-plates by the agency of electricity. This was the first news of the new art which appeared in England, and it evidently referred to the paper which Jacobi communicated to the St Petersburg Academy of Sciences on October 5, 1838, and in which he explained his process. In the *Athenæum* of May 4, 1839, there was a short paragraph relating to Jacobi's discovery, and public attention in this country was thus drawn to the subject. Only four days after the appearance of this paragraph, Mr Thomas Spencer, of Liverpool, gave notice to the local Polytechnic Society that he would read a paper on a similar discovery of his own. This paper was not read, however, until September 13; and although the author wished to describe his process before the British Association at Birmingham in August, it appears that his communication was never brought before the meeting. In Mr Spencer's paper, which was eventually published by the Liverpool Polytechnic, he states that his attention was first directed to the subject by mere accident: he had used a copper coin, instead of a plain piece of copper, in a

modification of Daniell's cell, and on removing the deposited metal he was struck with the faithful copy of the coin which it presented, though of course the copy was in intaglio instead of relief. Yet even this observation was allowed to remain unproductive until another accident called his attention to it afresh. Some varnish having been spilt upon the copper element of a Daniell's cell, it was found that no copper was thrown down upon the surface thus protected by a non-conducting medium; hence it was obvious that the experimentalist had it in his power to direct the deposition of the metal as he pleased; and this led Mr Spencer to prosecute a series of experiments by which he was at length able to obtain exact copies of medals, engraved copper plates, and similar objects. It should be mentioned that between the date on which he announced his paper and the date on which it was actually read, Mr C. J. Jordan, a printer, described experiments which he had made in the preceding year very similar to those of Spencer. This announcement was made in a letter published in the *London Mechanics' Magazine* for June 8, 1839. It thus appears that three experimentalists were close upon the same track about the same time, but it is generally admitted that among these competitors Mr Spencer has the merit of having been the earliest to bring his process to perfection, and to demonstrate its practical value.

Soon after the appearance of Mr Spencer's paper, it became a fashionable amusement to copy coins, seals, and medals by the new process. These copies in metal are termed *electrotypes*. The apparatus employed in the early days of the art, and which may still be conveniently used for small electrotypes, is similar in principle to a single Daniell's cell. It usually consists of a glazed earthenware jar containing a solution of sulphate of copper, which is kept saturated by having crystals of the salt lodged on a perforated shelf, so that they dip just below the surface of the solution. A smaller porous cylinder, containing very dilute sulphuric acid, in which a rod of amalgamated zinc is placed, stands in the jar, and is therefore surrounded by the solution of sulphate of copper. The object to be copied is attached by a copper wire to the zinc, and is immersed in the cupric solution. It thus forms the negative element of a galvanic couple, and a current of electricity passes from the zinc through the two liquids and the intervening porous partition to the object, and thence back to the zinc through the wire, thus completing the circuit. During this action, the zinc dissolves, and sulphate of zinc is formed; at the same time the copper solution is decomposed, and its copper deposited upon the metallic surface of the object to be coated,—the solution thus becoming weaker as it loses its copper, but having its strength renewed by consumption of fresh crystals of blue vitriol. To avoid the complete incrustation of the metal or other object, one side of it is coated with varnish or some other protective medium, so that the deposition of copper takes place only on such parts as are exposed. The deposit may be easily removed when sufficiently thick, and will be found to present an exact counterpart of the original, every raised line being represented by a corresponding depression. To obtain a facsimile of the original it is therefore necessary to treat this matrix in the same way that the original was treated, and this second deposit will of course present the natural relief. Another method consists in taking a mould of the original coin in fusible metal, and then depositing copper upon this die, so as to obtain at once a direct copy of the original.

Considerable extension was given to the process by a discovery, apparently trivial, which was first announced by Mr Murray at a meeting of the Royal Institution in January 1840. He found that an electro-deposit of metal

could be formed upon almost any material if its surface was rendered a conductor of electricity by a thin coating of graphite or "black-lead." Instead, therefore, of copying a coin in fusible metal, or indeed in any metallic medium, it is simply necessary to take a cast in plaster-of-Paris, wax, gutta-percha, or other convenient material, and then to coat the surface with finely-powdered black-lead, applied with a camel-hair pencil. Medals in high relief, with much undercutting, or busts and statuettes, may be copied in electrotype by first taking moulds in a mixture of glue and treacle, which forms an elastic composition capable of stretching sufficiently to permit of removal from the object, but afterwards regaining its original shape.

About the same time that Murray suggested the use of black-lead, Mr Mason made a great step in the art by introducing the use of a separate battery. Daniell's cell, in consequence of its regular and constant action, is the favourite form of electric generator. The copper cylinder of this arrangement is connected with a plate of copper placed in a trough containing a solution of sulphate of copper, to which a small quantity of free sulphuric acid is commonly added; whilst the zinc rod of the cell is connected with the objects on which the copper is to be deposited, and which are also suspended in the bath of cupric solution. The current enters the bath at the surface of the copper plate, which is the *anode* or positive pole of the combination, and passes through the solution to the suspended medals which constitute the *cathode* or negative pole. As fast as the copper is thrown down upon these objects, and the solution is therefore impoverished, a fresh supply is obtained by solution of the copper plate; this copper is consequently dissolved just as quickly as the electrotypes are produced, and no supply of crystals is needed, as in the case of the Daniell cell. The great advantage of using a separate battery is that several objects may be coated at the same time, since it is only necessary to attach them to a metal rod in connection with the battery. Almost any form of galvanic arrangement may be employed by the metallurgist as a generator of electricity. But as the exciting liquid in a battery needs to be replenished from time to time, and as the zinc plates also wear out, its use is attended with more or less inconvenience in the workshop, and the electro-metallurgist has therefore turned his attention to other sources of electricity. Indeed, as far back as 1842, when the art was but in its infancy, a patent was taken out by Mr J. S. Woolwich for the use of a magneto-electrical apparatus; and of late years powerful machines in which electricity is excited by means of magnetism have been introduced into electro-metallurgical establishments. When a bar of soft iron, surrounded by a coil of insulated copper wire, is rotated between the poles of a magnet, a current of electricity is induced in the coil at every magnetization and demagnetization of the core. By means of a commutator, these alternating currents in opposite directions may be converted into a constant stream of electricity, available for the deposition of metals by electrolysis. The armatures are rotated by mechanical means, such as the use of a steam-engine, and hence the electricity is ultimately produced by conversion of mechanical work.

In the machine constructed by Mr Wilde, which has been largely employed by electro-metallurgists, a small magneto-electric apparatus, with permanent magnet, is employed to excite the electromagnet of a much larger machine. The induced current of the second machine is stronger than that of the first in proportion as the electromagnet is more powerful than the permanent magnet; this second current may then be used to excite another electromagnet, and hence by means of this principle of accumulation, currents of great energy may be obtained. The

armatures in these machines are constructed on Siemens's principle, and consist of long bars of iron magnetized transversely, and having the wire wound longitudinally. During the rotation of the armature, so much heat is developed that special means are taken to prevent its accumulation. In another form of Wilde's machine, a vertical disk carrying a number of coils, each with its own core, is caused to rotate between two rings of magnets. A powerful machine, with multiple armatures of this kind, is used by Messrs Elkington at Birmingham, and is capable of depositing $4\frac{1}{2}$ cwt. of copper every 24 hours.

Another recent modification of the magneto-electric machine used by electro-metallurgists is that invented by M. Gramme. A ring of soft iron carrying a large number of coils of insulated copper wire is caused to rotate between the poles of a fixed horse-shoe magnet, and the currents induced in the coils are collected by two metallic disks, whence they may be drawn off for use in electro-deposition. As the core is circular, the magnetization proceeds continuously, and hence the current is uniform; but as both poles of the magnet are used, two opposite continuous currents are simultaneously produced.

Thermo-electricity is another source of electromotive power of which the practical worker has availed himself. In 1843 a patent was taken out by Moses Poole for the use of a thermo-electric pile in place of a voltaic battery, but it is only within the last few years that such a source of electricity has been introduced into the workshop. The best-known form of thermopile is that devised by M. Clamond of Paris. One element is formed of tinned sheet-iron, and the other of an alloy composed of two parts of zinc to one of antimony. A large number of these pairs, insulated from each other, are arranged in circular piles around a central cavity, in which their junctions are heated by means of a Bunsen burner. The ease with which such an apparatus can be manipulated recommends this source of electricity to the electro-metallurgist.

Having procured a supply of electricity from one or other of these sources, the electro-metallurgist applies it either to the deposition of a metal upon a matrix or to the coating of one metal by another. Hence the art of electro-metallurgy divides itself into two branches, one being called *electrotypy*, and the other being generally known as *electro-plating*. In an electrotype the reduced metal is separated from the mould on which it is deposited, and forms a distinct work of art; whilst in electro-plating the deposited metal forms an inseparable part of the plated object.

It has already been explained how electrotypes are generally taken. One of the most important branches of this art is that of producing copper duplicates of engravings on wood. A cast of the block is first taken in wax or in gutta-percha, and when cold the surface of this mould is brushed over with black-lead; by means of a wire, the black-leaded mould is suspended in a bath of sulphate of copper connected with a battery, and in the course of a few hours a sufficiently thick plate of copper is deposited. The copy, on removal from the mould, is strengthened by being backed with type-metal; it is then planed smooth at the back, and mounted for use on a wooden block. This process is now carried out on a large scale, since it is found that a greater number of sharp impressions can be obtained from the electro than from the wood. For rotary printing machines the electrotypes are curved. Set-up type is also sometimes copied thus instead of being stereotyped, the electro-deposited copper being harder than the stereo metal.

Copper is sometimes thrown down as a thin coating upon plaster busts and statuettes, thus giving them the appearance of solid metal. In Paris, too, it is now common to give a thin coat of electro-deposited copper to exposed iron-work, such as gas-lamps, railings, and fountains. The iron is

first painted, then black-leaded, afterwards electro-coppered, and finally bronzed. Cast-iron cylinders used in calico-printing are also coated with copper by a single-cell arrangement; and it has been suggested to coat iron ships in a similar manner. Usually, however, the electro-plater has to cover the baser metals with either silver or gold.

Electro-plating was introduced very soon after the discovery of the art of electro-metallurgy, the earliest investigators being Messrs G. R. and H. Elkington, Mr Alexander Parkes, and Mr John Wright in this country, and M. de Ruolz in France. It was Mr Wright who first employed a solution of cyanide of silver in cyanide of potassium, and this is the solution still in common use. It should be borne in mind that the cyanide of potassium is a very dangerous poison. The objects to be silver-plated are usually made of German silver, which is an alloy of copper, zinc, and nickel. Before being placed in the depositing vat, the articles must be thoroughly cleansed. Grease is removed by a hot solution of caustic potash, and mechanical cleaning is commonly effected by means of a bundle of fine brass wires, known as a "scratch-brush;" the brush is mounted on a lathe, so as to revolve rapidly, and is kept moist with stale beer. Articles of copper, brass, and German silver are usually prepared by being dipped in different kinds of "pickle," or baths of nitric and other acids. To insure perfect adhesion of the coating of silver, it is usual to deposit a thin film of quicksilver on the surface, an operation which is called "quicking." The quickening liquid may be a solution of either nitrate or cyanide of mercury. After being quickened, the articles are rinsed with water, and then transferred to the silver-bath, where they remain until the deposit is sufficiently thick. The quantity of silver must depend upon the quality of the article: one ounce of silver per square foot forms an excellent coating, but some electro-plated household goods are turned out so cheap that they must carry but the merest film of silver. The vats in which the electro-plating goes on were formerly made of wood, but are now usually of wrought iron. Plates of silver are suspended from a rectangular frame connected with the positive pole, whilst the articles to be plated are suspended by wires from a similar smaller frame communicating with the negative pole. Large articles are suspended from wires, looped at the end, and protected in tubes of glass or india-rubber, whilst small articles may be placed in wire cages or in perforated stoneware bowls. On removal from the depositing vat, the plated objects are usually dipped in hot water, then scratch-brushed with beer, again washed with hot water, and finally dried in hot sawdust. A bright silver surface, requiring no further treatment when removed, may be obtained by adding to the silver bath a very small proportion of bisulphide of carbon.

Electro-gilding is effected in much the same way as electro-silvering. It is found, however, that magneto-electricity cannot be employed with advantage. Various gilding solutions are in use, but preference is usually given to the double cyanide of gold and potassium, originally introduced by Messrs Elkington. The solution is generally used hot, its temperature ranging from 130° Fahr. to the boiling-point. If the object to be gilt is not of copper, it is usual to coat it with an electro-deposit of copper before submitting it to the gilding solution. The coating of gold is generally very thin, and only a few minutes' exposure to the hot solution is necessary to effect its deposition. When the solution is fresh, a copper anode may be employed, its place being taken by a small gold electrode after the solution has been in work for some time. The presence of copper in the solution imparts a full reddish colour to the electro-deposit of gold; and the tone of the metal may also be modified by the presence of salts of various other

metals, such as those of silver. Sometimes only part of an object is to be gilt, such as the inside of a silver-plated cream-jug; in this case the vessel would be filled with the gilding solution, in which the anode of the battery is immersed. Gold is sometimes deposited not as a coating upon other metals, but as an electrotype in gutta-percha or in plaster moulds; small objects of elaborate workmanship being thus produced in solid gold, without the workmanship of the chaser and engraver.

Although copper, silver, and gold are the metals to which the attention of the electro-metallurgist is usually restricted, it should be remembered that he is also able to obtain electro-deposits of a very large number of other metals. Many of these are not practically used, but one of them has of late years become of considerable importance. This is the metal *nickel*. In 1869 Dr Isaac Adams of Boston, United States, patented a process for depositing nickel from solutions of various double salts; but Dr Gore had many years previously employed similar salts in England, and had published the results of his experiments. The deposition of nickel, especially from the sulphate of nickel and ammonium, is now carried out on a large scale both in England and in the United States. The metal is deposited as a very thin but excessively hard coating, and has the advantage of not readily tarnishing or corroding even in a moist atmosphere. Hence it has become common to electro-nickel iron and steel objects for use on board ship, as well as gun-barrels, sword-scabbards, harness furniture, gas-burners, and various articles for household use.

Iron, like nickel, may be deposited from its double salts, and excellent results have been obtained by Klein, of St Petersburg, with the double sulphate of iron and ammonium. Engraved copper-plates are much harder when faced with electro-deposited iron than when unprotected, and they consequently yield a much larger number of impressions before losing their sharpness. Plates for printing bank-notes have been treated in this way.

Not only can the electro-metallurgist deposit simple metals, such as those noticed above, but he is able likewise to deposit certain *alloys*, such as brass, bronze, and German silver. The processes by which this can be effected are not, however, very generally used.

Among the minor applications of electro-metallurgy we may mention the process of electrotyping flowers, insects, and other delicate natural objects. These are first dipped for a moment in a warm solution of nitrate of silver in alcohol, and then exposed to a reducing liquid, such as a solution of phosphorus in bisulphide of carbon; an electro-deposit may then be thrown down upon this metallized surface. Daguerreotypes are sometimes improved by coating them with a very delicate film of electro-deposited gold. Again, in some of the modern photographic processes for printing, copper electrotypes are taken directly or indirectly from the bichromatized gelatine. Of late years, too, a method of refining crude copper by means of electro-metallurgy has been introduced, and is now successfully carried out on a large scale. Slabs of blister-copper are plunged into a solution of sulphate of copper, and form the anodes of a battery; the copper then dissolves, and is deposited in a condition of great purity at the opposite pole, most of the impurities sinking to the bottom of the depositing vat. The process should be restricted to copper which is free from any metals likely to be deposited along with the metal under purification.

It has been considered desirable not to include within the limits of this article any of the numerous formulæ for preparing the solutions used by electro-metallurgists. For these, and for other details, see the treatises of G. Gore (1877), J. Napier (5th ed., 1876), A. Watt (5th ed., 1874), A. Smee (3rd ed., 1851), and G. Shaw (1844); C. V. Walker's *Electrotypes Manipulation* (1850); and H. Dirck's *History of Electro-metallurgy* (1863). (F. W. R.)*

ELECTROMETER. An electrometer, according to Sir Defin-
Wm. Thomson, who is the greatest living authority on this tion of
subject, and has done more than any one else to perfect terms.
this kind of physical apparatus, is "an instrument for measuring differences of electric potential between two conductors through the effects of *electrostatic* force." A galvanometer, on the other hand, which might also be defined as an instrument for measuring differences of electric potential, utilizes the *electromagnetic* forces due to the currents produced by differences of electric potential. An instrument designed merely to *indicate*, without measuring, differences of electric potential is called an *electroscope*. It is obvious that every electrometer may be used as an electroscope, and it is also true that all electroscopes are electrometers more or less; but the name electrometer is reserved for such instruments as have a scale enabling us, either directly or by appropriate reduction, to refer differences of potential to some unit.

The modern electrician is far more concerned with measurements of electric potential than with measurements of electric quantity; and consequently all modern electrometric instruments are suited for direct measurements of the former kind. It is only indirectly that such instruments measure electric quantity. With the older electricians it was otherwise; and some of the earliest electrometers were designed for the direct measurement of quantity.

Such was the measuring jar of Lane,¹ represented in fig. 1 (after Lane's Riess). D is a Leyden jar, fastened to a stand in such a way that jar.

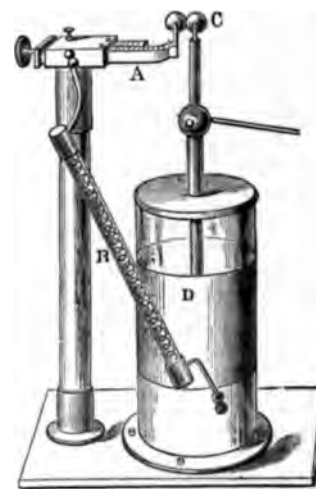


FIG. 1.—Lane's Jar.

its outer armature can be insulated or connected to earth at will. The inner armature is in good metallic connection with the knob C. A horizontal metal piece A is mounted on a glass pillar, and carries another knob, which can be set at any required distance from C by means of a screw and graduation. The piece A is connected with the outer armature of the jar by a thin wire B contained in a glass tube. This last piece was added by Riess,² whose arrangement of the apparatus we have been describing. One way of using the instrument is as follows. The balls are set at a convenient distance apart, the stand is carefully insulated, and the outer armature of the jar connected with the battery of jars or other system to be charged, and the inner armature with the source of electricity, say the prime conductor of an electric machine. The electricity accumulates on the inner armature till a certain difference of potential between C and A is reached, and then a certain quantity q of electricity passes from C to A in the form of a spark, after which a quantity q remains distributed between the outer armature and the accumulator which is being charged. This process is continued, and as each spark passes, a quantity q is added to the charge on the outer armature and accumulator. Hence if the capacity of the outer armature be negligible compared with that of the accumulator, the charge of the latter will be proportional to the number of sparks between the balls. The measuring jar may also be used to measure the overflow of electricity from one armature of an accumulator when the other is connected with an electric machine. In this case the outer coating of the jar is connected with the earth, and C is connected with the armature of the accumulator. There is no occasion to discuss minutely here the corrections necessary in the latter method of using the apparatus; on these and kindred points

¹ *Phil. Trans.*, 1769.

² The object of the fine wire is to absorb the energy of the discharges, and prevent the disintegration of the metal of the balls which renders the action of the apparatus irregular (see Riess, *Reibungselectricität*, § 386).

consult the account given by Mascart, *Traité d'Electricité Statique*, tom. i. §§ 313-316, and Riess, *l.c.*

The torsion balance of Coulomb is another instrument suited for the direct measurement of electrical quantity. For its construction and use see the article ELECTRICITY, p. 18.

Discharging electro-
scope. The discharging electroscope of Gaugain belongs to the present class of instruments. It consists (fig. 2) of an ordinary (old-fashioned) gold-leaf electroscope, with the addition of a small knob B, connected with the metal sole of the instrument, and standing a little to one side of one of the leaves. The charge on any conductor is measured by connecting it with the knob A through a sufficient length of wet cotton to retard the discharge properly. When a certain amount of electricity has reached the gold leaf, it is attracted to the knob B and is discharged; it then falls back, is recharged, then discharged by contact with B a second time, and so on. It is found that the same quantity of electricity is discharged at each contact if the process be properly regulated; so that the whole charge on the conductor is measured by the number of oscillations of the gold leaf required to discharge it completely.¹



FIG. 2. — Discharging Electro-scope.

The rest of the instruments (save one) to be described may be classified under the three heads given by Sir Wm. Thomson in his valuable report on electrometers,² viz., (1) repulsion electrometers, (2) attracted disc electrometers, and (3) symmetrical electrometers.

I. Repulsion Electrometers.—The electroscopic needle of Gilbert is the oldest specimen of a repulsion electroscope. The linen threads of Franklin, and the double pendulum used by Canton, Du Fay, and others, which was an improvement thereon, are typical of another species of electroscope coming under the same genus.

Cavallo's electro-
scope. Cavallo's electroscope³ (fig. 3) embodies the double pendulum principle. It consists of two fine silver wires loaded with small pieces of cork or pith, and suspended inside a small glass cylinder. Through the cap which closes the cylinder passes the stout wire from which the pendulums are suspended. This wire ends in a thimble-shaped dome A, which comes down very nearly to the cap; the outside of the cap and part of the wire are covered with sealing wax, and the object of the dome is to keep moisture from the stem, so that the electroscope could be used in the open air even in rainy weather. To add to the sensitiveness of the instrument two strips of tinfoil are pasted on the glass at B and C opposite the pith balls. An electroscope similar to this was used by Saussure.⁴ Volta used a pair of straws instead of the pith ball pendulums.

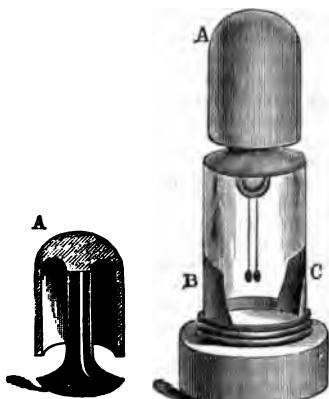


FIG. 3. — Cavallo's Electroscope.

Bennet's
gold-leaf
electro-
scope. By far the most perfect form of electroscope on the double pendulum principle is the gold-leaf electroscope of Bennet.⁵ Fig. 4 represents a modern form of this instrument. The gold leaves are gummed on the two sides of a flat piece of metal carried by a stout stem, which passes through the top of a glass shade and ends in a flat disc. By means of this disc we may convert the instrument into Volta's condensing electroscope (already described, see ELECTRICITY, p. 34). Inside the glass shade, and rising well over the leaves, stands a cylinder of wire gauze, which ought to be in metallic connection with the earth, or with some conductor whose potential is taken as the standard of reference. The introduction of the wire cylinder is due to Faraday, and is an essential improvement; it is absolutely necessary, in fact, to convert the instrument into a trustworthy indicator of differences of potential. It serves the double purpose of protecting the leaves from external disturbing influences, and of ensuring that the instrument always indicates the difference between the potential of the body connected with the

leaves and another definite potential. Thus, if we insulate the sole of the electroscope, and connect A with the leaves, and B with the gauze, the divergence of the leaves corresponds to the difference between the potentials of A and B, and will always be same for the same potential difference.⁶ Hence, if the divergence of the leaves were read off by means of a properly constructed scale, the instrument might be used as a rough electrometer. The value of



FIG. 4. — Bennet's Electroscope.

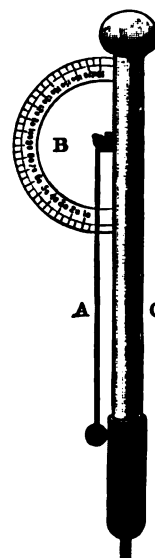


FIG. 5. — Henley's Electrometer.

the graduation would of course have to be determined by experiment. Peccet did, as a matter of fact, use the gold-leaf electroscope in this way.

The electrometer of Henley,⁷ sometimes called Henley's quadrant electrometer (fig. 5), may be taken as the type of single pendulum electroscopes. It consists essentially of a pendulum A hinged to meter, a vertical support C, which carries a vertical graduated semicircle B, by means of which the deviation of A from the vertical can be read off. This form of electroscope is, or was, much used for indicating the state of electrification of the prime conductors of electric machines. The stem is screwed into the conductor, and the divergence of the pendulum indicates roughly the charge.

The sine electrometer of August, represented in fig. 6, is a modification of the single pendulum electroscope, analogous in principle to Pouillet's sine compass. A is a pendulum suspended by two threads to secure motion in one plane; B is a ball fixed to the case, and connected with a suitable electrode. Any charge is given to A; B is charged with q units of electricity; the case is turned through an angle ϕ in a vertical plane until the distance between A and B is the same as it was when both were neutral; then, if the charge on A be always the same,

$$q \propto \sin \phi.$$

This instrument is interesting on account of the principle employed in its construction; but we are not aware that it has ever been used in practice.

Another class of instruments, in which the movable part is a horizontal arm turning about a vertical axis, may be looked upon as the descendants of Gilbert's electroscopic needle. The electrometer of Peltier and its modification into a sine electrometer (by Riess) are instruments of this class. Descriptions of both will be found in Mascart, §§ 291 and 292.

Dellmann's electrometer (fig. 7) is constructed on a principle similar to that applied in the two instruments last named. D is a needle, formed of light silver wire, suspended by a fine glass fibre from a torsion head A. Below the needle is a piece of sheet metal NE, divided half through by a notch in the middle, and then bent in opposite directions on both sides of the notch, so that, when looked at end on, it appears like a Y. Underneath NE is a



FIG. 6. — Sine Electrometer.

¹ There is a correction for residue, see Mascart, t. i. § 317, &c.

² *Brit. Assoc. Rep.* 1867, or *Reprint of Papers on Electrostatics and Magnetism*, § 343.

³ Riess, §§ 49 and 50.

⁴ 1777.

⁵ *Phil. Trans.*, 1787.

⁶ It was by no means safe to take this for certain in the old instruments, owing to the electrification of the glass.

⁷ *Phil. Trans.*, 1772.

graduated disc PL, through the centre of which passes a glass tube F supporting NE, so that it can be raised or depressed by a lever G. Inside F is a spring by means of which the lever H, which serves as electrode, can be connected or disconnected at will with the metal piece NE. The whole contained in a metal case B, the lid of which is of glass, so that the position of the needle D on the graduation PL can be read off by means of the lens M. To use the instrument, the case is connected with the earth, the needle is brought nearly at right angles to NE, and NE is raised by

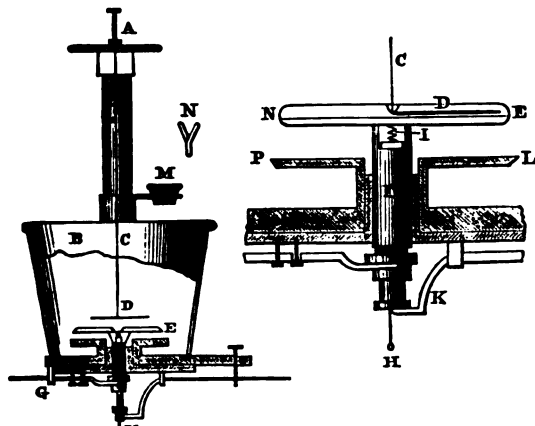


FIG. 7. — Dellman's Electrometer.

means of G till the needle is in contact with it; then the electrode K is brought into communication with NE, and the body whose charge or potential is to be measured is connected with K. The connection with K is then suppressed, and NE lowered; and the needle, now free, is repelled by NE. If, by means of the torsion head, we bring the needle along to a fixed position relative to NE, the electrical couple will be proportional to the square of the charge communicated to NE and D, i.e., to the square of the potential of the body connected with K, provided the capacity of the electrometer be negligible compared with that of the body. Hence the potential is measured by the square root of the torsion on the fibre when the needle is in a given position.

The form of Dellmann's electrometer we have just described was that used by Kohlrausch.¹ It has been simplified by its inventor, and applied in his important investigations on atmospheric electricity.

Coulomb's balance might be used as an electrometer on the repulsion principle. Special care would, however, be necessary to avoid or to allow for disturbances arising from the case of the instrument, which ought under any circumstances to be coated wholly or partially with tinfoil on the inside, according to Faraday's plan. Sir Wm. Thomson did, in fact, design an electrometer of this description, and has given tables (*Reprint of Papers*, § 142) for reducing its indications. This type of electrometer has not come into general use.

II. *Attracted Disc Electrometers.*—The first idea of this kind of

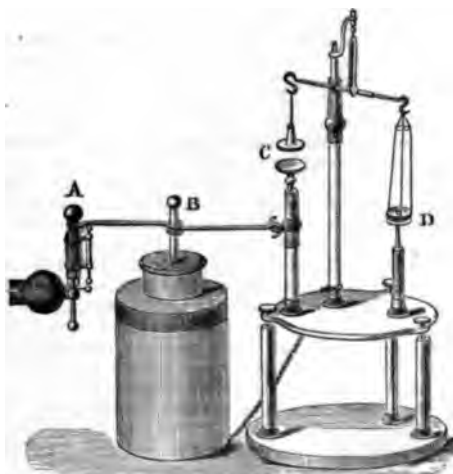


FIG. 8. — Snow Harris's Disc Electrometer

instrument is due to Sir Wm. Snow Harris. One of the instruments in which he carried out the principle and the mode of using

it will be understood from fig. 8. C is an insulated disc, over which is suspended another disc, hung from the arm of a balance, and connected with the earth. A weight w is put in a scale attached to the other arm of the balance. The insulated disc is connected with the internal armature B of a Leyden jar, whose outer armature is in connection with the suspended disc. Electricity is conveyed to B, and the quantity q measured by a small Lane's jar A, until the electric attraction at C is just sufficient to turn the balance. Snow Harris found that $w \propto q^2$. This and other laws established by him agree with the mathematical theory as developed in the article ELECTRICITY.²

Great improvements have been effected in this kind of electrometer by Sir Wm. Thomson—(1) by his invention of the "guard ring" or "guard plate;" (2) by using the torsion of a platinum wire for the standard force; (3) by devising proper means for attaining a definite standard potential, and by protecting the vital parts of the electrometer from extraneous disturbance; and (4) by introducing sound kinematical principles into the construction of the movable parts.

In order to illustrate these points it will be well to describe the Thomson's portable electrometer (fig. 9), one of his simpler instruments, in detail.

The principal electrical parts of this electrometer are sketched in fig. 10. HH is a plane disc of metal (called the "guard plate") meter.

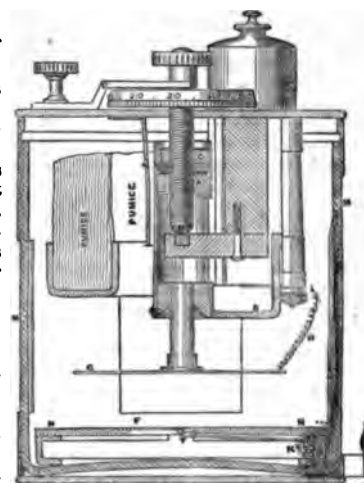


FIG. 9. — Section of Thomson's Portable Electrometer.

HH is a plane disc of metal (called the "guard plate") meter. kept at a constant potential by being fixed to the inner coating of a small Leyden jar MM (fig. 9), which forms the case of the instrument. At F a square hole is cut out of HH, and into this fits, as nearly as it can without danger of touching, a square piece of aluminium foil as light as is consistent with proper stiffness. One side of this disc is bent down, and then runs out horizontally into a narrow stem ending in a stirrup L—the whole being not unlike a spade. The sole of the stirrup consists of a fine hair, which moves up and down before a vertical enamelled piece bestridden by the fork of the stirrup. On the enamel are two small dots very near each other. When the hair seen through a small convex lens appears straight, and bisects the distance between the dots, the stirrup is said to be in the sighted position. The aluminium spade is suspended on a horizontal platinum wire stretched by platinum springs at its two ends, and is carefully balanced with its centre of gravity in the line of suspension, so that the only force other than electric that can affect it is the torsion of the wire, which acts like the string in the toy called the "jumping frog," or like the hair rope in the catapults of the ancients. The spade is so arranged that F is as nearly as possible in the same plane with the guard plate when the hair is in the sighted position. It is the torsional couple exerted by the wire in this position that forms the standard force. The remaining important electrical part is the plane horizontal disc G.

It is essential to the action of the instrument that we should be able to move the disc G parallel to itself and to HH through measured distances. The mechanism by which this is accomplished is a remarkable instance of the application of geometrical principles to mechanism, and the reader will do well to read Thomson's "Lesson to the instrument makers" on this subject in the *Reprint* of his papers, § 369. The glass stem which carries G is fixed into the lower end of a hollow brass cylinder; in the upper end of the cylinder is fixed a nut AC, through which works a carefully cut screw ending in a rounded point B of polished steel. The point B rests on a horizontal agate plate let into a foot which projects from

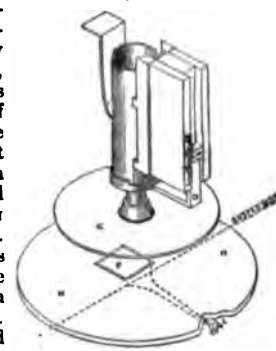


FIG. 10.

¹ *Pogg. Ann.*, 1842 and 1852.

² See also *Reprint of Sir Wm. Thomson's Papers*, § 158.

a strong vertical support fastened to the brass lid of the jar MM (fig. 9), and passes through a slit in the hollow cylinder. This vertical piece is fitted on one side with two V notches, into which the hollow cylinder is pressed by a spring fastened to the lid and bearing half way between the Vs, and on the other side with a rectangular groove in which slides the vertical part of a knee-piece D, in rigid connection with the hollow cylinder. D prevents the cylinder from turning round, but allows it to move vertically; it also carries a fiducial mark running opposite a graduation on one edge of the groove, by means of which whole turns of the screw are read off, fractions being estimated by means of a drum head. The nut AC is arranged in two parts, with a spring between them, to prevent "lost time" and secure steadiness (for details, see paper cited above.)

The disc G is connected by a spiral of fine platinum wire with the main electrode S, which is insulated from the lid of the box by a glass stem. The arrangement of this electrode is worthy of notice, and will be understood from fig. 11. The dome T is called the umbrella; its use is obvious. A similar, only less perfect, device was noticed in Cavallo's electroscope. The vital parts of the instrument are all inside the coated jar, and therefore removed from disturbing influences; only it is necessary to remove some of the tinfoil opposite the hair in order to see it. The effect of this is counteracted by means of a screen of fine wire.

The use and the theory of the instrument are very simple. The body whose potential is to be measured is connected with the umbrella, which is raised in order to insulate the main electrode from the case, the last being supposed to be in connection with the earth. Let v be the potential of the inner coating of the jar, the disc, and guard plate, V that of the body and G, and d the distance between G and H when the hair is in the sighted position. Then, since F may be regarded as forming part of an infinite plate,¹ if its surface be S its potential energy will be $\frac{1}{2}Sv(v \cdot V)$ (see ELECTRICITY, p. 34), i.e.,

$$\frac{S(v - V)^2}{8\pi d}.$$

Hence the attraction f on F will be given by

$$f = \frac{S(v - V)^2}{8\pi d^2} \dots \dots \dots (1).^1$$

Here f is a constant, depending on the torsion of the suspending wire of the aluminium balance; hence, A^2 standing for $8\pi f \div S$, i.e., A being a constant depending on the construction of the instrument, we have

$$v - V = Ad \dots \dots \dots (2).$$

If we now depress the umbrella, so as to bring G to the potential of the earth, and work the screw till the hair is again in the sighted position, we have, d' being the new reading of the screw,

$$v = Ad' \dots \dots \dots (3).$$

Hence, from (2) and (3),

$$V = A(d' - d) \dots \dots \dots (4).$$

We thus get V in terms of A and the difference of two screw readings, so that uncertainties of zero reading are eliminated. The value of A must be got by comparison with a standard instrument, if absolute determinations be required.

Absolute
electro-
meter.

Thomson's absolute electrometer (fig. 12) is an adaptation of the attracted disc principle for absolute determinations. We give merely an indication of its different parts, referring to Thomson's paper (*l.c.*) for details. B is an attracting disc, which can be moved parallel to itself by a screw of known step ($\frac{1}{10}$ in. or thereby). A is a guard plate, in the centre of which is a circular balance-disc of aluminium suspended on three springs, and connected by a spiral of light platinum wire with A. The disc can be raised or depressed into definite positions by means of a screw (the kinematical arrangements in connection with these screws are similar to that in the portable electrometer). A hair on the disc, an object lens h , a fiducial mark, and an eye lens l enable the observer to tell when

¹ Those who desire to know the degree of approximation here should consult Maxwell, *Electricity and Magnetism*, vol. i. § 217.

this disc is in such a position that its lower surface is plane with lower surface of A. $y y$ are the halves of a box which screens the disc from electric disturbances. An idiostatic guage (consisting of an aluminium lever with guard plate, hair, and lens, as in the portable electrometer), placed over a plate F in connection with the guard plate, enables the observer to tell when the guard plate and the inside coating of the instrument (which forms a Leyden jar as in the portable instrument) are at a certain definite potential. And finally, a small instrument called the "replenisher" enables

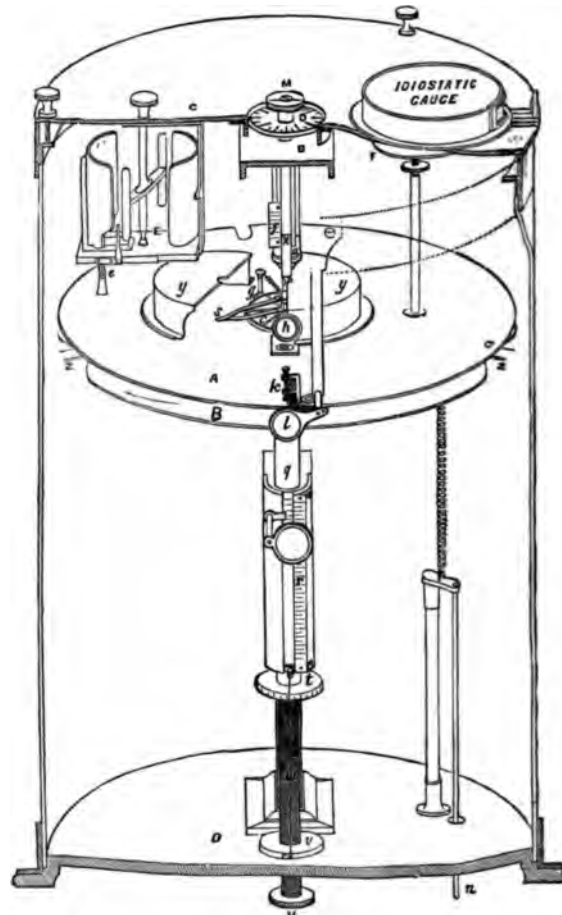


FIG. 12.—Thomson's Absolute Electrometer.

him to raise or lower the potential of A till this definite potential is reached.

A short description of the replenisher will be in place here. It *Repl* is represented pretty clearly at E (fig. 12). Two metal shields, *isab* in the form of cylindrical segments, are insulated from each other by a piece of ebonite; the left hand one is in connection with the guard plate, the right hand one with the case of the instrument (and therefore with the outer coating of the jar). A vertical shaft, which can be spun round by means of a milled head, carries two metal flies on the ends of a horizontal arm of vulcanite. Two small platinum springs (the front one is seen at e) are arranged so as to touch the flies simultaneously in a certain position just clear of the shields. Let us suppose the left shield along with A to be positively electrified, and the flies to be in contact with the springs; e being close to the left shield, the front fly will be electrified - and the back fly +. Suppose the shaft to revolve against the hands of a watch lying face up on the cover of the electrometer. The front fly carries off its - charge, and, when near the middle of the right shield, comes in contact with a spring connected with the shield. Being thus practically inside a hollow conductor, it gives up its - charge to the shield. At the same time the back fly gives up its + charge to the left shield. The result of one revolution therefore is to increase the + and - charges on the respective shields, or, in other words, to increase the difference of potential between them. By giving the machine a sufficient number of turns, the potential of A may be raised as much as we please; and, by spinning in the opposite direction, the potential can be lowered; so that, once A is charged, it is easy to adjust its potential till the hair of the gauge is in the sighted position.

To work the instrument, the electrode π of the lower plate B is

connected with the guard plate to avoid all electrical forces on the balance; the hair of the balance is brought to the sighted position,

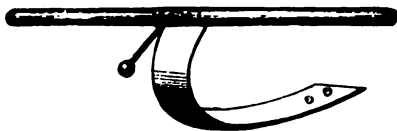
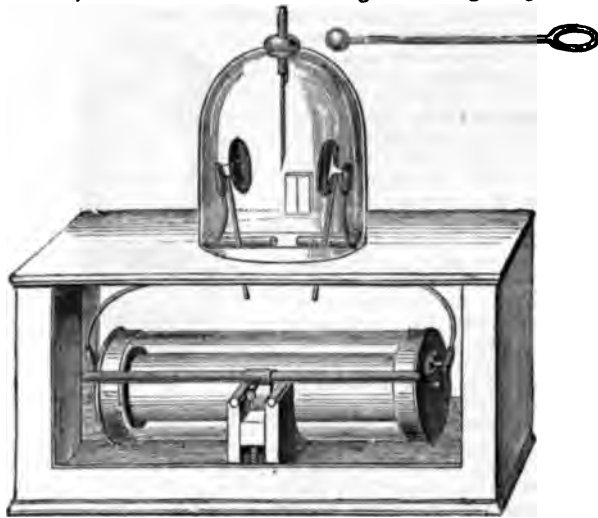


FIG. 13.—Dry Pile Electroscop.

and the upper screw reading taken; then a weight of w grammes is distributed symmetrically on the disc, the balance brought up again by working the screw, and the reading again taken. We

thus ascertain how far the weight of w grammes depresses the balance. The weight is now removed, and the balance left at a distance above A equal to that just found. A is now charged, and its potential adjusted till the hair of the gauge indicates that the standard potential v is reached. Let it now be required to measure the difference between the potentials V and V' of two conductors. Connect first one and then the other with a , and work the lower screw till the hair of the balance is sighted in each case, and let the screw readings reduced to centimetres be d and d' . Then, since the force on the disc in each case is gw , where g is the acceleration produced by gravity in a falling body in centimetres per second, we have by (1)

$$V - V' = (d' - d) \sqrt{\frac{8\pi gw}{S}} \quad \dots \quad (5),$$

where S denotes the area of the balance disc, or rather the mean of the areas of the disc and the hole in which it works. We thus get the value of $V - V'$ in absolute electrostatic C. G. S. units.

III. *Symmetrical Electrometers.*—Two instruments fall to be Dry pile described under this head,—the dry pile electroscop, and Thomson's electro-quadrant electrometer. The idea common to these instruments scope. is to measure differences of potential by means of the motions of an electrified body in a symmetrical field of force. In the dry pile electroscop, a single gold leaf is hung up in the field of force, between the opposite poles of two dry piles, or, in later forms of the instrument, of the same dry pile. The original inventor of this apparatus was Behrens, but it often bears the name of Bohnenberger, who slightly modified its form. Fechner introduced the important improvement of using only one pile, which he removed from the immediate neighbourhood of the suspended leaf. The poles of the pile are connected with two discs of metal, between which the leaf hangs. This arrangement makes it easier to secure perfect symmetry in the electric field, and allows us to vary the sensitiveness of the instrument by placing the metal plates at different distances from the leaf. In order to make the attainment

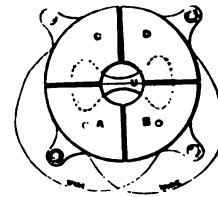


Fig. 14.

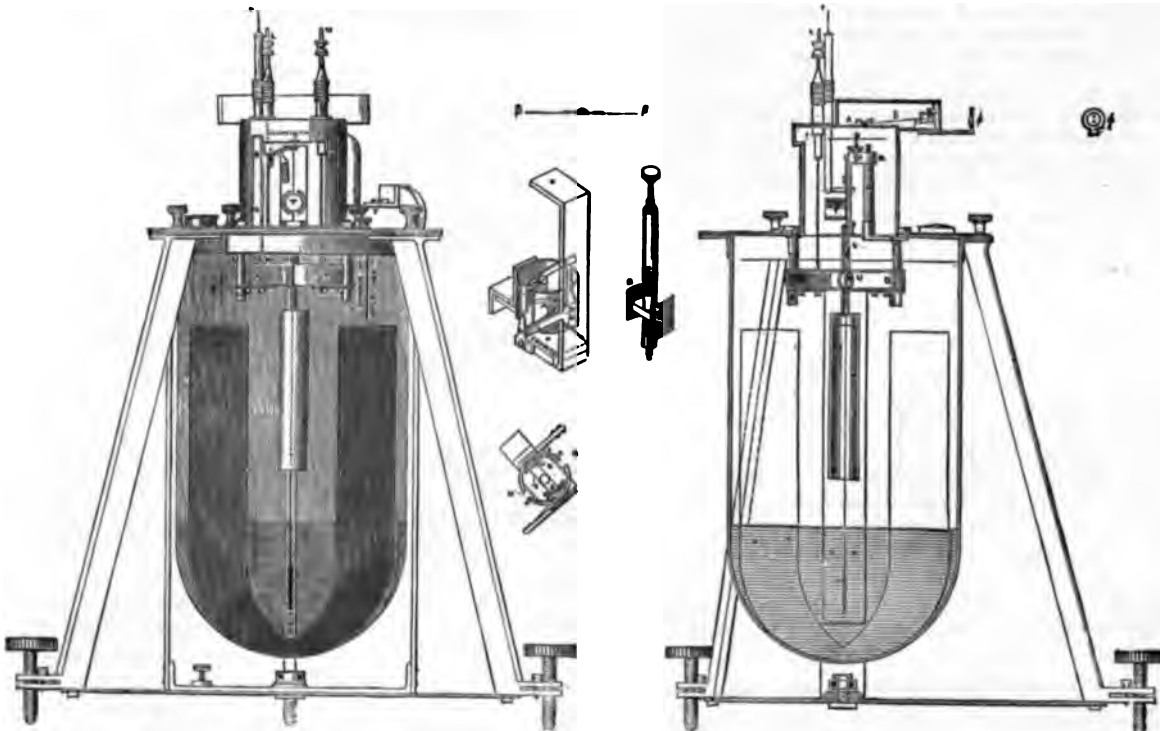


FIG. 15.—Elevation and Section of Thomson's Quadrant Electrometer.

of perfect symmetry still more easy and certain Riess¹ added a metal rod to the apparatus, which can be made to touch the two metal caps of the dry pile simultaneously, and then be removed, leaving the pile symmetrically electrified. This form of the electroscop, with the various improvements, is represented in fig. 13.

¹ *Reibungselectr.*, § 16.

Hankel² still further improved the dry pile electroscop by giving a micrometric movement to the plates, by substituting a galvanic battery with a large number of cells for the uncertain and varying dry pile, and by using a microscope with a divided scale to measure the motions of the gold leaf. With these improvements it became an *Electrometer* of great delicacy and considerable range. Some of the

² Mascart, § 272, or *Pogg. Ann.*, 1858.

Quadrant electro- meter.

experiments in which Hankel used it are alluded to in the article ELECTRICITY.

In the quadrant electrometer of Sir Wm. Thomson, which is the most delicate electrometric instrument hitherto invented, the moving body is a horizontal flat needle of aluminium foil, surrounded by a fixed flat cylindrical box (fig. 14), which is divided into four insulated quadrants A, B, C, D. The opposite pairs A, D and B, C are connected by thin platinum wires. The two bodies whose potentials are to be compared are connected with the two pairs of quadrants. If A and B be their potentials, and C the potential of the needle, it may be shown (see Maxwell, *Electricity and Magnetism*, § 219) that the couple tending to turn the needle from A to B is

$$a(A - B)\{C - \frac{1}{2}(A + B)\} \dots \dots (6),$$

where a is a constant depending on the dimensions of the instrument. If C be very great compared with $\frac{1}{2}(A + B)$, as it usually is, then the couple is

$$aC(A - B) \dots \dots \dots (7)$$

simply; in other words, the couple varies as the difference between the potentials of the quadrants. Some idea of the general distribution of the parts of the actual instrument may be gathered from fig. 15, which gives an elevation and a section of the instrument. The case forms a Leyden jar as usual in Thomson's electrometers; the internal coating in this instance is formed by a quantity of concentrated sulphuric acid, which also keeps the inside of the instrument dry. The quadrants are suspended by glass pillars from the lid of the jar, and one of these pillars is supported on a sliding piece, arranged on strict kinematical principles, so as to be movable in a horizontal direction by means of a micrometer screw Y. This motion is used to adjust the position of the needle, when charged, so that its axis may fall exactly between the quadrants A, C, and B, D. A glass stem C, rising from the lid of the jar into a superstructure called the "lantern," supports a metal piece Z, to which is fastened a metal framework fitted with supports and adjustments for the bifilar suspension of the needle. A fine platinum wire drops from the needle into the sulphuric acid, thus connecting the needle with the inside coating of the jar. This tail wire is also furnished with a vane, which works in the acid and damps the oscillations of the needle. A stout aluminium wire rises from the needle, carries a light concave mirror T, and ends in a cross piece to which are attached the suspension fibres. The aluminium stem and the platinum tail wire are defended from electrical disturbances by a guard tube, which is in metallic connection with the piece Z, and also, by means of a platinum wire, with the acid; it is through this, by means of the "temporary electrode" P, that the inside of the jar is charged. The two principal electrodes are P and M. Connected with Z is a metal disc S, attracting the aluminium balance of a gauge like that of the absolute electrometer. This gauge is well seen in the bird's-eye view given in fig. 16. A

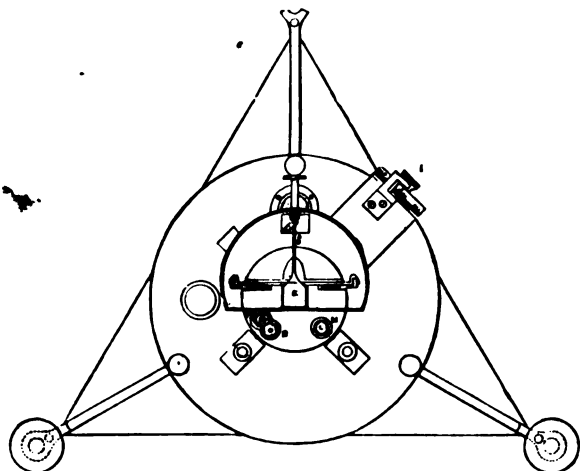


FIG. 16.—Thomson's Quadrant Electrometer—Bird's-eye view.

replenisher, like that in the absolute electrometer, is fitted to the lid of the jar, and by means of it the potential of the needle can be adjusted till the hair of the gauge is in the sighted position.

The deflections of the instrument are read off by means of an image formed by the mirror T on a scale at the distance of a metre or so, the object being a wire which is stretched below the scale in a slit illuminated by a lamp. Within certain limits the deflections are proportional to the deflecting couple, i.e., to the difference between the potentials of the quadrants A, D and B, C; but where this is not so, the instrument can easily be graduated experimentally.

For many purposes, especially in the lecture room, an instrument

so complicated as the above is unnecessary and undesirable. A simpler form (fig. 17) of quadrant electrometer is now manufactured by Elliot Brothers, and answers most ordinary purposes very well.

Capillary Electrometers.—Electrometers have recently been constructed by taking advantage of the fact that the surface tension of mercury is greatly affected by the hydrogen deposited on it when it is the negative electrode in contact with dilute sulphuric acid (see ELECTROLYSIS, p. 109). A quantity of mercury is placed in the bottom of a test tube, and communicates with a platinum electrode let in through the bottom of the tube; on the mercury is poured dilute sulphuric acid, and into this dips a tube drawn out into a capillary ending. This tube contains mercury down to a certain mark on the capillary part, the remainder being occupied with acid which is continuous with that in the test tube. So long as the mercury in the test tube is simply in metallic connection with that in the upper tube, the position of the mercury in the capillary part is stationary; but if an electromotive force be introduced into the external circuit, acting towards the test tube, then hydrogen is deposited on the small mercury surface, its surface tension increases, and the pressure in the tube must be considerably increased to maintain the mercury at the mark. This increase of pressure is proportional to the electromotive force within certain limits, hence we can use this arrangement as an electrometer.

Electrometric Measurement.—Several examples of electrometric measurement will be found in the article ELECTRICITY (pp. 18, 37, 38, 42, 46, &c.). We recommend in this connection the study of the sections on atmospheric electricity in Sir Wm. Thomson's *Report of Papers on Electricity and Magnetism*, and sections 220 and 229 in Clerk Maxwell's *Electricity and Magnetism*. We have been drawing throughout on Thomson's *Report on Electrometers and Electrometric Measurements*, but it will not be amiss to draw attention to it once more.

(G. CH.)



FIG. 17.—Quadrant Electrometer.

Lipp
mann's
capilla
electro
meter.

MAGNETISM

THE word magnetism is derived from the Greek word *μάγνηξ*, which was applied to an ore of iron possessing a remarkable attractive power for iron, and supposed to have been originally found near the town of Magnesia, in Lydia.¹ Thus Lucretius writes:—

Quem Magneta vocant patrio de nomine Graii,
Magnetum quia fit patriis in finibus ortus.

This name is said by Plato² to have been given to it by Euripides, and he adds that most call it the Heracleian stone. It is needless here to criticize the above or other derivations that have been given for the word; we merely remark that it is now applied to all the phenomena kindred to that which first drew attention to the magnetic iron ore, viz., a selective attraction for iron.

In the following article we shall give, in the first place, a sketch of the leading phenomena of strongly magnetic bodies. We shall then describe a provisional theory sufficient to render a general account of these phenomena, and shall afterwards proceed to render this theory more precise, to develop it to its necessary conclusions, and to compare these with experiment, indicating where the theory is either incorrect or incomplete. Then we shall discuss the paramagnetic and diamagnetic properties of all bodies, as expounded by Faraday; an account will be given of the connexion between the magnetic and the other physical properties of bodies; and, lastly, we shall endeavour to give some idea of the different physical theories that have been proposed in order to give something more than a mere short-hand record of the facts of observation.

LEADING PHENOMENA.

It appears, from what Lucretius says in the passage above quoted,³ that the Greeks and Romans were aware, not only that the loadstone, or magnetic iron ore, attracted iron, but also that it endued iron in contact with it with its own peculiar property. Thus an iron ring will hang suspended by the attraction of a loadstone, and from that ring another, and so on, up to a certain number, depending on the power of the stone and the weight, &c., of the rings. They were also aware that the attraction was confined to iron, or at all events was not indiscriminate, and that it was not destroyed by the intervention of other bodies, such as brass, between the magnet and the iron. It appears, too, from the passage—

Fit quoque ut a lapide hoc ferri natura recedat
Interdum, fugere atque sequi consueta vicissim, &c.—

that they had an idea that, under certain circumstances, the attraction might be replaced by a repulsion. If, however, we understand aright the latter part of Lucretius's somewhat obscure description of what seems to have been an actual experiment of his own, this notion was in reality a hasty generalization, not justified by the observed facts.⁴ In any case there seems no warrant for assuming, as some have done, that the ancients had any definite conception of magnetic polarity.

What they wanted in definite experimental knowledge they supplied by an abundant use of the imagination.

¹ Gilbert, *De Magnete*, lib. i. chap. ii., says, "Magnesia ad Mæandrum"; but it is uncertain whether this or Magnesia ad Sipylum is meant.

² *Ἐν τῇ λιβῇ ἣν Εὐριπίδης μὲν Μαγνήτιν ἀνέμασεν, οἱ δὲ πολλοὶ Ἡράκλειαν (Ion, 533 D).* See Munro's *Lucretius*, vol. i. p. 662. The other name is from Heraclea in Lydia.

³ Bk. vi. line 906 sq., and 1042 sq.; comp. Plato, *Ion*, *ut supra*, whom there is reason to think he is quoting.

⁴ See below, p. 225.

We are told, for instance, that the magnet attracts wood and flesh, which was certainly beyond their powers of observation; that it is effective in the cure of disease; that it affects the brain, causing melancholy; that it acts as a love philtre; that it may be used in testing the chastity of a woman; that it loses its power when rubbed with garlic, but recovers it when treated with goat's blood; that it will not attract iron in the presence of a diamond, and much else that was eagerly copied by the wonder-loving writers of the Middle Ages.

The science of magnetism made no real progress till the invention of the mariner's compass. The early history of this instrument is very obscure. According to some authorities it was invented in China, and found its way into Europe probably through Arabian sources. The light thrown by recent researches on the literature of the Chinese has apparently thrown doubt upon their claim to this invention,⁵ although the knowledge of the loadstone and its attractive property may have been older among them than even among the Greeks. The first accounts of the compass in Europe go back to the 12th century, and, although the instrument described is very rough, it is not spoken of as a new invention. In its earliest form it seems to have consisted simply of an iron needle which was touched with the loadstone and placed upon a pivot, or floated on water, so that it could turn more or less freely. It was found that such a needle came to rest in a position pointing approximately north and south (some accounts say east and west, in which case there must have been a cross piece on the needle to indicate what was probably the important direction for the mariner). As these compasses were made of iron (steel was not used till much later), and were probably ill-pivoted, they must have been very inaccurate; and the difficulty of using them must have been much increased by the want of a card, which was a later addition made apparently by the Dutch.

It is unnecessary to enter into more detail here respecting the early history of the compass, as the matter has been very fully treated in the article COMPASS.⁶ We proceed therefore to show the bearing of the invention upon the science of magnetism. It will at once be seen that it involves two scientific discoveries of capital importance:—first, that the loadstone can transmit to iron with which it comes in contact a permanent property like its own; and, secondly, that a loadstone or magnet if suspended freely will turn so that a certain direction in it assumes a fixed position relative to the geographical meridian, a certain part of the magnet turning always towards the north, and the part opposite towards the south. These opposite parts of the magnet are called its "poles."

To fix our ideas we shall describe a process by which we might definitely determine this direction in the magnet. Following the example of Gilbert, let us consider a spherical magnet. Our reason for dealing with this form in the first instance is to make it perfectly clear that the phenomena depend essentially on something apart from the form of the body. We shall suppose that the magnet is homogeneous as to its mass, so that its centre of gravity

Axis of a magnet experimentally defined.

⁵ See Möllendorff, *Z. D. M. G.*, xxxv. 76.

⁶ It may be mentioned that the statement that Peter Adsaiger, in a letter written in 1296, mentions the magnetic declination, appears to be a mistake, arising from the mistranscription of a title. See Wenckebach, quoted by Lamont, *Handbuch des Magnetismus*, p. 449. The passage from Are Frode, quoted by Hansteen, and alluded to in last edition of this encyclopædia, appears also to be of doubtful antiquity. See Poggendorff, *Geschichte der Physik*, p. 99.

coincides with its centre of figure. Suspend this spherical magnet by a fine thread of untwisted silk, attached to any point of its surface, say P. After the magnet has come to rest, mark the vertical plane through the centre which falls in the geographical meridian; this may be done by tracing a great circle on the surface of the magnet. Next find the point P' in which the vertical through P cuts the surface again, and suspend the magnet by P', again marking the plane which falls in the meridian. Now, find the plane which bisects the acute angle between the two former planes, mark it by a great circle, and call it the axial plane of P. If we thus find the axial planes of any number of points, we shall find that they all intersect in one common line passing through the centre of the sphere. We may call this line the "axis" of the magnet. Let us mark the points where it cuts the surface; we may call these the "poles" of the magnet. We shall then observe that, however we suspend it, the magnet will always come to rest so that the vertical plane through the axis makes a definite angle with the meridian. This angle (δ) is called the "declination" (also, by sailors, the "variation"); it varies from place to place, and from time to time, but very slowly, so that throughout a limited area of the earth's surface, and for a limited time, it may be regarded as constant.¹

Poles,
north
and
south.

Magnetic
meridian.

One end of the axis always turns northwards, and the other always southwards; we shall call the former the "north" and the latter the "south pole," although, for reasons to be afterwards explained, it would be more appropriate to invert the order of these names. Henceforth the vertical plane in which the axis of the magnet comes to rest will be called the magnetic meridian, and the two horizontal directions in this plane magnetic north and magnetic south respectively.

It must be carefully noticed that there is a certain amount of arbitrariness in our definition of the axis and poles of a magnet. In reality it is only the direction of the axis that is fixed in the body, and not its absolute position. This will be made plain if we repeat all our experiments with the spherical magnet after fastening to it a piece of wax or other non-magnetic body, so as to leave its magnetic properties unchanged, but to throw its centre of gravity out of the centre of figure. Everything will fall out as before, only the axial planes of the different points of suspension will now meet in a line, parallel, it is true, to the axis determined before, but passing through the new centre of gravity. In point of fact, therefore, we might choose any point in the body, draw a line through it in the proper direction, and call this the axis. Hereafter we shall, unless the contrary is stated, draw the axis through the centre of gravity of the body, or through its centre of figure if it has one; and we define the poles, for the present, as the points in which the axis cuts the surface of the magnet, supposing, as will be generally the case, that the line cuts the surface in two points and no more.

Magnetic
needle.

Having now obtained a definite idea of the axis of a magnet, and seen that it has, in the first instance at least, nothing to do with the external form of the body, let us proceed to make an artificial magnet of the particular kind usually called a "magnetic needle," and briefly examine its properties. Take a tolerably thin flat piece of pretty hard-tempered steel, of the elongated symmetrical form NS shown in fig. 1. We suppose it, in the first place, in an unmagnetized condition. Let it be pierced by a well-turned axis *ab*, passing accurately through its centre of gravity, and perpendicular to its plane, so that, when the

axis is placed on two horizontal knife edges, the needle will rest in any position indifferently. Further, let four very small hooks, *c, d, e, f*, be attached, two (*c, d*) to the ends of the axis, and other two (*e, f*) to the edges of the needle in a line perpendicular to NS. Now rub the half of the needle

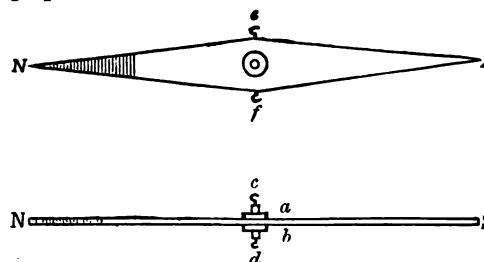


Fig. 1.

towards N with the south pole of the spherical magnet whose properties we have just discussed, beginning the stroke at the middle and ending it at the point of the needle, and for symmetry's sake let us do the same to the other side of the needle, and then repeat this process with the north pole of the sphere on the other half towards S. Let us examine the properties of the needle thus "magnetized." If we suspend it first by the hook *c* and then by the hook *d*, we shall find that in both cases the line joining NS² makes very nearly the same angle with the geographical meridian. Hence the magnetic axis must lie in a plane through NS perpendicular to the plane of the needle. A similar experiment with the two hooks *e, f* will show that the magnetic axis lies approximately in the plane of the strip, which we may suppose for the present to be infinitely thin. Hence the magnetic axis may be taken to be coincident with the line NS joining the points of the needle. This coincidence is, however, in general only approximate, and in delicate measurements corrections have to be made on that account, of which more hereafter. If we now mount our magnetized needle on a piece of cork or two straws, and float it in a basin of water, or replace its axle by a small cap and set it on a pivot, we have the mariner's compass in its early form. We shall call it a magnetic needle, to distinguish it from the more elaborate compass of the present day. A favourite way of showing the directive property of a magnet, described by Gilbert, is to magnetize a sewing-needle, and lay it very gently, by means of a fork of wire, on the surface of water; it will float and turn until it takes up its position in the magnetic meridian.

A needle mounted in this way, so as to have great freedom to move in a horizontal plane, is of great use in magnetic experiments. Gilbert calls it a "versorium." When very delicate applications are in view, the point of the pivot on which it is mounted must be very hard (say of hard tempered steel or iridium), and the cap should be fitted with an agate or other hard stone having a polished cavity of the form of a blunted cone to receive the pivot. A still better arrangement, also used by Gilbert, is to suspend a short and very light piece of steel wire—a fine sewing needle may be used—by means of a single fibre of silk. The most delicate arrangement of all is to use one of Sir W. Thomson's light galvanometer mirrors with the magnets attached, and follow its movements by means of the lamp and scale as usual. See GALVANOMETER.

Such, with as much of modern accuracy imported into them as was necessary for clearness of exposition, were the facts of magnetism as known up to the beginning of the 16th century.

Another experiment with our magnetized needle will enable us to describe the next important magnetic discovery. In its unmagnetized condition the needle rested indifferently in any position when its axis was placed on

¹ For the early observations on the declination the reader is referred to the treatment of the subject of terrestrial magnetism in the article METEOROLOGY. At the present time the declination at Greenwich is a little over 18°; at Edinburgh it would be about 4° more.

² Or the vertical plane through it, should it happen to be not quite horizontal.

two horizontal knife edges. In the magnetized state this is no longer the case. The axis of the needle now takes up a fixed position, with its north end pointing downwards (fig. 2), and if disturbed will oscillate about that position, and finally settle into it again. The angle which the axis NS makes with the horizon is least when the plane of rotation of the needle is in the magnetic meridian: the angle (α) in this case is called the "dip," or (by Continental writers) the "inclination." It is greatest, viz., 90° , when the plane of rotation of the needle is vertical and perpendicular to the magnetic meridian. At Greenwich the dip is about $67^\circ 30'$ at the present time. If we place the needle with its plane of rotation perpendicular to the line of dip, the equilibrium will be indifferent, as it was in all positions before magnetization; but there is no other position of the magnetized needle for which this is true.

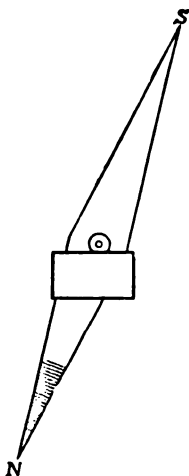


Fig. 2.

The remarks which we made as to variation in space and time of the declination apply also to the dip. The variation from place to place differs, however, in nature from that of the declination. Along a line running in the neighbourhood of the geographical equator, partly north and partly south of it, the dip is zero. North of this line, which is called the magnetic equator, the north end of the needle dips below the horizon; and the angle of dip increases as we go northwards, until, at a point in the Hudson's Bay Territory, the needle dips with its north pole vertically downwards. South of the magnetic equator the south end dips below the horizon; and there is again a point in the southern hemisphere where the south end dips vertically downwards. These points are called the "magnetic poles" of the earth. For further details on this subject we refer the reader to the discussion of terrestrial magnetism in the article METEOROLOGY.

rt.
an.

It was in the accurate observation of the declination and dip of the magnetic needle that the science of magnetism arose. The dip appears to have been first observed by Georg Hartmann, vicar of the church of St Sebaldus at Nuremberg (1489-1564), who seems to have been in advance of his age in magnetical matters. In a letter¹ to Duke Albrecht of Prussia, dated 4th March 1544, he writes:—

"Besides, I find this also in the magnet, that it not only turns from the north and deflects to the east about 9° more or less, as I have reported, but it points downwards. This may be proved as follows. I make a needle, a finger long, which stands horizontally on a pointed pivot, so that it nowhere inclines towards the earth, but stands horizontal on both sides. But as soon as I stroke one of the ends (with the loadstone), it matters not which end it be, then the needle no longer stands horizontal, but points downwards (*fällt unter sich*) some 9° more or less. The reason why this happens was I not able to indicate to his Royal Majesty."

From this it will be seen that Hartmann had unquestionably observed the tendency of the magnetized needle to dip. His method of observing is of course unsuited for

¹ Brought to light by Moser. See Dove's *Repertorium der Physik*, ii., 1838. It does not appear that Hartmann's letter was ever before published. Moser is therefore scarcely justified in attacking Norman's priority in this matter, still less in attempting to deny him the credit of first observing the dip by a sound method. Had he read the *Novae Attractiones* he could scarcely have fallen into such an error; for in respect of clearness and scientific precision Hartmann's letter, interesting as it is, cannot for a moment be compared with Norman's little work.

measurement, and it is not surprising that he got a result of 9° instead of somewhere about 70° .

In 1576 the dip was independently discovered by Robert Norman. Norman, a skilful seaman and an ingenious artificer, according to Gilbert. He was in the habit of making compass needles, and carefully balancing them so as to play horizontally on their pivots before magnetization. He found that, after they were magnetized, they constantly dipped with the north end downwards, so that a counterpoise had to be added to bring them back to the horizon. This led him to construct a special instrument, the prototype of the modern dipping needle, to show this new phenomenon. With this instrument he made the first accurate measurement of the dip, and found it to be $71^\circ 50'$ at London.²

The early English magnetic observers, of whom Norman and Burroughs (who wrote an able supplement to Norman's work) were admirable examples, must have done much for the introduction of precise ideas into magnetism. But their fame was speedily eclipsed by William Gilbert of Colchester³ Gilbert (1540-1603), whom Poggendorff has justly called the Galileo of magnetism, and whom Galileo himself thought enviably great. In his great work entitled *De Magnete Magneticisque Corporibus et de Magno Magnete Tellure Physiologia Nova*, first published in 1600, we find a complete account of what was known of magnetic phenomena up to his time, with a large number of new ideas and new experimental facts added by himself. We find in Gilbert's work, in a more or less accurate form, nearly all that we shall lay before the reader in the first section of this article, described very much in the language that we shall use. "How far he was ahead of his time is best proved by the works of those who wrote on magnetism during the first few decades after his death. They contributed in reality nothing to the extension of this branch of physical science."⁴

Mutual Action of Like and Unlike Poles.—If we take a Like magnet whose poles N' , S' have been determined and marked as above explained, and bring its north pole N' near the north pole N of a magnetic needle, N will move in a direction indicating repulsion between N and N' . The same result will follow if the south pole S' of the magnet be brought near the south pole S of the needle. But if S' be brought near N , or N' near S , attraction will be indicated. Hence the following fundamental law of the action between two magnets:—*Like poles repel each other; unlike poles attract each other.* It would appear, therefore, that the whole action of one magnet upon another is of a somewhat complicated character, even if we take the simplest view of it that the experimental facts will allow, viz.,

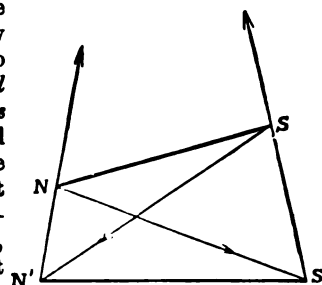


Fig. 3.

that the action may be represented by forces acting between the two pairs of points in each magnet which we have defined as north and south poles. On this assumption, the action of $N'S'$ upon NS would consist of the four forces represented in fig. 3, for all these must exist in accordance with the law just established. Whether this is a sufficient

² He published a work, of which the following description is given in the *Ronalds' Catalogue*:—"The *Novae Attractiones*, containing a short discourse of the Magnes or Lodestone, and amongst other his virtues, of a new discovered secret and subtill propertie concernyng the Declining of the Needle, touched therewith, under the plaine of the Horizon. Now first found out by Robert Norman, Hydrographer. 4to (black letter, scarce), London, 1581."

³ For details as to his life, see art. GILBERT.

⁴ Poggendorff, *Geschichte der Physik*, p. 286.

representation of the most general case, and what the exact law of the forces ought to be, we are not yet in a position to decide. One thing, however, is clear, that the action between two poles must diminish when the distance between them increases; otherwise we should not have been able to make the action of N or S upon N' prevail, by bringing the one or the other nearer.

It was perhaps the complexity of this analysis (along with the fact that the action of the magnet upon soft iron, which was the earliest discovered magnetic phenomenon, is not a pure case of this action, but involves also another phenomenon, viz., magnetic induction) that prevented for so long the discovery of the elementary law we are now discussing. At all events, it seems to have been a new discovery in the 16th century, if we may judge from a passage in the letter of Hartmann above alluded to. He was certainly aware of the existence of magnetic repulsion in some form or other. It is somewhat difficult to gather from his description what it was exactly that he observed, and he nowhere states the law fully and explicitly. In Norman's *Neue Attractiv*¹ we find it clearly stated, and demonstrated by means of a needle floating on water or suspended by a thread;² yet he does not appear to claim the fact as his discovery. If, therefore, Hartmann was not the actual discoverer, we may at least conclude that the law became familiar to magnetic philosophers during the thirty years that separated him from Norman.

Mapping out the magnetic field. *The Magnetic Field.*—We next introduce a method of conceiving and describing magnetic actions which was invented and much used by Faraday. Since a magnet acts upon a magnetic needle placed anywhere in the surrounding space,³ we call that space the magnetic field of the magnet. Neglecting the earth's magnetism, we may map out this field as follows. Conceive any plane drawn through the axis of the magnet, and place it so that this plane shall be horizontal. Then at any point in this plane place a very small magnetic needle, and note the direction which its axis assumes under the action of the magnet; then proceed to move the centre of the needle in the direction in which its north pole points, and continue the motion so that at each point the centre is following the direction indicated by the north pole. The line thus traced will at last cut the surface of the magnet at some point lying towards its south pole; and if we continue the line backwards, by following the direction continually indicated by the south pole of the needle, it will cut the surface of the magnet at some point lying towards the north pole. Such a line is called a line of magnetic force; and, since one such line can be drawn through every point of the plane, and any number of planes can be taken through the axis of the magnet, we can conceive the whole magnetic field filled with such lines. Fig. 4, taken from Faraday, gives an idea of the distribution of the lines of force in the field of a bar magnet; fig. 5 represents the lines in the field due to two neighbouring like poles.

These diagrams were not obtained by the method we have just described, but by a much simpler process which we shall describe by and by. Their use, so far as we have gone, is to tell us how a small needle, free to move about its centre in any direction, will place itself at any part of the field, viz., it will place its axis along the tangent to the line of force which passes through its centre, its north pole pointing in that direction which ultimately leads to the south pole of the magnet producing the field.

Suppose we apply these ideas to a spherical magnet (a terrella, or earthkin, as Gilbert calls it). The lines of force

in any plane through its axis would be found to run something like the curves in fig. 6. If, therefore, we carried a small needle (suspended from a silk fibre so as to be perfectly free to move in all directions) round the magnet

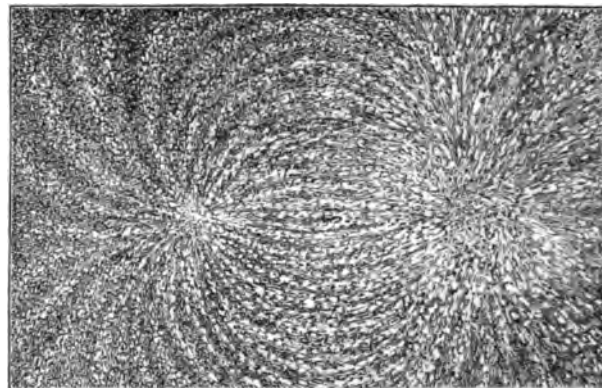


Fig. 4.

in a meridian plane, its axis would constantly remain in the meridian plane, its north pole always point towards the south pole of the spherical magnet, but dip more and more

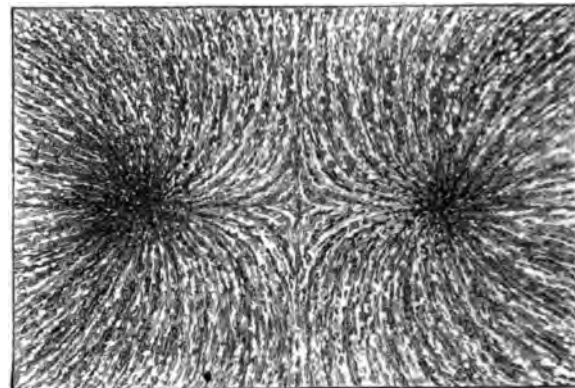


Fig. 5.

below the tangent plane to the sphere as the centre recedes from the equator, and end by pointing straight towards the south pole when the centre reaches the magnetic axis (see fig. 6).

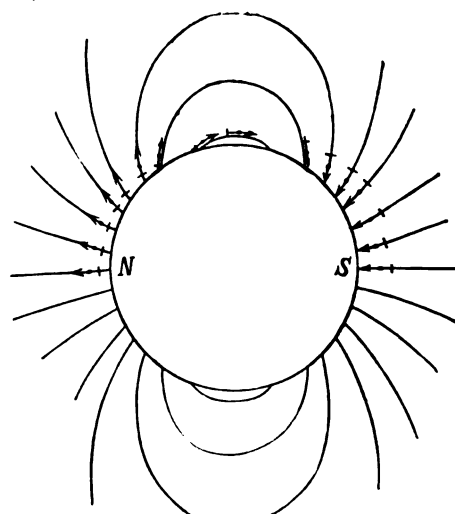


Fig. 6.

When we reflect that in all our experiments the properties of magnets, whether native, such as the loadstone, or artificial, such as the needles magnetized by rubbing with the loadstone, have proved alike, and that every

¹ Chap. i.

² See also Gilbert, *De Magnete*, lib. i. cap. v.

³ Gilbert uses the phrase *orbis virtutis* in a somewhat similar sense.

purely magnetic action on a magnet has its source in some other magnetic body, we are naturally led to the conclusion that the reason why at every point of the earth's surface the axis of a freely suspended magnet assumes a definite position is simply that the earth itself is a great magnet, and that in observing the declination and dip we are simply exploring the magnetic field of the earth. It is true that, according to the experiment above described, the declination would every where be zero, and the magnetic equator would coincide with the geographical, but that arises merely because we assumed our earthkin, for simplicity of explanation, to be symmetrically magnetized, so that its lines of force ran in planes passing through its axis. It remains to be discussed whether the most general assumption, viz., that the earth is a magnetic body, will not account for the facts of terrestrial magnetism. The answer to this question has been given, as we shall see, by Gauss.

This idea, whose simplicity is the truest measure of its greatness, is due to Gilbert, and was by him made the foundation of his work on magnetism. The boldness of his theory will be appreciated when we remind the reader that in his day the dip was but newly discovered, and had been measured only at London, so that Gilbert's very full and clear exposition of this phenomenon, which we have given above, was in fact a scientific prediction, which was not fully verified till long afterwards.¹ Before Gilbert a variety of wild conjectures had been made as to the cause of the directive property of the magnet.² Many, like Columbus, Cardan, and Paracelsus, believed that the magnet was attracted by a point in the heavens, possibly some magnetic star. Others supposed that the attracting point was situated in the earth; Fracastorius imagined hyperborean mountains of loadstone situated near but not quite at the north pole; and to this theory others contributed the detail that the magnetism of these mountains was so powerful that ships in these regions have to be built with wooden nails instead of iron ones, which would be instantly drawn out by the magnetic attraction.

It is clear that, if we call that magnetic pole of the earth which lies in the northern hemisphere its north pole, we ought, in accordance with our fundamental law of magnetic action, to call the north-seeking pole of an ordinary magnet a *south* pole. When it is necessary to speak of magnets from this point of view, the difficulty is got over by calling the north-seeking pole the austral pole, and the south-seeking pole the boreal pole. In reality the danger of confusion is more imaginary than real. The reader should be warned, however, that in some French works the ordinary nomenclature is reversed, and that Faraday uses "marked" and "unmarked," and Airy "red" and "blue," in the sense in which north and south are commonly used.

The Earth's Action on a Magnet is a Couple.—Norman in his *Newe Attractive* (chapters v. and vi.) discusses very acutely the question whether there is any force of translation exerted upon a magnet. He advances three conclusive experiments to prove the negative. First, he weighed several small pieces of steel in a delicate gold balance, and then magnetized them, but could not detect the slightest alteration in their weight, "though every one of them had received virtue sufficient to lift up his fellow." Secondly, he pushed a steel wire through a spherical piece of cork, and carefully pared the latter so that the whole sank to a certain depth in a vessel of water and remained there, taking up any position about the centre indifferently. After the wire was magnetized very carefully, without disturbing its position in the cork, it sank to the same

depth as before, neither more nor less, the only difference being that now the wire set itself persistently in a definite fixed direction parallel to the magnetic meridian, the north end dipping about 71° or 72° below the horizon. Thirdly, he arranged a magnetized needle on a cork so as to float on the surface of water, and found that, although it set in the magnetic meridian, there was not the slightest tendency to translation in any direction.³ He concludes that there is no force of translation on the magnet, either vertical or horizontal. He was evidently somewhat puzzled how to put this result into a positive form, and his "point respective," as he calls it, is not a very clear explanation of the earth's action. What he wanted was the modern idea of a "couple," i.e., a pair of equal but oppositely directed parallel forces acting on the two ends of the needle; but such an idea was not conceived in Norman's day. Gilbert adopts Norman's result in this matter, adding nothing essential, reproducing even Norman's diagram of the spherical cork with the wire through it. It is clear therefore that Gilbert had a forerunner in the practice, as Bacon had in the theory, of inductive science; for Norman says, speaking of the mass of fables that had passed for truth in geography, hydrography, and navigation before his time, "I wish experience to be the leader of Writers in those Artes, and reason their rule in setting it downe, that the followers bee not led by them into errors, as oftentimes have beene seene."

The Magnetic Property is Molecular.—Apart altogether from the question as to how we are to represent the action of a magnet upon other magnets, there arises another quite distinct question, as to where the cause of this action resides. That these two questions are really distinct, although there has always been a tendency in the more superficial treatises on the subject to confuse them, will be obvious from the fact that we shall afterwards obtain more than one perfectly general way of representing the action of a magnet at external points, whereas there must be one and only one cause of this action. A very old experiment⁴ at once throws considerable light on this point. If we break a bar magnet into two

pieces, it will be found that each of these is itself a magnet, its axis being in much the same direction as that of the original magnet, and its poles in corresponding positions, see fig. 7. The same holds if we break the bar into any number of pieces; and, quite generally, if we remove any piece, however small, from a magnet, this piece will be found to be magnetic, the direction of its axis usually bearing a distinct and easily recognizable relation to the direction of the axis of the whole magnet. We are therefore driven to the conclusion that the magnetic quality of a body is related to its ultimate structure, and not simply to its mass as a whole, or to its surface alone; and this conclusion is not to be invalidated by the fact that we can in general, as will afterwards appear, represent the action of the magnet at external points by means of a proper distribution of centres of attractive and repulsive forces upon its surface merely.

Temporary Magnetism of Soft Iron and Steel in the Magnetic Field.—Bodies which possess permanent magnetic

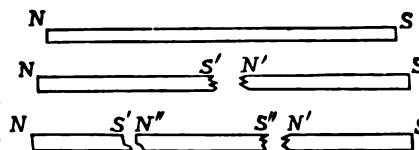


Fig. 7.

¹ The first verification was by Hudson, who, in 1608, found the dip in $75^\circ 22'$ N. lat. to be $89^\circ 30'$. Gilbert found 72° at London in 1600. The place of vertical dip in the northern hemisphere was first reached by Sir James Ross in 1831. It was found about $70^\circ 5' 17''$ N. lat. and $96^\circ 45' 48''$ W. long.

² See Gilbert, *De Magnete*, lib. i. cap. i.

³ Hartmann (see his letter above cited) was in error on this subject. He describes a somewhat similar experiment, and distinctly states that the needle has a motion of translation. "Schwimmt mit dem Ort welcher ist mitternächtlich am Stein, bis er kam an den Port der Schlüssel, da das Wasser in war."

⁴ Cf. Gilbert, *De Magnete*, lib. i. cap. v.

Magnetic induction. properties, not depending on the circumstances in which they are placed, we shall henceforth call "permanent magnets." The law of the action of one permanent magnet upon another, as we have seen, is that like poles repel and unlike poles attract each other. The action of a permanent magnet on pieces of soft iron is, at first sight, different, for either pole attracts them alike.

Experiments illustrating induction. To fix our ideas let us take a small thin bar of soft iron or of steel, and test it with a delicate magnetic needle. It will usually be found, more particularly if a steel bar is taken, that one end of the bar will *repel* one or other of the poles of the needle. This is a sure sign of permanent magnetism. If, however, we heat the bar to whiteness and allow it to cool in a position perpendicular to the earth's magnetic force, all permanent magnetism will be found to have disappeared. If we now place the bar in a horizontal plane (fig. 8) with its axis perpendicular to the axis of the needle, and one of its ends A, B near either pole of the needle, that pole will be attracted, no matter whether it be the north pole or the south pole of the needle, or which end of the bar be used.

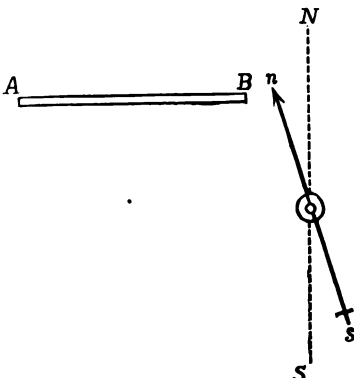


Fig. 8.

Care must be taken in this experiment to avoid using a too strongly magnetized needle, and to keep the needle from touching the bar, otherwise the bar may receive traces of permanent magnetism which will disturb the result. It is very easy, by repeating the above experiment with an unmagnetized needle, to show that the power that the bar acquires of attracting the poles of the needle is temporary and depends on the presence of a magnetized body.

Keeping to our principle that a magnetic cause is to be sought for every magnetic action, we are led to explain the above experiment by saying that in the magnetic field a bar of soft iron or of unmagnetized steel becomes magnetic in such a way that its north pole points as nearly as may be in the positive direction of the lines of force passing in its neighbourhood (or, in other words, in the direction, as nearly as may be, in which a magnetic needle would point if placed in its neighbourhood). A body which becomes magnetic in this way by the magnetic action of another body is said to be "magnetized" by "induction." We shall suppose, in the meantime, that it loses all the magnetism thus acquired when the inducing action is withdrawn; although this is not necessarily, and in fact not generally, the case, as we shall see by and by. The reason why soft iron is attracted by a permanent magnet is therefore now said to be that the iron becomes magnetic by induction, and is then acted upon by the magnet like any other magnet similarly placed. The accuracy of this analysis of the phenomenon may be confirmed by many simple but striking experiments, such as the following.

In the experiment above described, instead of placing the non-magnetic bar in a horizontal plane, place it in the plane of the magnetic meridian with its axis in the direction of the earth's force (*i.e.*, parallel to the line of dip). The lower end of the bar will then be found to repel and the upper end to attract the north pole of the needle (figs. 9, 10). This is at once explained on the above hypothesis; for the bar will be magnetized inductively by the earth's force, so that its lower end becomes a north pole, and its upper end a south pole.

Let NS (fig. 11) be a bar magnet placed horizontally so that its axis produced passes through O, the centre of suspension of the needle *sn*, then the needle will be deflected

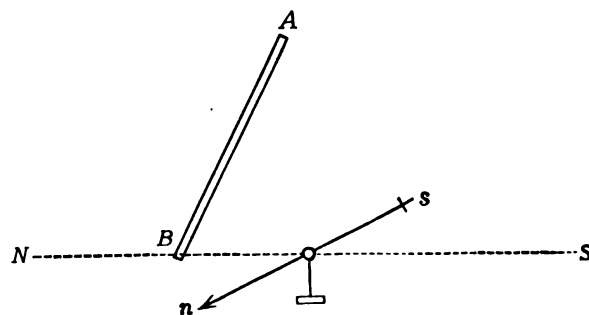


Fig. 9.

in the direction of the arrow. If now we place between S and O a small sphere of soft iron, this deflexion will be

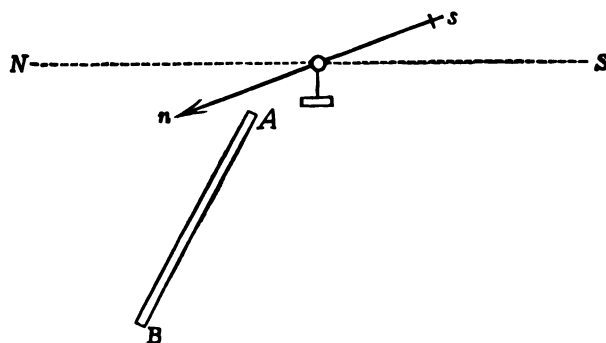


Fig. 10.

increased, the reason being that the sphere is magnetized by induction, having a south pole towards O and a north pole

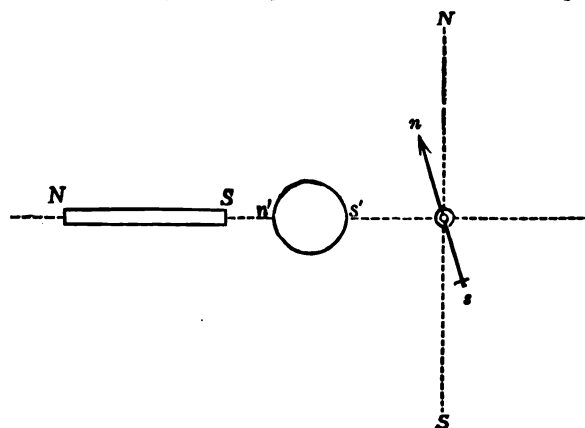


Fig. 11.

towards S, and the action of these is added to that of NS.

Let NS (fig. 12) be a magnet placed in the magnetic meridian, *n's'* a small magnetic needle in the same horizontal plane, with its centre in the line bisecting NS at right angles. When acted on by NS alone, *n's'* will place itself parallel to NS, with its north pole pointing S in the direction NS

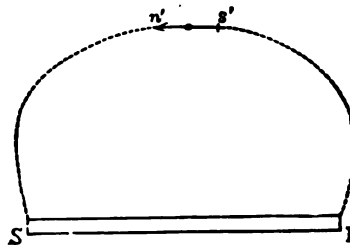


Fig. 12.

Let the dotted line represent a line of force. If we move a small piece of soft iron *ns* along this line in the direction from N towards S, it will first deflect the needle as

in fig. 13, and finally as in fig. 14, and in each position reversing it end for end will not alter the effect. All this is at once explained by the above hypothesis.

A variation of the last experiment may be made thus.

Place a magnet vertically, in the neighbourhood of a magnetic needle; by moving it up and down a position will be found in which the action of the magnet on the needle is wholly vertical, so that the needle is not deflected from the magnetic meridian.

Now take a small piece of soft iron and move it along a line of force passing near the needle, proceeding from the north to the south pole of the vertical magnet. It will then be found, in accordance with our hypothesis, that the north pole of the needle is first repelled, and finally attracted by the soft iron.

If we hang two short pieces of iron wire alongside of each other by parallel threads, they will be found to repel one another, and to hang separated by a considerable interval when a magnet is brought under them (see figs. 15 and 16). This experiment is due to Gilbert, who rightly explains it by saying that the two ends nearer the magnetic pole S become like poles of opposite kind to S, while the two farther ends are like poles of the same kind as S. The experiment may be varied by placing some little distance below the pole of a magnet S a piece of mica or thin cardboard M, and placing below that a short piece of soft iron wire; it will remain adhering to the mica, and so long as it is alone will hang more or less nearly vertical, but when another is placed alongside of it the two will diverge as in fig. 17.

One of the most interesting examples of magnetic induction is furnished by the action of a magnet on iron filings. If we plunge a magnet into a quantity of iron filings and then remove it, we find it thickly fringed around the poles, where the filings adhere to the magnet and to one another so as to form short bushy filaments; the thick-

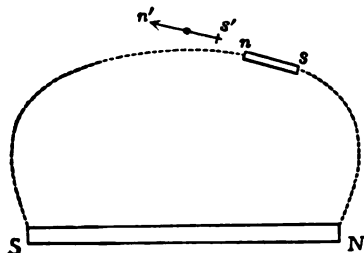


Fig. 13.

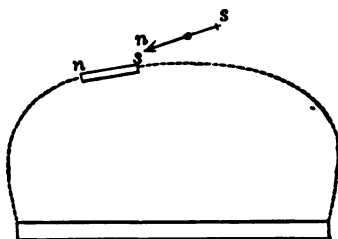


Fig. 14.

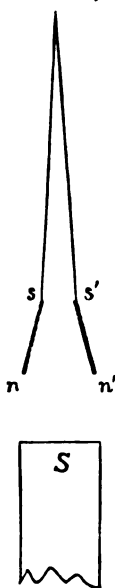


Fig. 15.

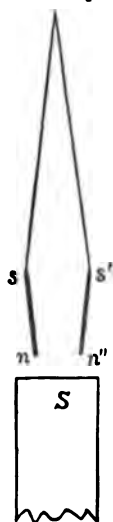


Fig. 16.

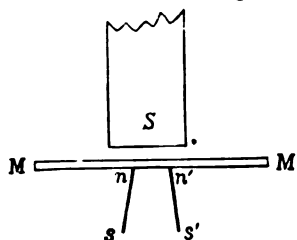


Fig. 17.

ness of the fringe diminishes very rapidly towards the middle of the magnet, where very few adhere at all. These filaments are composed of magnetized particles of iron adhering by their unlike poles.

If we place a small bar magnet under a piece of moderately rough drawing paper, strewn as uniformly as possible with fine iron filings, and then tap the paper very gently so as to relieve the friction, and allow each filing to follow the magnetic action, then the filings will be seen to arrange themselves in a series of lines, passing, roughly speaking, from pole to pole, as in fig. 4 (p. 222). The explanation of this phenomenon is simply that each filing becomes magnetized by induction, and, if it were quite free to move about its centre, it would not be in equilibrium until it set its longest dimension along the line of force through its centre. The roughness of the paper effectually prevents translation, but does not hinder rotation, especially when the friction is relieved by tapping; hence every filing does actually set as if it were a little magnetic needle, subject of course to some slight disturbance from the neighbouring filings. The whole therefore assumes a grained structure, and the graining runs in the direction of the lines of force. We have thus an extremely convenient way of representing these lines to the eye, which lends itself in a variety of ways to the illustration of magnetic phenomena. In fig. 5 are shown the lines formed in the field near two like magnetic poles. These magnetic figures may be fixed in a great variety of ways, and projected on a screen so as to be visible to a large audience, but it is scarcely necessary to dwell here upon details of this kind.

These magnetic curves seem to have fixed the attention of natural philosophers at a very early period. They were originally called the magnetic currents, from an idea that they represented the stream lines of magnetic matter, which explained the magnetic action according to the theory then in vogue. La Hire mentions them, *Mém. de l'Acad.*, 1717. Bazin gives an elaborate account of them in his *Description des Courans Magnétiques dessinés d'après Nature*, Strasbourg, 1753. Musschenbroek seems to have been the first to give the correct explanation depending on magnetic induction, *Diss. de Magnete*, 1729.

If the filings be laid very thickly on the paper, and one pole of the magnet be brought under them at a short distance off, they will arrange themselves in a pattern, and at the same time bristle up so as to stand more or less erect, according as they are nearer or farther from the magnet. They have thus the appearance of being repelled from the magnet. It was, in all probability, this phenomenon that was observed by Lucretius when he says (vi. 1042):—

“Exultare etiam Samothracia ferrea vidi,
Ac ramenta simul ferri furere intus ahenis
In scaphiis, lapis hic Magnes cum subditus esset.”

His conclusion, therefore, that iron sometimes flies and sometimes follows the magnet, was scarcely justified by his experimental facts, and it is a mistake to suppose, as some have done, that he was aware of the polarity of permanent magnets.

If we tap the card in the last experiment a curious Magnetic result may sometimes be observed.¹ The lines of filings will be seen to recede from the point of the card immediately over the pole of the magnet. If, however, the magnet be held over, instead of under, the card, tapping will cause the filings to approach the point under the pole of the magnet. The most probable explanation² of this is to be found in the fact that the erected filings stand in the

¹ Æpinus, *Tentamen Theoriæ Electricitatis et Magnetismi*, 1759; Cavallo, *Treatise on Magnetism*, 1787.

² Roget, *Library of Useful Knowledge*, 1832.

former case as shown in fig. 18, and in the latter as shown in fig. 19; that is, in both cases, owing to the action of gravity, they are more acutely inclined to the card than

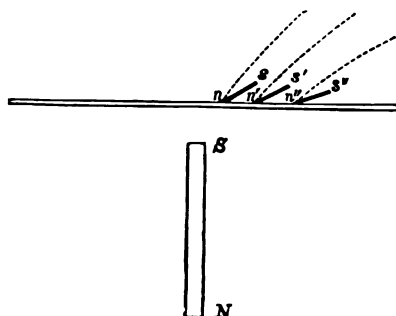


Fig. 18.

the lines of force (represented by dotted lines in the figure). Consequently, when the filing springs up into the air, and is thus free to follow the magnetic couple, it turns more

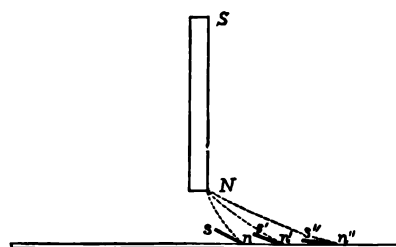


Fig. 19.

into the direction of the line of force; the effect of this is to carry its lower end each time a little farther from the axis of the magnet in the one case, and a little nearer to it in the other.

Electro-magnets. By far the most important case of magnetic induction is the electromagnet. Whenever an electric current flows in a closed circuit, the surrounding space becomes a field of magnetic force, and any piece of iron in it will be inductively magnetized. Such an arrangement of an electric circuit and iron is called an electromagnet. The variety of form and of application of such instruments in modern science is endless. A few of the more important modifications will be considered below.

Induced and permanent magnetism in the same body. *Co-existence of Induced and Permanent Magnetism.*—The fact that a body is already a permanent magnet does not prevent its being susceptible to magnetic induction. If we take any piece of iron at random, the chances are that one end or other of it will repel the north pole of a magnetic needle,—in other words, it will be to some extent permanently magnetic; but if we bring it slowly nearer and nearer to the pole of the needle, provided its magnetism be not too strong, it will by and by attract the pole which it at first repelled. Again, if we take two steel magnets, which may be as powerful as we please, provided at all events that they are unequally powerful, and bring two like poles together, these poles will at first repel each other in accordance with the fundamental law of permanent magnets; but, when the distance is less than a certain amount, the repulsion passes into an attraction, and when the poles are in contact this attraction may be very considerable. These phenomena are at once explained by the law of induction. The induced or temporary magnetism is superposed on the permanent magnetism, and, when the poles are near enough, the opposite magnetism induced by the pole attracts it more than the permanent like magnetism repels it; and this happens even with steel, whose susceptibility for magnetic induction is considerably less than that of iron. This phenomenon was observed pretty early in the history

of magnetism, but was not fully explained until the idea of magnetic induction was fully developed. Michell, in his *Treatise of Artificial Magnets*,¹ gives a tolerably clear account of it. Musschenbroek mentions it,² along with the fact that a magnet attracts iron more than it does another magnet, but offers no explanation of either fact. The latter result, so far as it is true, can of course be explained by the smaller susceptibility of steel, particularly of hard steel, to magnetic induction, which is the main factor in attraction at small distances. Poggendorff³ and others have experimented on the subject in later times. The reader should notice the close analogy between these phenomena and the repulsion and attraction at different distances between two similarly electrified conductors. See article ELECTRICITY, vol. viii. p. 33.

Induction of Permanent Magnetism.—The case above supposed, in which the induced magnetism is wholly temporary, although it can be easily realized with small magnetizing forces, is not the general one, but in fact the exception. Usually a certain proportion of the magnetism remains after the inducing force is removed. This happens even with the softest iron, when the inducing force is very great. Just as bodies differ very much in their susceptibility for induced magnetism, so they differ greatly in their power of retaining this magnetism when the inducing force ceases, or, as the phrase is, in “coercive force.” Thus, while the inductive susceptibility of steel is less than that of iron, it retains much more of the magnetism imparted to it, and is therefore said to have much greater coercive force; and the coercive force is greater the harder the steel is tempered.

It is obvious, therefore, that the principle of “induction,” along with the idea of “retaining power” or “coercive force,” furnishes us with the key to the explanation of the communication of permanent magnetism, whether by means of natural magnets or of artificial magnets, or of the electric current. In particular, we see at once the reason why the end of a needle which has been touched by the north pole of another magnet becomes a south pole, and *vice versa*,—a fact which greatly puzzled the earlier magnetic experimenters, and indeed all who were inclined to think that, in the process of magnetization, something was communicated from the one magnet to the other.

MATHEMATICAL THEORY OF THE ACTION OF PERMANENTLY MAGNETIZED BODIES.

In this section we shall suppose the bodies considered to be rigidly magnetized; i.e., we shall suppose that magnetic action exerted on any body produces no change in its magnetization. It is further to be observed that we are merely establishing a compendious representation of observed facts, and foreclosing nothing as to their physical theory or ultimate cause. Our method is therefore to some extent tentative, and its success is to be judged by the agreement of the results with experiment.

There are two main facts to be borne in mind:—(1) that a magnet is polarized, and (2) that the properties of its smallest parts are similar to those of the whole. Adopting the mathematical fiction of action at a distance, we may represent the action of such a body by a proper distribution of imaginary *positive* and *negative* attracting matter through-^{Positive and negative magnetism} out its mass. This imaginary matter, following Sir W. Thomson, we shall call “magnetism,” as we thus avoid suggesting other properties of matter than attraction, of which in the present case experience has given no evidence. We assume that *magnetism of any sign repels magnetism*

¹ Cambridge, 1750.

² *Philosophia Naturalis*, §§ 953, 954, 1762.

³ *Pogg. Ann.*, xlv. p. 375, 1838.

of the same sign and attracts magnetism of the opposite sign. Magnetism is supposed to be so associated with the matter of the body that magnetic force exerted on the magnetism is ponderomotive force exerted on the matter. On the other hand, magnetic force is always supposed to be exerted by magnetism upon magnetism, and never directly by or upon matter. Into the nature of this association of magnetism with matter there is no pretence, indeed no need, to enter.

The elementary law of action assumed is that the attraction or repulsion (as the case may be) between two quantities m and m' of magnetism supposed concentrated in two points at a distance r apart is $\frac{mm'}{r^2}$, and is in the line joining the two

points. This supposes that the unit quantity of magnetism is so chosen that two units of positive magnetism at unit distance apart repel each other with unit force. This definition, which is fundamental in the electromagnetic system of units, gives for the dimensions of a quantity of magnetism $[L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}]$. If the electrostatic system be adopted the result would of course be different.

An accurate meaning can now be given to the phrase "strength of a magnetic field," or its equivalent "resultant magnetic force at a point in the field;" it is defined to be the force exerted upon a unit of positive magnetism supposed concentrated at the point. The force exerted on a unit of negative magnetism would of course be equal in magnitude, but oppositely directed; and in general, if R denote the resultant magnetic force at the point, the magnetic force exerted on a quantity κ of magnetism concentrated there is κR .

We may, as in the corresponding theory of electricity, introduce the ideas of volume density (ρ) and surface density (σ),—so that ρdv and σdS denote the quantities of magnetism in an element of volume and on an element of surface respectively; ρ and σ may of course be positive or negative according to circumstances.

It will now be seen that, mathematically speaking, the theories of action at a distance for electricity and magnetism are identical, and every conclusion drawn will have, so far as the physical diversity of the two cases may allow, a double application.¹ In particular it will be found that the theory of magnetism, when properly interpreted, gives the theory of dielectrics polarized in the way imagined by Faraday.

The fact of magnetic polarity requires the conception of negative as well as positive magnetism; the fact that the properties of the smallest parts of a magnet are similar to those of the whole requires that in every element of the body there shall be both negative and positive magnetism. From the fact that in a uniform field, i.e., one in which the resultant magnetic force has at every point the same magnitude and direction, the force of translation upon a magnet is nil, it follows that the algebraic sum of all the magnetism in any magnet must be zero; for, if R denote the strength of the field, by the theory of parallel forces the whole force on the magnet will be $\sum(\kappa R) = R\sum\kappa$; hence $\sum\kappa = 0$. In other words, in every magnet there must be as much negative as positive magnetism; and this conclusion also must be extended to the smallest parts of every magnet, so long as we do not go behind the mere facts of observation. The positive and negative magnetism cannot be coincident throughout, otherwise there would be no external magnetic action, but the separation is in the elements of the body. Thus, although there is no force of translation in a uniform field, there will in general be a couple. Consider the positive and negative magnetism

separately, and let κ denote any element of the former and κ' any element of the latter. Let N be the centre of mass of the positive, S the centre of mass of the negative magnetism; so that, if the magnet be referred to a set of rectangular axes, the coordinates of N and S are

$$\left. \begin{aligned} \frac{\sum \kappa x}{\sum \kappa}, \quad \frac{\sum \kappa y}{\sum \kappa}, \quad \frac{\sum \kappa z}{\sum \kappa} \\ \frac{\sum \kappa' x'}{\sum \kappa'}, \quad \frac{\sum \kappa' y'}{\sum \kappa'}, \quad \frac{\sum \kappa' z'}{\sum \kappa'} \end{aligned} \right\} \dots \dots (1).$$

Let the distance $NS = l$, and let $K = l\sum\kappa = -l\sum\kappa'$; this Magnetic quantity K is called the "magnetic moment." By the theory of parallel forces, if we suspend the magnet in a uniform field of strength R , the action upon it reduces to two forces $R\sum\kappa$ and $-R\sum\kappa'$, each parallel to the direction of the field, acting respectively at N and S , in other words to a couple Magnetic whose moment is $R\sum\kappa l \sin \chi$ or $KR \sin \chi$, where χ is the angle between SN and the direction of the field. Hence, if the magnet be perfectly free to follow the magnetic action of the field, it will set so that the line SN or the line NS is parallel to the direction of the field, the equilibrium being stable in the former case, but unstable in the latter. The line NS is therefore parallel to what we have already defined on experimental grounds as the axial direction in the magnet. N , S , and NS are sometimes called *poles* and the axis of the magnet; we have adopted the looser definition given above because it is more convenient and nearer the popular usage.

The above results may be applied to some cases very important in Theory practice. Let the magnet whose centres of positive and negative of magnetism are N and S be suspended by the middle point of NS , dipping which, for simplicity, may be assumed to be also its centre of gravity. Let OX (fig. 20) be a horizontal line drawn northwards, OZ

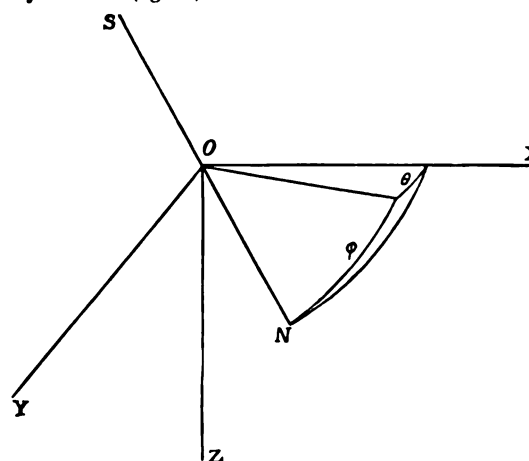


Fig. 20.

a vertical drawn downwards, both in the magnetic meridian. Let the vertical plane through NS make an angle θ with the magnetic meridian, and let ON make an angle ϕ with the horizon. If R be the strength of the earth's magnetic field, and i the angle of dip, then the horizontal and vertical components of the earth's force are $H = R \cos i$ and $Z = R \sin i$.

First, suppose the angle ϕ fixed, and the magnet free to rotate about OZ only; then the couple tending to diminish the angle θ is $2 \cdot \sum \kappa \cdot R \cos i \cdot \frac{l}{2} \cos \phi \sin \theta$, or $KR \cos i \cdot \cos \phi \sin \theta$.

In other words the directive couple varies as the sine of the angle of deviation from the magnetic meridian. This conclusion was verified experimentally by Lambert, and also by Coulomb² by means of his torsion balance. It will be seen that, *ceteris paribus*, the directive couple is greatest when the magnetic axis is horizontal.

¹ To prevent needless repetition, we shall adopt henceforth, without further explanation, the definitions, terminology, and results given in the article ELECTRICITY, vol. viii. p. 24 sq.

² Mém. de l'Acad., 1785.

Next suppose the angle θ fixed, and the magnet free to rotate about a horizontal axis inclined at an angle $90^\circ - \theta$ to OX. The couple tending to diminish the angle ϕ is $KR(\cos \phi \cos \theta \sin \phi - \sin \phi \cos \phi)$. The position of equilibrium is given by the equation $\tan \phi = \sec \theta \tan \iota$.

The angle at which the axis is depressed below the horizon is therefore least when $\theta = 0$, and greatest when $\theta = 90^\circ$, its value being ι in the former case, and 90° in the latter, as stated above, p. 221.

In general, if λ', μ', ν' and λ, μ, ν be the direction cosines of the direction of the field and of the axis of the magnet respectively, then, resolving the forces acting at N and S, we see at once that the three components of the magnetic couple are

$$KR(\nu'\mu - \mu'\nu), KR(\lambda'\nu - \nu'\lambda), KR(\mu'\lambda - \lambda'\mu) \quad (2).$$

Magnetic
moment
resolved
as a
vector.

These are clearly the same as the components of the couple on a system of three magnets whose axes are parallel to OX, OY, OZ, and whose magnetic moments are $K\lambda, K\mu, K\nu$. Hence, so far as the action of a uniform field is concerned, we may resolve the magnetic moment like a vector, and replace a given magnet by others the resultant of whose moments is the moment of the given magnet.

It appears therefore that in a uniform field every magnet behaves as if it were made up of a certain quantity of positive magnetism and an equal quantity of negative magnetism placed at such a distance apart on a line parallel to the magnetic axis that the product of the quantity of magnetism into that distance has a value equal to the magnetic moment of the magnet. It is very important to observe that the magnetic moment alone appears in the above formulæ for the magnetic action. We cannot therefore separately determine from observations in a uniform field either the quantity of positive or negative magnetism in a magnet or the distance between the magnetic centres of mass.

Finite
magnet
replaced
by an
infinite
number
of in-
finitely
small
magnets.

Let M be any magnet, and P a point whose distance from any point of M is infinitely great compared with the linear dimensions of M. Then, since all the lines drawn from P to different points of M are sensibly parallel and equal in length, we may suppose the positive and negative magnetism of M to be collected at their mass centres, i.e., M to be replaced by an ideal magnet. It is also obvious that, throughout a region around P whose linear dimensions are of the same order as those of M, the field due to M may be regarded as uniform. Hence we conclude that in calculating the mutual action of two magnets M and M' we may replace each of them by an ideal magnet, provided the distance between them be infinitely great compared with the linear dimensions of either. This condition may be satisfied either by making the distance between the magnets very great if their dimensions be finite, or by making their dimensions infinitely small if the distance between them be finite. The second alternative suggests at once a method for representing the magnetic action of magnetized bodies at finite distances. We may divide up the body into portions whose linear dimensions are infinitely small compared with their distance from any point at which their action is to be considered; each of these portions is itself a magnet, and may be replaced by an ideal magnet having the same axis and moment. The whole magnetic action is obtained by integrating the action of all the ideal magnets of which the body is thus supposed to be composed.

Intensity
of
magnet-
ization.

Let λ, μ, ν be the direction cosines of the magnetic axis of any element dv of a magnet, and I such that $I dv$ is the magnetic moment of the element, and let $I\lambda = A, I\mu = B, I\nu = C$; then I is called the "intensity of magnetization" at the point where the element is taken. I may be regarded as a vector which specifies the magnetization of the body; in general it varies continuously from point to point; if it has the same value and direction at every point, the body is said to be uniformly magnetized. A line drawn so that the direction of I at every point of it is tangential to it is

called a "line of magnetization." It is clear from what has already been shown that we may if we choose replace the element dv by three ideal magnets whose axes are parallel to the coordinate axes, and whose moments are Adv, Bdv, Cdv respectively.

If then K be the magnetic moment of the whole magnet, δK the magnetic moment of any element δv , and p, q, r the direction cosines of the magnetic axis of the whole magnet, we have $K = \int \delta K \lambda, - \int \delta K \mu, - \int \delta K \nu$; and, remembering that $\kappa' = -\kappa$ for every element, and

$$p = \left(\frac{\sum \kappa \lambda}{\sum \kappa} \right) \frac{1}{l} = \frac{\sum (\kappa \lambda \cdot \frac{x-x'}{l})}{\sum \kappa} = \frac{\sum (\delta K \lambda)}{K} = \frac{\sum I \lambda \delta v}{K} = \frac{\sum A \delta v}{K}.$$

We may therefore write, replacing summation by integration,

$$Kp = \iiint Adv, Kq = \iiint Bdv, Kq = \iiint Cdv \quad (3).$$

Let SN be an ideal magnet of infinitely small length l , let m be its positive magnetic moment, and $m = \kappa l$. Let Q be its middle point, and the angle PQN = θ , N being the positive or north-seeking pole; and let QP = D . Then the potential at P due to this magnet is

$$\kappa \left\{ D^2 - D^2 \cos \theta + \frac{1}{2} l^2 \right\}^{-\frac{1}{2}} - \kappa \left\{ D^2 + D^2 \cos \theta + \frac{1}{2} l^2 \right\}^{-\frac{1}{2}}.$$

Expanding and neglecting powers of $\frac{l}{D}$ above the first, we get for the potential

$$\frac{m \cos \theta}{D^2} \quad (4).$$

Hence the potential at P (ξ, η, ζ) of an infinitely small magnet Pota Adv at (x, y, z), having its axis parallel to the axis of x , is of the form $A(\xi - x)/D^2$, and similarly for the other two. We therefore obtain mag

$$V = \iiint \left\{ A(\xi - x) + B(\eta - y) + C(\zeta - z) \right\} \frac{1}{D^3} dv \\ - \iiint \left\{ A \frac{d\left(\frac{1}{D}\right)}{dx} + B \frac{d\left(\frac{1}{D}\right)}{dy} + C \frac{d\left(\frac{1}{D}\right)}{dz} \right\} dv \quad (5) \\ - \iiint I \left\{ \lambda \frac{d}{dx} + \mu \frac{d}{dy} + \nu \frac{d}{dz} \right\} \frac{1}{D} dv$$

Taking the second of these expressions and integrating by parts in the usual way, we get

$$V = \iint \frac{\sigma}{D} dS + \iiint \frac{\rho}{D} dv; \quad (6) \\ \text{where } \sigma = I A + m B + n C - I \cos \theta \\ \rho = - \left(\lambda \frac{dA}{dx} + \mu \frac{dB}{dy} + \nu \frac{dC}{dz} \right)$$

λ, m, n being the direction cosines of the outward normal to any element dS of the surface of the magnet, and θ the angle between the normal and the direction of magnetization at dS .

Hence the action of any magnet may be represented by Poisson's means of a certain volume distribution (ρ) and a certain surface distribution (σ) of free magnetism. This important proposition is due to Poisson.¹

The fact, in itself obvious, that the sum of all the magnetism of Poisson's distribution must be zero, gives the theorem

$$\iiint \left(\lambda \frac{dA}{dx} + \mu \frac{dB}{dy} + \nu \frac{dC}{dz} \right) dv = \iint (I A + m B + n C) dS \\ - \iint I \cos \theta dS \quad (7),$$

which admits of course of direct analytical proof.

The magnet may also be replaced, so far as its external action is concerned, by a distribution wholly on its surface, as was shown by Gauss.² This will be seen at once if we replace the positive and negative magnetism throughout the body by positive and negative electricity, and suppose the surface of the magnet covered with a conducting layer in connexion with the earth. The surface will thus become charged with a distribution of positive and negative electricity whose total sum is zero, such that the potential of the surface is zero, and hence the potential at every external point zero. The potential of this surface layer

¹ *Mém. de l'Institut*, tom. v., 1821.

² *Intensitas Vis*, § 2 (1832), and *Allgemeine Lehrsätze*, § 36 (1839).

at every point external to the body is therefore equal and opposite to that of the internal electricity. If, therefore, we change the sign of the surface density at every point, we obtain a surface distribution whose potential at every external point is the same as that of the body. There is of course only one such distribution: we may call it Gauss's distribution.

Poisson's distribution will coincide with that of Gauss provided the magnetization be such that

$$\frac{dA}{dx} + \frac{dB}{dy} + \frac{dC}{dz} = 0 \quad (8);$$

when this condition is satisfied at every point of the body, A is said to be "solenoidally" magnetized; a particular case is that of uniform magnetization.

So long as the point considered is external to the magnet there is no difficulty in attaching a definite meaning to the resultant magnetic force (\mathfrak{H}) at a point; its components are given by

$$\alpha = -\frac{dV}{dx}, \quad \beta = -\frac{dV}{dy}, \quad \gamma = -\frac{dV}{dz} \quad (9);$$

and the values obtained will be the same whether V be calculated by means of Poisson's or of Gauss's distribution. Inside the body the result is otherwise, for reasons that are not difficult to understand, when we examine the nature of our fundamental assumptions. It is therefore necessary to be careful to define what we mean by resultant magnetic force in the interior of a magnet. It is defined by the above equation (9) on the understanding that V is calculated from Poisson's distribution. We can show that \mathfrak{H} thus defined is the resultant force in an infinitely small cylindrical cavity within the magnet, whose axis is parallel to the line of magnetization, and whose radius a is infinitely small compared with its axis $2b$.

The removal of the matter filling such a cavity will affect Poisson's volume distribution to an infinitely small extent; the alteration of the force if any, will therefore arise simply from the surface distribution which we must place on the walls of the cavity in order to make up the complete representation of the action of the magnet in the cavity. This distribution reduces to two circular disks of radius a at the two ends, the densities of the magnetism on which are $-I$ and $+I$ respectively. The action due to these is a force $4\pi I(1 - b/\sqrt{a^2 + b^2})$ in the direction of magnetization. If a be infinitely small compared with b , this force becomes zero, which proves our proposition.

If, on the other hand, the cavity in the magnet be disk-shaped—say a narrow crevasse perpendicular to the line of magnetization—then the force due to the distribution on its walls becomes $4\pi I$, and the resultant force in the cavity is no longer \mathfrak{H} , but a force \mathfrak{J} , whose components are

$$\alpha = \alpha + 4\pi A, \quad \beta = \beta + 4\pi B, \quad \gamma = \gamma + 4\pi C \quad (10).$$

\mathfrak{J} is called the "magnetic induction" at the point (x, y, z) .

From the definition of \mathfrak{J} it follows that outside the magnet

$$\frac{d\alpha}{dx} + \frac{d\beta}{dy} + \frac{d\gamma}{dz} = 0 \quad (11).$$

Inside

$$\left. \begin{aligned} \frac{d\alpha}{dx} + \frac{d\beta}{dy} + \frac{d\gamma}{dz} &= 4\pi\rho \\ &= -4\pi \left(\frac{dA}{dx} + \frac{dB}{dy} + \frac{dC}{dz} \right) \end{aligned} \right\} \quad (12).$$

At the surface of a magnetized body the tangential component of \mathfrak{H} is continuous, but the normal component increases abruptly by $4\pi I \cos \theta$ in passing from the inside to the outside of the surface.

Outside magnetized matter the magnetic force and the magnetic induction are coincident. Inside we have

$$\frac{d\alpha}{dx} + \frac{d\beta}{dy} + \frac{d\gamma}{dz} = \frac{d\alpha}{dx} + \frac{d\beta}{dy} + \frac{d\gamma}{dz} + 4\pi \left(\frac{dA}{dx} + \frac{dB}{dy} + \frac{dC}{dz} \right) = 0. \quad (13).$$

Hence the magnetic induction satisfies the solenoidal condition both inside and outside magnetized matter. It has normal continuity, and, in general, tangential discontinuity, at the surface of a magnetized body.

For if ν , τ and n , t be the normal and tangential components of \mathfrak{H} and \mathfrak{J} just inside, and ν' , τ' and n' , t' the corresponding components just outside the surface near any point, we have $n = \nu + 4\pi I \cos \theta$, and $n' = \nu'$; but $\nu' = \nu + 4\pi I \cos \theta$, therefore $n = n'$. On the other hand $\tau' = \tau$, whereas τ is the resultant of t and $4\pi I \sin \theta$, which is parallel to the surface, but otherwise may have any direction according to circumstances; hence, since $t' = t$, in general τ' is not equal to τ .

In fact there will be tangential discontinuity of the magnetic induction unless the line of magnetization be perpendicular to the surface of the magnet; in this case there is complete continuity of the magnetic induction. When the magnetization at the surface is tangential, there is, on the other hand, complete continuity of the magnetic force.

It follows from the above that the surface integral of the magnetic induction taken over any closed surface S vanishes.

First, let the surface be wholly within or wholly without continuously magnetized matter. We have, integrating all over S and all over the space enclosed by S, the analytical theorem

$$\iint (la + mb + nc) dS = \iiint \left(\frac{da}{dx} + \frac{db}{dy} + \frac{dc}{dz} \right) dv \quad (14);$$

hence the result follows, for every element of the right-hand integral vanishes. Next, suppose S to be partly within and partly without a magnetized body. Divide it into two parts by a double partition one of whose walls runs outside the surface of the body and infinitely near it, the other inside and infinitely near it; then, on account of the normal continuity of \mathfrak{J} , the surface integral will be the same in absolute value over each of these walls. Hence the integral over the whole of S differs infinitely little from the sum of the integrals over the two surfaces into which it is broken up by the double partition, each of which vanishes by the former case. Hence the theorem holds in this case also.

We may therefore apply to lines and tubes of magnetic induction without restriction all the theorems proved for lines and tubes of electric force in space free from electrified bodies. We may speak of the number of lines of magnetic induction instead of the surface integral if we choose. And we have this important theorem:—

The number of lines of magnetic induction that pass through an unclosed surface depends merely on its boundary.

There must therefore be a vector \mathfrak{A} , whose line integral round the boundary is equal to the surface integral of \mathfrak{J} over the surface.

The components F, G, H of \mathfrak{A} are connected with those of \mathfrak{J} by the equations

$$a = \frac{dH}{dy} - \frac{dG}{dz}, \quad b = \frac{dF}{dz} - \frac{dH}{dx}, \quad c = \frac{dG}{dx} - \frac{dF}{dy} \quad (15),$$

as has been shown in the article ELECTRICITY, vol. viii. p. 69.

Mutual potential energy and mutual action of two magnetic systems.—The potential energy of a small magnet is $\kappa(V_2 - V_1)$, where V_1 and V_2 are the values of V at its negative and positive poles. If the magnet be infinitely small, of length ds say, the direction of two cosines of ds being λ, μ, ν , this may be written $\kappa ds dV/ds$, i.e., mdV/ds , or, if we are considering a magnetized element of volume dv ,

$$I \left(\lambda \frac{dV}{dx} + \mu \frac{dV}{dy} + \nu \frac{dV}{dz} \right) dv \quad (16).$$

Hence the potential energy of the whole magnetic system in a field whose potential is given by V is

$$\left. \begin{aligned} W &= \iiint \left(A \frac{dV}{dx} + B \frac{dV}{dy} + C \frac{dV}{dz} \right) dv \\ &= - \iiint (A\alpha + B\beta + C\gamma) dv \end{aligned} \right\} \quad (17),$$

the integration being extended all over the magnetized masses supposed to be acted upon. Integrating by parts, we get at once

$$W = \iint V \sigma dS + \iiint V \rho dv \quad (18),$$

σ and ρ being the surface and volume densities of Poisson's distribution, a result that might have been expected. W may also be expressed as a sextuple integral; for, if $\lambda', \lambda'', \mu', \mu'', \nu', \nu''$ refer to the acting system, then

$$V = \iiint \Gamma \left(\lambda' \frac{d}{dx} + \mu' \frac{d}{dy} + \nu' \frac{d}{dz} \right) \frac{1}{D} dx' dy' dz'.$$

Whence

$$W = \iiint dxdydz \left(\lambda \frac{d}{dx} + \mu \frac{d}{dy} + \nu \frac{d}{dz} \right) \left\{ \lambda \frac{d}{dx} + \mu \frac{d}{dy} + \nu \frac{d}{dz} \right\} \frac{1}{D} \quad (19).$$

A remarkable expression for W may be obtained by supposing the integration in (17) extended throughout the whole of space, on the understanding that A, B, C are zero where there is no magnetized matter, and then integrating by parts. We get, since the surface integral at infinity may be shown to vanish,

$$W = - \iiint \left(A \frac{dV}{dx} + B \frac{dV}{dy} + C \frac{dV}{dz} \right) dv - \iiint V \left(\frac{dA}{dx} + \frac{dB}{dy} + \frac{dC}{dz} \right) dv \quad (20),$$

where it must be understood that A, B, C vary continuously, however rapidly. In point of fact, where, as at the surface of a magnetized body, there is discontinuity, a finite portion of the integral will arise from an infinitely thin stratum near the surface. The proper representation of this part will be a surface integral, as may be seen by referring to (18), from which we might have started.

If now V' be the potential of the magnet acted upon, then

$$\frac{d^2 V'}{dx^2} + \frac{d^2 V'}{dy^2} + \frac{d^2 V'}{dz^2} = 4\pi \left(\frac{dA}{dx} + \frac{dB}{dy} + \frac{dC}{dz} \right);$$

whence

$$W = - \frac{1}{4\pi} \iiint V' \left(\frac{d^2 V'}{dx^2} + \frac{d^2 V'}{dy^2} + \frac{d^2 V'}{dz^2} \right) dv - \frac{1}{4\pi} \iiint \left(\frac{dV}{dx} \frac{dV'}{dx} + \frac{dV}{dy} \frac{dV'}{dy} + \frac{dV}{dz} \frac{dV'}{dz} \right) dv - \frac{1}{4\pi} \iiint R R' \cos \theta \, dv \quad (21),$$

where R and R' are the resultant forces at any point of space due to the acting and acted-upon systems respectively, and θ the angle between their directions.

Force and couple on a magnet in given field.

In practice W is expressed as a function of the variables (equal in number to the degrees of freedom) that determine the relative position of the two systems; differentiation with respect to any one of these then gives the generalized force component tending to decrease that variable.

We may also calculate the forces directly. For, the components of force on the element dv , being the differences of the forces acting on the two poles of the element, are

$$\left(A \frac{da}{dx} + B \frac{da}{dy} + C \frac{da}{dz} \right) dv, \text{ \&c.};$$

and the components of couple, in calculating which the field may be supposed uniform, are (see above, p. 228)

$$(\gamma B - \beta C) dv, \text{ \&c.}$$

Hence, integrating, we get, with the chosen origin, for the components of the whole force and couple,

$$\begin{aligned} \mathfrak{F} &= - \iiint \left(A \frac{da}{dx} + B \frac{da}{dy} + C \frac{da}{dz} \right) dv, \\ \text{and similarly for } \mathfrak{H} \text{ and } \mathfrak{Z}. \\ \mathfrak{X} &= - \iiint \left\{ \gamma B - \beta C + y \left(A \frac{d\gamma}{dx} + B \frac{d\gamma}{dy} + C \frac{d\gamma}{dz} \right) - x \left(A \frac{d\beta}{dx} + B \frac{d\beta}{dy} + C \frac{d\beta}{dz} \right) \right\} dv, \\ \text{and similarly for } \mathfrak{Y} \text{ and } \mathfrak{H}. \end{aligned} \quad (22).$$

In the important case of a uniform field whose components are a, β, γ , we have

$$W = -K(la + m\beta + n\gamma) \quad (23),$$

K being the moment of the magnet, and l, m, n the direction cosines of its axis. From this formula the results given above (p. 227) can be deduced with great ease.

Examples.—Some examples of the application of the foregoing theory are here given, partly on account of their intrinsic value as types enabling us to conceive the different varieties of magnetic action, partly for the sake of the light they throw on the theory itself. The reader who

desires more such should consult Maxwell's *Electricity and Magnetism*, or Mascart and Joubert, *Leçons sur l'Électricité et le Magnétisme*.

Solenoidal Magnets have already been defined as such that the Solenoidal condition

$$\frac{dA}{dx} + \frac{dB}{dy} + \frac{dC}{dz} = 0.$$

Solenoidal magnetization.

The lines of magnetization, therefore, have all the properties of lines of magnetic induction or electric force. In particular, if we consider a portion of the magnet enclosed by a tube of the lines of magnetization, the product of the intensity of magnetization by the section at each point is the same. Such a portion of magnetized matter taken by itself is called a "magnetic solenoid," and the product mentioned is called its "strength." It is clear (from the general definition, or it may be proved directly from the secondary property just mentioned) that the action of the solenoid may be represented by the distribution of a certain quantity ωI of positive magnetism on the one end and an equal quantity of negative magnetism on the other, I being the intensity of magnetization, ω the normal section at the end. The action therefore depends merely on the strength of the solenoid and on the position of its ends. The shape

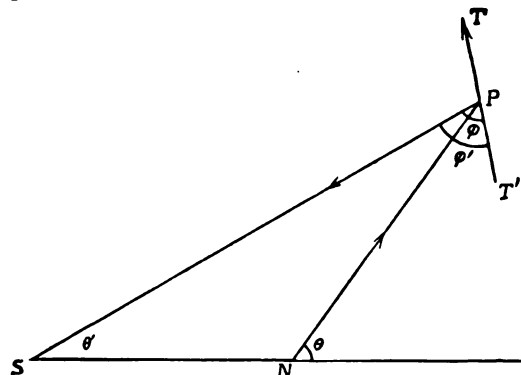


Fig. 21.

of the intervening portion is immaterial. If we suppose it straight, Equip and if the section be infinitely small so that the magnetism at the tentia ends may be regarded as condensed at two points, we have an ideal lines magnet of finite length. The equipotential lines of such a magnet lines in any plane through its axis are of course given by the equation

$$\frac{1}{r} - \frac{1}{r'} = \text{const.} \quad (24),$$

where r and r' are the distances of any point P on the line from the poles.

The equation to the lines of force is easily obtained; for, if NP Ideal and SP (fig. 21) make angles θ and θ' with the axis of the magnet, bar and ϕ and ϕ' with the line of force, we must have

$$\sin \phi / r^2 - \sin \phi' / r'^2 = 0;$$

hence, since

$$\sin \phi = r d\theta / ds, \quad \sin \phi' = r' d\theta' / ds,$$

we get

$$d\theta / r - d\theta' / r' = 0; \quad \text{i.e., } \sin \theta d\theta - \sin \theta' d\theta' = 0;$$

which gives for the equation to a line of force

$$\cos \theta - \cos \theta' = \text{const.} \quad (25).$$

We may imagine a magnet of this kind so long that the action Two of one of its poles may be altogether neglected at points which are like at a finite distance from the other. We thus effectively realize what poles never occurs in nature, viz., a magnet with one pole only. If we place the like poles of two such magnets near each other, we get a field the equipotential lines and lines of force in any axial plane of which are given by the equations

$$\frac{1}{r} + \frac{1}{r'} = \text{const.} \quad (26).$$

$$\cos \theta + \cos \theta' = \text{const.} \quad (27).$$

The lines of force given by equations (25) and (27) may be traced in a diagram by means of the following simple and elegant construction.³ Draw two circles A and B , having equal radii and N and S respectively for centres; produce the line NS both ways, and, starting from the centre, divide it into any number of equal parts; through these draw perpendiculars to meet the circles A and B ;

³ The first mathematical investigations of the equation to the lines of force of an ideal magnet appear to have been made by Playfair at the request of Robison, and by Leslie, *Geom. Analysis*, 1821. They had previously been very carefully considered from an experimental point of view by Lambert, *Mém. de l'Acad. de Berlin*, 1766.

⁴ Roget, *Jour. Roy. Inst.*, 1831.

¹ See Thomson, *Reprint of Papers on Electricity and Magnetism*, p. 433.

from N draw a series of lines to the points of division on B, and from S a similar series to the points of division on A. These lines will form a network of lozenges the loci of the vertices of which will be lines of force, corresponding to (25) or (27) according as we

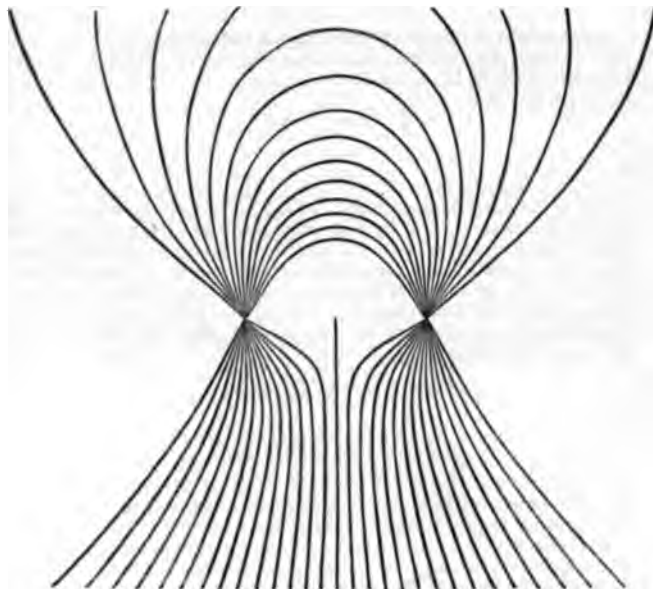


Fig. 22.

pass from point to point along one set of lozenge diagonals or along the other. Fig. 22 will give the reader an idea of the general appearance of the two sets of lines. He may compare the ideal with the actual cases by referring to figs. 4 and 5, p. 222.

In the case of an infinitely small magnet, the equipotential lines are of course given by the polar equation $r^2 = c^2 \cos \theta$, c being a variable parameter. It is easily shown that the lines of force, which are necessarily orthogonal to these, have for their equation $r = c \sin^2 \theta$.¹ If ϕ be the angle between r and the tangent of the line of force, we have $\tan \phi = r d\theta/dr = \frac{1}{2} \tan \theta$; hence the following construction for the direction of the line of force at P due to a small magnet at O:—let K be the point of trisection of OP nearest O, and let KT, perpendicular to OP, cut the axis of the magnet in T; then TP is the tangent to the line of force at P. This construction in a slightly different form was given by Hansteen² and by Gauss³; the latter adds that the resultant force at P is given by $M \cdot PT/OT \cdot OP^2$ where M is the magnetic moment of the magnet, a proposition which the reader will easily verify. These propositions are of considerable use in rough magnetic calculations. As this is an important case we give a diagram of the equipotential lines and lines of force in fig. 23.

We may, if we choose, consider a filament of matter magnetized longitudinally at every point, but so that the strength ρl ($-J$, say) is variable. Such a filament is called a complex solenoid. It may clearly be supposed made up of a bundle of simple solenoids whose ends are not all coincident with the ends of the filament. If ds be an element of such a filament, the potential is given by

$$V = \int ds J \frac{1}{D} = \frac{J_1}{D_1} - \frac{J_2}{D_2} - \int ds \frac{1}{D} \frac{dJ}{ds} \dots (28).$$

That is, its action may be represented by two particles of magnetism J_1 and J_2 at its two ends, and by a continuous distribution of free magnetism along its length whose density is $-dJ/ds$. This is of course merely a particular case of Poisson's distribution.

When a body is solenoidally magnetized, the magnetic force both external and internal depends solely on the surface distribution, i.e., merely on the ends of the solenoids of which the body is composed. We may therefore suppose the two ends of any solenoid joined by a solenoid of equal strength lying in the surface of the body. Proceeding thus, we may in an infinite number of ways construct a surface layer of tangentially magnetized matter which will represent the magnetic action of a solenoidally magnetized body. Thomson has shown by means of a highly interesting piece of analysis how to find the components of this tangential magnetization. See *Reprint of Papers on Electricity and Magnetism*, p. 401.

The magnetic theorems just stated will suggest at once to the

mind of the reader acquainted with the analysis employed in hydrokinetical problems the close analogy that subsists between the two methods. In fact, by proper arrangement, every problem in the one subject can be converted into a problem in the other. For details we refer the reader to Thomson, who was, so far as we know, the first to work out this matter fully; in the present connexion he should consult more particularly §§ 573 *sq.* of the *Reprint*.

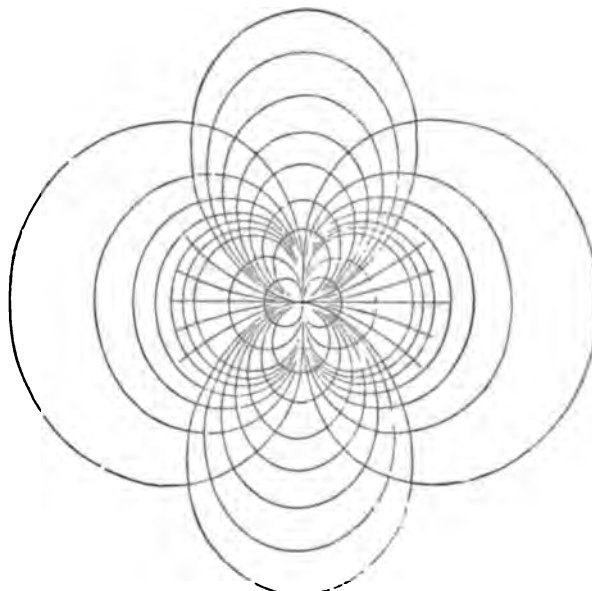


Fig. 23.

Uniformly Magnetized Bodies constitute in practice the most important case of solenoidal magnets. In the first place it is obvious of unit that the whole magnetic moment of such a body is simply its volume formly multiplied by the intensity of magnetization, and that the axis of magnet- the whole is parallel to the axis of each of its infinitely small parts. ized The method usually applied to calculate the potential in this case bodies. may be presented in two ways. The potential is calculated accord-

ing to Poisson's method in this case merely from a surface distribution of varying density $I \cos \theta$. We may replace this by a layer of uniform density ρ and varying normal thickness. Let the thickness at any point measured parallel to the magnetic axis be t ; then the normal thickness is $t \cos \theta$; hence $\rho t \cos \theta = I \cos \theta$, and $\rho t = I$; i.e., t is constant. We may therefore suppose the magnet replaced by itself (fig. 24) with a uniform volume distribution ρ of positive magnetism, and itself displaced through a distance t in a direction opposite to that of magnetization with a uniform volume distribution $-\rho$; or, which comes to the same thing, the potential of the magnet at P is $\rho(U - U')$, where U is the potential at P of a uniform volume distribution of density +1 throughout the magnet, and U' the potential of the same at a point P' displaced through a distance t in the direction of magnetization.

If l, m, n be the direction cosines of the magnetic axis, this gives at once

$$\left. \begin{aligned} V &= -\rho \left(\frac{dU}{dx} l + \frac{dU}{dy} m + \frac{dU}{dz} n \right) \\ &= -I \left(l \frac{dU}{dx} + m \frac{dU}{dy} + n \frac{dU}{dz} \right) \\ &= AX + BY + CZ \end{aligned} \right\} \dots (29);$$

where X, Y, Z are the components of the resultant force due to volume distribution $\rho = +1$ throughout the body, and A, B, C the components of the magnetization.

The same result may also be arrived at thus. The part of the potential due to the element dv is $I dv \cos \theta / r^2$, but this is the component parallel to the direction of I of the resultant force at P of a volume distribution whose density in dv is I; hence, since the

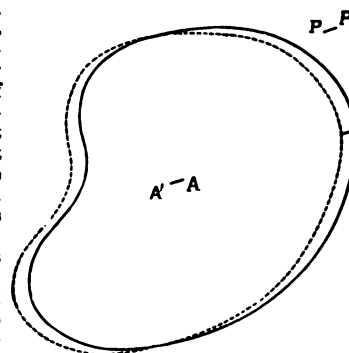


Fig. 24.

¹ Hansteen, *Magnetismus der Erde*, p. 208 (1819).

² *Magnetismus der Erde*, p. 208.

³ *Resultats d. Mag. Forretn.*, 1837 and 1840.

direction of I is everywhere the same, the whole potential is the component parallel to the magnetic axis of the body of the resultant force at P of a volume distribution $\rho = I$ throughout its whole extent. This gives at once the expression of (29) for V .

Sphere. In the case of a uniformly magnetized sphere of radius a , the axis being parallel to the axis of x , and the centre at the origin, we get, if r be the distance of P from the origin, for external points,

$$V = \frac{1}{2} \pi I a^2 x / r^3 \dots (30);$$

in other words, the external action is that of a magnet of infinitely small dimensions, having the same moment and axis, placed at the centre.

For internal points

$$V = \frac{1}{2} \pi I x \dots (31);$$

whence it appears that the magnetic force inside the sphere is constant in magnitude and in direction, being opposite to the uniform magnetization, and equal to $-\frac{1}{2} \pi I$.

Ellipsoid. The potential of a uniformly magnetized ellipsoid may be similarly treated. Let the origin be at the centre, and the axes along the principal diameters of the ellipsoid, whose lengths are $2a$, $2b$, $2c$, and let l , m , n be the direction cosines of its magnetic axis. Consider first an external point. Then,¹ if

$$L = 2\pi abc \int_0^\infty \frac{d\phi}{\sqrt{(a^2 + \phi)(b^2 + \phi)(c^2 + \phi)}}, \quad M = \&c., \quad N = \&c. \quad (32),$$

where a is the positive root of

$$\frac{x^2}{a^2 + \phi} + \frac{y^2}{b^2 + \phi} + \frac{z^2}{c^2 + \phi} = 1,$$

we have

$$X = Lx, \quad Y = My, \quad Z = Nz,$$

and

$$V = ALx + BMy + CNz \dots (33),$$

where it must be remembered that L , M , N are functions of x , y , z , inasmuch as a is so.

If (x, y, z) be an internal point, X , Y , Z are the components of the force due to a similar and similarly situated ellipsoid through (x, y, z) . Let its axes be pa , pb , pc ; we now have

$$X = Lx, \quad Y = My, \quad Z = Nz,$$

where

$$L = 2\pi p a b c \int_0^\infty \frac{d\phi}{\sqrt{(p^2 a^2 + \phi)(p^2 b^2 + \phi)(p^2 c^2 + \phi)}},$$

or, writing $\phi = p^2 \psi$,

$$L = 2\pi abc \int_0^\infty \frac{d\psi}{\sqrt{(a^2 + \psi)(b^2 + \psi)(c^2 + \psi)}}, \quad M = \&c., \quad N = \&c. \quad (34).$$

We thus obtain for V ,

$$V = ALx + BMy + CNz, \dots (35),$$

where L , M , N are now constants, which remain the same so long as the ratios of the axes remain unaltered. The components of the force inside the ellipsoid are

$$\alpha = -AL, \quad \beta = -BM, \quad \gamma = -CN \dots (36).$$

The force is therefore uniform; but its direction does not coincide with that of the magnetization, unless the latter be parallel to one of the principal diameters, and then the force is opposite in direction to the magnetization. It will be observed that the force inside similar ellipsoids similarly magnetized to the same intensity is always the same.

For an oblate ellipsoid of revolution, in which $b = c = a/\sqrt{1 - e^2}$,

$$L = 4\pi \left(\frac{1}{e^2} - \frac{\sqrt{1 - e^2}}{e^3} \sin^{-1} e \right), \quad M = N = 2\pi \left(\frac{\sqrt{1 - e^2}}{e^3} \sin^{-1} e - \frac{1 - e^2}{e^2} \right).$$

For a very flat oblate ellipsoid of revolution $L = 4\pi$, $M = N = \pi^2 a/c$. For a prolate or ovary ellipsoid of revolution, in which

$$a = b = c\sqrt{1 - e^2},$$

$$L = M = 2\pi \left(\frac{1}{e^2} - \frac{1 - e^2}{2e^3} \log \frac{1 + e}{1 - e} \right),$$

$$N = 4\pi \left(\frac{1}{e^2} - 1 \right) \left(\frac{1}{2e} \log \frac{1 + e}{1 - e} - 1 \right).$$

From the formulæ for an ellipsoid we could easily deduce those for an infinitely long elliptic or circular cylinder; we have merely to make one of the axes infinite. We find in this way, for instance, that the force inside a circular cylinder of infinite length magnetized transversely is $-2\pi I$.

The reader will find it interesting to examine the values of the magnetic induction in the foregoing cases, and to verify its normal continuity at the surface of the magnet.

Lamellar Magnets form another very important class. In them the components of magnetization are derivable by differentiation

from a function $\phi(x, y, z)$, which is sometimes called the "potential of magnetization,"² so that

$$A = -\frac{d\phi}{dx}, \quad B = -\frac{d\phi}{dy}, \quad C = -\frac{d\phi}{dz} \dots (37).$$

It is obvious at once that the family of surfaces $\phi(x, y, z) = \text{const.}$ cut the lines of magnetization at right angles; for, if dx, dy, dz be the projections of the element of any line on the surface, we have by differentiation

$$\frac{d\phi}{dx} dx + \frac{d\phi}{dy} dy + \frac{d\phi}{dz} dz = 0,$$

$$\text{i.e., } Adx + Bdy + Cdz = 0,$$

which is the analytical expression of the property in question. We may therefore suppose a lamellar magnet divided up by these surfaces of magnetization into an infinite number of infinitely thin normally magnetized shells or lamellae. It can be shown that the product of the intensity of magnetization by the thickness at each point of any such shell is the same; for, if $\phi(x, y, z) = c$ and $\phi(x, y, z) = c + \delta c$ be the equations to the two surfaces bounding the shell, δv the normal distance between them at any point, we have

$$\frac{d\phi}{dx} \delta x + \frac{d\phi}{dy} \delta y + \frac{d\phi}{dz} \delta z = \delta c,$$

hence

$$\left(\frac{d\phi}{dx} \frac{dx}{dv} + \frac{d\phi}{dy} \frac{dy}{dv} + \frac{d\phi}{dz} \frac{dz}{dv} \right) \delta v = \delta c,$$

i.e., $I \delta v = \delta c = \text{constant}$ for the same shell, which was to be proved. This product is called the strength of the shell.

A shell, which is everywhere normally magnetized, but whose strength is not constant, is called a "complex shell"; a magnet made up of such shells is called a "complex lamellar magnet." The condition to be satisfied by A, B, C in this case is simply that the lines of magnetization must be orthogonal to a family of surfaces, i.e., $Adx + Bdy + Cdz$ must be convertible into a perfect differential by multiplication by a factor; otherwise that

$$A \left(\frac{dB}{dz} - \frac{dC}{dy} \right) + B \left(\frac{dC}{dx} - \frac{dA}{dz} \right) + C \left(\frac{dA}{dy} - \frac{dB}{dx} \right) = 0.$$

The potential at P of a simple magnetic shell of strength i is given by the formula

$$V = \iint \frac{i \cos \theta dS}{D^2} = i \iint \frac{dS \cos \theta}{D^2} = i\omega \dots (38),$$

where ω is the solid angle subtended at the point P .³ There is a convention here as to sign, viz., that side of the shell is positive towards which the lines of magnetization pass, and the solid angle subtended at points infinitely near that side is positive, while that subtended at points infinitely near the other side is negative. If we cause P to move from the positive side away to infinity, then back from infinity to the negative side, or to move anyhow from infinitely near the positive side to a point infinitely near the negative side without cutting through the shell, it will decrease continuously by $4\pi i$; if we pass through the shell from a point infinitely near on the negative side to a point infinitely near on the positive side, there will be a sudden increase of $4\pi i$; tangentially to the shell there is continuity. The potential of a closed shell is evidently zero for any external point, $\pm 4\pi i$ for an internal point according as the positive or negative side is innermost. It appears also that the potential of a simple magnetic shell depends merely on its strength and on its boundary, just as that of a magnetic solenoid depends merely on its strength and the position of its ends.

A lamellar magnet will in general be made up partly of closed potential shells, and partly of shells whose boundaries lie on the surface; of lam only the latter of course can influence the potential at external points. The general expression for the potential at any point not $P(\xi, \eta, \zeta)$ is

$$V = \iiint \left(\frac{d\phi}{dx} \frac{d}{dx} \left(\frac{1}{D} \right) + \frac{d\phi}{dy} \frac{d}{dy} \left(\frac{1}{D} \right) + \frac{d\phi}{dz} \frac{d}{dz} \left(\frac{1}{D} \right) \right) dv \\ - \iint \phi \left(l \frac{d}{dx} \left(\frac{1}{D} \right) + m \frac{d}{dy} \left(\frac{1}{D} \right) + n \frac{d}{dz} \left(\frac{1}{D} \right) \right) dS - \iiint \phi \nabla^2 \left(\frac{1}{D} \right) dv \\ - \iint \frac{\phi \cos \theta}{D^2} dS + 4\pi \phi'$$

where θ is the angle between D and the outward normal to dS , and ϕ' the value of ϕ at the point ξ, η, ζ (zero of course if ξ, η, ζ be outside the magnet). The value of V thus found is not discontinuous at the surface as might be supposed, for both the surface integral and

¹ See Thomson and Tait, *Natural Philosophy*, vol. I. § 522.

² To be distinguished of course from the magnetic potential.

³ Gauss, *Allgemeine Theorie des Erdmagnetismus*, § 22.

$4\pi\phi'$ have discontinuities there, and they are equal in amount and of opposite sign.

For an external point the potential is

$$\iint \frac{\phi \cos \theta}{D^3} dS.$$

The immediate interpretation of this is that the potential is the same as that due to a normally magnetized layer on the surface of the body whose strength at dS is ϕ ; in other words, *qua* external action, every lamellar magnet may be replaced by a complex shell on its surface.

There is, however, another way of looking at the result. Since A, B, C are derivable from a potential ϕ , the difference between the values of ϕ at any two points is simply the value of the line integral $\int (A dx + B dy + C dz)$ along any path between those points. Hence if the tangential component of magnetization be given in direction and magnitude all over any surface, the value of ϕ , *a constant pris*, is given all over that surface. We conclude therefore that, if a body be lamellarly magnetized, and we know the tangential component of its magnetization all over its surface, its external action is determined; ¹ for a constant c added to ϕ will simply add to the surface integral

$$c \iint \frac{\cos \theta}{D^3} dS,$$

which, being c times the whole solid angle subtended by the surface at any external point, vanishes. For another very interesting proof of this result, see Thomson, *Reprint*, p. 398 sq.

The vector potential of a lamellar magnet may be expressed by means of the formulae

$$\left. \begin{aligned} F &= \iiint \left(\frac{d\phi}{dy} \frac{d}{dz} \left(\frac{1}{D} \right) - \frac{d\phi}{dz} \frac{d}{dy} \left(\frac{1}{D} \right) \right) dv \\ &\quad - \iint \phi \left(m \frac{d}{dz} \left(\frac{1}{D} \right) - n \frac{d}{dy} \left(\frac{1}{D} \right) \right) dS' \\ &\quad - \iint \frac{1}{D} \left(m \frac{d\phi}{dz} - n \frac{d\phi}{dy} \right) dS'; \quad G = \&c.; \quad H = \&c. \end{aligned} \right\} \quad (40).$$

These formulae furnish an immediate proof of the theorems of Thomson above stated.

By means of the last of them, it has been shown in the article ELECTRICITY ² that the vector potential of a simple magnetic shell can, as might be expected, be expressed by means of a line integral taken round its boundary; in the same place it has also been shown, &c. that the potential energy of such a shell in a magnetic field reduces to a similar line integral; and that the mutual potential energy of two such shells reduces to a double line integral taken round their boundaries.

An important approximate expression for the potential of a magnet at a point P , whose distance r from some chosen point in the magnet is great compared with the greatest linear dimension of the magnet, may be obtained as follows. Let the coordinates of P with respect to the chosen point and any axes through it be ξ, η, ζ ; and let the coordinates of any point in the body referred to the same axes be x, y, z . Also let $D = \{(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2\}^{\frac{1}{2}}$, and

let $r = (\xi^2 + \eta^2 + \zeta^2)^{\frac{1}{2}}$, $s = (x^2 + y^2 + z^2)^{\frac{1}{2}}$, $t = \xi x + \eta y + \zeta z$. Then the potential U at (x, y, z) of a unit pole placed at (ξ, η, ζ) is given by

$$U = \frac{1}{D} = \frac{1}{r} + \frac{t}{r^3} + \frac{3t^2 - r^2 s^2}{2r^5} + \frac{5t^3 - 3tr^2 s^2}{2r^7} + \&c. \\ = \frac{1}{r} + U_1 + U_2 + U_3 + \&c.,$$

by a well-known theorem, where U_1, U_2, U_3 , &c., are spherical harmonics of degrees 1, 2, 3, &c., in x, y, z , and $-2, -3, -4$, &c., in ξ, η, ζ . Now, by the theorem of mutual potential energy, the potential V of the magnet at (ξ, η, ζ) is the potential energy of the magnet in the field due to a unit pole at (ξ, η, ζ) ; hence by (17)

$$V = \iiint \left(A \frac{dU}{dx} + B \frac{dU}{dy} + C \frac{dU}{dz} \right) dv \quad \dots (41), \\ = V_1 + V_2 + V_3 + \&c.$$

where V_1 arises from U_1, V_2 from U_2 , and so on. V_1, V_2, V_3 , &c., will be spherical harmonics in ξ, η, ζ of the most general kind, involving essentially 3, 5, 7, . . . $2i+1$ constants respectively, their degrees being $-2, -3, \dots -i$ respectively. These constants will, however, depend in each case on a larger number of integrals taken throughout the magnetized body, thus the constants in V_i will depend upon $\frac{1}{2}(i+1)(i+2)$ integrals. ³ There is no diffi-

culty in writing down these terms except the length of the formulae. Putting

$$\left. \begin{aligned} \iiint A dv &= Kl, \quad \iiint B dv = Km, \quad \iiint C dv = Kn \\ L &= \iiint A x dv, \quad M = \iiint B y dv, \quad N = \iiint C z dv \\ P &= \iiint (Bx + Cy) dv, \quad Q = \&c., \quad R = \&c. \end{aligned} \right\} \quad (42),$$

we get

$$V = K \frac{l\xi + m\eta + n\zeta}{r^3} + \frac{(2L - M - N)\xi^2 + \&c. + 3P\eta\xi + \&c.}{r^5} + \&c. \quad (43).$$

It may be shown ⁴ that, in the most general case, if we take the axis of x parallel to the magnetic axis, and the origin at the point

$$\left\{ \frac{(2L - M - N)2K}{R/K}, \frac{R/K}{Q/K} \right\},$$

and turn the axes about an angle $\tan^{-1} P/(M - N)$, the above reduces to

$$V = \frac{K\xi}{r^3} + \frac{3}{2} \frac{(M - N)(\eta^2 - \zeta^2)}{r^5} + \&c. \quad \dots (44).$$

An interesting particular case is that in which the magnet is symmetrical with respect to the three coordinate planes. If we take its axis to be in the axis of x , then, since all the integrals L, M, N, P, Q, R vanish, V_2 disappears, of the next set only

$$A_1 = \iiint A x^2 dv, \quad A_2 = \iiint A y^2 dv, \quad A_3 = \iiint A z^2 dv \quad \dots (45)$$

remain, and we get

$$V = \frac{K\xi}{r^3} + \frac{3\{A_1(2\xi^2 - 3\eta^2 - 3\zeta^2) + A_2(4\eta^2 - \xi^2 - \zeta^2) + A_3(4\xi^2 - \eta^2 - \zeta^2)\}\xi}{2r^7} \quad (46).$$

The potential to the same degree of approximation of a positive and negative pole of strength μ , placed on the magnetic axis at distances $+L$ and $-L$ from the origin (centre of symmetry), is

$$V' = \frac{2\mu L \xi}{r^3} + \frac{2\mu L^3 \xi (2\xi^2 - 3\eta^2 - 3\zeta^2)}{2r^7}.$$

If we attempt now to find μ and L , so that the two magnetic Ideal systems may be equivalent, we find different values for L for representing different positions of the external point. If, however, the magnetative be symmetrical about its axis, so that $A_1 = A_2$, then the expression magnet. for V reduces to

$$V = \frac{K\xi}{r^3} + \frac{3(A_1 - A_2)\xi(2\xi^2 - 3\eta^2 - 3\zeta^2)}{2r^7} \quad \dots (47),$$

we then get $2\mu L = K$, and $2\mu L^3 = 3(A_1 - A_2)$, whence $L^2 = 3(A_1 - A_2)/K$. In other words, in the case of a magnet which is symmetrical about its axis and also about an equatorial plane, we can represent the external action by means of a fixed ideal magnet, provided higher powers of the ratio of the greatest linear dimension of the magnet to the distance of the point considered than the fourth can be neglected. It is to be observed, however, that if $A_1 < A_2$, the length of the ideal representative magnet will be imaginary. ⁵

A convergent series for the mutual potential energy of two Series for magnets M and M' may be obtained from the sextuple integral of mutual (19). Let the origin be a fixed point O in M , and let the coordinates of a fixed point O' in M' with reference to a set of axes fixed in O be ξ, η, ζ ; further, let x, y, z and x', y', z' be the coordinates of any elements dv and dv' in M and M' , the axes being in the former case the system already indicated, in the latter a parallel system through O' ; then, if r denote $(\xi^2 + \eta^2 + \zeta^2)^{\frac{1}{2}}$, and $\delta_1, \delta_2, \delta_3$ stand for

$$\frac{d}{d\xi}, \frac{d}{d\eta}, \frac{d}{d\zeta}, \text{ we have} \\ W = - \int dv \int dv' (A\delta_1 + B\delta_2 + C\delta_3)(A'\delta_1 + B'\delta_2 + C'\delta_3) \left(\frac{1}{r} + \sum u_n \right),$$

$$\text{where} \quad u_n = \frac{1}{n!} \left\{ (x' - x)\delta_1 + (y' - y)\delta_2 + (z' - z)\delta_3 \right\}^n \frac{1}{r};$$

or

$$W = W_1 + W_2 + W_3 + \dots \quad (48),$$

where W_1, W_2, W_3 , are spherical harmonics in ξ, η, ζ of degrees $-2, -3, -4$, &c.

If we neglect all the terms except those of the first order, which amounts to supposing M and M' infinitely small, we get

$$W = -KK' \left\{ u\delta_1^2 + \dots + (mn' + m'n)\delta_2\delta_3 + \dots \right\} \frac{1}{r}.$$

If θ_1, θ_2 be the angles between the axes of M and M' and the line

⁴ See Sir W. Thomson, *Reprint*, p. 368.

⁵ This point is sometimes called the centre of the magnet, and the new axes of Y and Z its secondary axes. It should be observed, however, that this "centre" is not necessarily the middle point of the line joining the mass centres of the positive and negative magnetism. On this subject see a paper by Beltrami, "Sul Potenziale Magnetico," *Ann. d. Matem.*, 1883.

⁶ Cf. Riecke in *Pogg. Ann.*, p. 149, 1873; and *Wied. Ann.*, p. 8, 1879.

from the centre of M to the centre of M' , and θ_{12} the angle between the axes, this reduces to

$$W = \frac{KK'}{r^3} (\cos \theta_{12} - 3 \cos \theta_1 \cos \theta_2) \dots (49).$$

Action between two infinitely small magnets. From this formula we can derive at once by differentiation the force of translation and the couple about the centre of M' , which represent the action of M upon it. An elegant synthesis of this action has been given for the most general case by Tait.¹ It will be sufficient to confine ourselves here to the case where the magnetic axes are in one plane. In this case $\theta_{12} = \theta_1 - \theta_2$, and W becomes $KK'(\sin \theta_1 \sin \theta_2 - 2 \cos \theta_1 \cos \theta_2)/r^3$. Denoting by X, Y, L the forces

of translation parallel to MM' and perpendicular to MM' (so as to decrease θ_1) and the couple tending to decrease θ_2 , we have

$$\left. \begin{aligned} X &= -\frac{dW}{dr} = \frac{3KK'}{r^4} (\sin \theta_1 \sin \theta_2 - 2 \cos \theta_1 \cos \theta_2) \\ Y &= \frac{dW}{r d\theta_1} + \frac{dW}{r d\theta_2} = \frac{3KK'}{r^4} \sin(\theta_1 + \theta_2) \\ L &= \frac{dW}{d\theta_2} = \frac{KK'}{r^3} (\sin \theta_1 \cos \theta_2 + 2 \cos \theta_1 \sin \theta_2) \end{aligned} \right\} \dots (50).$$

Force of translation. One most important conclusion follows at once from these formulae, viz., that the translatory forces vary inversely as the fourth power of the distance, whereas the directive couple varies only inversely as the third power. Hence the couple may be quite sensible at distances for which the force of translation is inappreciably small. These conclusions apply of course equally to any pair of magnetized bodies, provided the distance between them be sufficiently great as compared with their linear dimensions. This, applied to the case of the earth, at once explains the phenomena that puzzled Norman and the earlier magnetic philosophers so greatly. The following particular cases are important (fig. 25):—

$$(A) \quad \theta_1 = \theta_2 = 0, \quad X = -\frac{6KK'}{r^4}, \quad Y = 0, \quad L = 0.$$

$$(B) \quad \theta_1 = \theta_2 = \frac{\pi}{2}, \quad X = \frac{3KK'}{r^4}, \quad Y = 0, \quad L = 0.$$

$$(C) \quad \theta_1 = 0, \quad \theta_2 = \frac{\pi}{2}, \quad X = 0, \quad Y = \frac{3KK'}{r^4}, \quad L = \frac{2KK'}{r^3}.$$

$$(D) \quad \theta_1 = \frac{\pi}{2}, \quad \theta_2 = 0, \quad X = 0, \quad Y = \frac{3KK'}{r^4}, \quad L = \frac{KK'}{r^3}.$$

Deflecting magnet. The last two cases are especially important: the position of the deflecting magnet in (C) is described as "end on" (erster Hauptlage), in (D) as "broadside on" (zweiter Hauptlage); it will be noticed that the couple in the former case is double that in the latter.

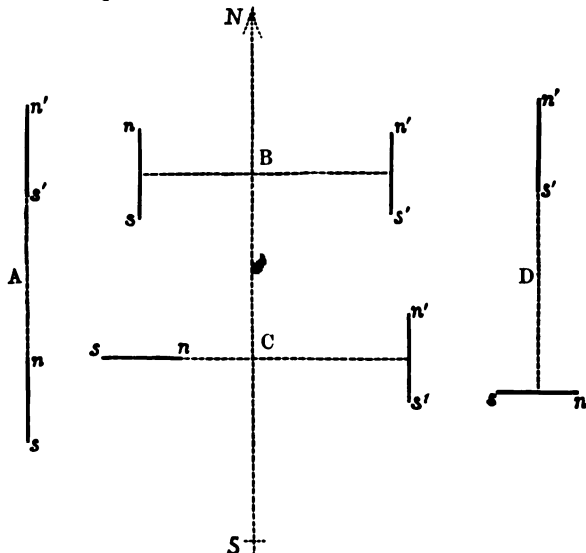


Fig. 25.

Closer approximation for deflecting couple end on and broadside on. If the terms of the second and third order be taken into account, and the magnet $n's'$ be deflected through an angle ϕ from its original position by a deflecting magnet (I.) originally end on and (II.) originally broadside on, we get for the couples

$$\left. \begin{aligned} \text{I.} \quad & \cos \phi \left[\frac{2KK'}{r^3} - \frac{T_1}{r^4} + \frac{T_2}{r^5} + \dots \right] \\ \text{II.} \quad & \cos \phi \left[\frac{KK'}{r^3} + \frac{T_1}{r^4} + \frac{T_2}{r^5} + \dots \right] \end{aligned} \right\} \dots (51),$$

¹ *Quart. Jour. Math.*, 1860; and *Quaternions*, § 414.

where T_1 and T_2 are odd functions of the relative position of M and M' , but T_3 and T_4 are even. In the case where M and M' are symmetrical about three orthogonal planes, O and O' being the centres of symmetry, T_1 and T_2 vanish, and the writer has obtained for the values of T_3 and T_4

$$\left. \begin{aligned} T_3 &= -6 \{ K(3A_1 - 4A_2 + A_3) - K'(2A_1 - A_2 - A_3) \} \\ T_4 &= -\frac{1}{2} \{ K(12A_1 - 11A_2 - A_3) - K'(3A_1 - 4A_2 + A_3) \} \end{aligned} \right\} \dots (52),$$

where A_1, A_2 , &c., have the meanings above assigned in (45).²

Sphere Magnetized in any Manner.—This is the most interesting of all the cases that fall under the present theory section, both from its being amenable to mathematical treatment and on account of its historical interest. It was first discussed in the beautiful memoir, entitled *Allgemeine Theorie des Erdmagnetismus*,³ in which Gauss laid the foundation of the rational theory of terrestrial magnetism. The following is a brief account of the theory, which has not been greatly added to since he left it.

Let X, Y, Z be the components of the earth's resultant magnetic force at any point on its surface, in the directions of geographical north, geographical west, and vertically upwards respectively. The force is completely known when these are given, since it depends on three elements only. If H, δ, ι have the meanings formerly assigned (p. 220, 221, 227), we have of course

$$H = \sqrt{X^2 + Y^2}, \quad \tan \delta = Y/X, \quad \tan \iota = Z/\sqrt{X^2 + Y^2}.$$

Again, if V be the magnetic potential of the earth, ι the latitude, and λ the longitude of any point on its surface, then, supposing the earth to be a sphere of radius a , we have

$$X = -\frac{1}{a} \frac{dV}{d\lambda}, \quad Y = -\frac{1}{a \cos \iota} \frac{dV}{d\lambda}, \quad Z = -\frac{dV}{dr}, \quad \dots (53),$$

r denoting the distance of any point from the centre of the earth. When V is known, therefore, the force is completely determined.

If now we suppose all the magnetized matter (or its equivalent—say, electric currents) to be within the earth, it follows, from the theory of spherical harmonics, that we can write down a convergent series for its potential at all external points, when the potential at every point of its surface is given.⁴ In fact, if the expansion of this surface potential in terms of surface harmonics be

$$S_1 + S_2 + \dots + S_i + \dots,$$

we have for all external points

$$V = S_1 \left(\frac{a}{r} \right)^2 + S_2 \left(\frac{a}{r} \right)^3 + \dots + S_i \left(\frac{a}{r} \right)^{i+1} + \dots \dots (54).⁵$$

The number of terms of this series that must be retained in order to obtain a sufficiently accurate representation of the phenomena will of course depend on circumstances, and can only be ascertained by trial. S_1, S_2, \dots, S_i are functions of known form, containing respectively 3, 5, $\dots, 2i+1$ constants; hence, if terms beyond the i^{th} order may be neglected, the expression for V will contain i^2+2i arbitrary constants. These must be determined by observation, and then the magnetic action at all points on the surface or outside the earth is known irrespective of the internal distribution of the magnetic causes.

If we look at the matter from the general point of view that V is determined when its surface value is known, we have the following propositions.

I. V is determined when the vertical force is known at every point of the earth's surface.

For, let the surface value of Z be expanded in a series of surface harmonics of which the i^{th} is Z_i ; then, equating this to the i^{th} harmonic in the surface value of $Z = -dV/dr$ derived from (54), we have $(i+1)S_i = aZ_i$, which determines S_i . Thus the proposition is proved.

² Cf. Riecke, *l.c.*

³ *Res. d. Mag. Veveys*, 1838.

⁴ See Thomson and Tait, vol. i. chap. 1, App. A and B.

⁵ The term S_0 of course vanishes, since the sum of the positive and negative magnetism within the earth is zero.

II. The surface value of V , and hence its general value for external points, is determined if the northward component of the magnetic force be known at every point of the earth's surface.

This follows at once from the fact that the difference of the values of V at any two places is the line integral of the magnetic force along any line joining them; thus, if V_0 be the value of V at the geographical north pole, we have

$$V = -a \int_0^l X dl + V_0 \dots \dots \dots (55).$$

But the constant V_0 does not affect the general value of V ; hence the proposition is established.

III. The same conclusion follows if the westward horizontal component be known all over the earth's surface and the northward component along any one meridian.

In fact, if V be the potential at any place whose latitude is l and longitude λ , then

$$V = -a \int_0^l X dl - a \int_0^\lambda Y \cos l d\lambda + V_0,$$

the first integration being performed along the given meridian, the second along the parallel of latitude corresponding to the place.

From I., II., and III. we have the remarkable conclusion that, if the vertical component be given all over the earth, or the northward component, or the westward component and the northward along one parallel, then in each case the other two elements are determined.

Gauss gives another interesting application of the line integral of magnetic force. If this integral be taken all round any closed curve or polygon, the result is zero. Let us express this for any geodesic triangle ABC, at whose vertices the horizontal force has the values

H_1, H_2, H_3 . If the inclinations of H to BC at B and C be α and α' , to CA at C and A β and β' , to AB at A and B γ and γ' , then, if the arcs BC, CA, AB be not too long, we may replace the component along BC at every point by the average of its values at B and C, and so on. We thus get

$$\frac{1}{2} BC (H_2 \cos \alpha + H_3 \cos \alpha') + \frac{1}{2} CA (H_3 \cos \beta + H_1 \cos \beta') + \frac{1}{2} AB (H_1 \cos \gamma + H_2 \cos \gamma') = 0 \dots \dots \dots (56).$$

If we suppose the values of H at B and C to be known, and the values of the declination to be known at all three places, the above equation determines the value of H at A. Calculating in this way from observed values at Göttingen, Milan, and Paris, Gauss found for H at Paris 0.51696, the observed value being 0.51804.

It has been supposed hitherto that the magnetic causes are entirely internal to the earth. The foregoing theory enables us to test how far this assumption is correct.

If we suppose that there are external causes, then the potential at internal points due to these will be

$$T_0 + T_1 \frac{r}{a} + T_2 \left(\frac{r}{a}\right)^2 + \dots T_i \left(\frac{r}{a}\right)^i + \dots,$$

$T_0, T_1, T_2, \dots T_i$ being the different harmonics in the surface value of the part of the potential due to external causes. Suppose now the whole vertical force deduced from observation for all parts of the earth's surface, and expanded in a series of surface harmonics, the i^{th} of which is Z_i ; then, since this is the sum of the i^{th} harmonics in the parts due to internal and to external causes, we have

$$-(i+1)S_i + iT_i = aZ_i \dots \dots \dots (57).$$

Further, suppose the surface value of V determined from observations of horizontal force, and let the i^{th} harmonic in it be V_i , then we have

$$S_i + T_i = V_i \dots \dots \dots (58).$$

From equations (57) and (58) we can determine S_i and T_i , and thus settle how much is due to external and how much to internal causes. It does not appear from observation that any sensible part of the mean value of V arises from causes external to the earth.

We have seen already that the action of any body can be represented at external points by an ideal layer of positive and negative magnetism. Gauss finds for the surface density of the layer in the case of a spherical body like the earth, the expression $(V/a - 2Z)/4\pi$, which may be deduced immediately from the formulae already given.

If we draw a series of equipotential surfaces correspond-

ing to small equidifferent values of V , these will cut the earth's surface in a series of equipotential lines, which are called the "magnetic parallels." These lines obviously have the following properties. The horizontal force is everywhere perpendicular to them, and is at any point inversely proportional to the distance between two consecutive lines there. So that, if these lines were drawn upon a terrestrial globe, their crowding would indicate increase of horizontal force. The lines of horizontal force, or "magnetic meridians," the tangent at every point of which is parallel to the horizontal component, are everywhere orthogonal to the magnetic parallels, and their positive direction is from parallels of greater potential to parallels of less potential. If, as has been tacitly assumed hitherto in accordance with the results of observation, the potential on the earth's surface have but one maximum and one minimum, then the parallels will be closed curves expanding successively from the maximum point and then closing again round the minimum point, and the magnetic meridians will all run between these two points. It is clear that at each of these points the equipotential surface and the earth's surface touch; at the minimum point the line of total resultant force will pass to the earth, at the maximum point from it; at the former, therefore, the north end of a freely suspended needle will dip vertically downwards, at the latter the south end will do the same. This is the simplest possible case for a magnetized sphere. It is easy to see that, if we define a north pole¹ as a point on the earth's surface at which the horizontal intensity vanishes, and the dip is 90°, there might be more than one such point. Consider the series of equipotential surfaces 1, 2, 3, 4, 5, 6 in fig. 26,² each of which has two eminences with a depression

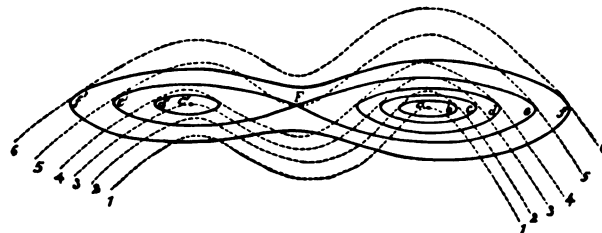


Fig. 26.

between them. The lines a, b, c, d, e, f are the sections of these by the earth's surface. 1 just touches the surface in a ; and, if the potential increase in the order in which the surfaces are numbered, a will be a north pole. The section by 2 is the single oval b . 3 touches the surface in c' , which is clearly another north pole, and also meets the surface in a single oval c equipotential with c' . The section by 4 is the double oval d, d' . The depression on 5 touches the surface at F , and meets it in a figure of 8, e, e' , poles, on which F is the double point. F is therefore yet another north pole according to our definition; it differs, however, from an ordinary north pole in one important respect; for the law that the north end of the compass points from parallel of greater to parallel of less potential shows at once that near F and inside the 8-shaped parallel the south end will point to F , whereas at a neighbouring point outside the north end will point to F . Such a point is called a false north pole, and we see that the existence of two true north poles necessitates the existence of a false north pole; and in general it may be established³ that,

¹ Of course pole as thus defined has nothing to do with pole in any of the former senses, e.g., the line joining its N and S poles is not parallel to the earth's magnetic axis.

² Gauss, *l.c.*, § 12. Cf. Mascart and Joubert, *Leçons sur l'Électricité et sur le Magnétisme*, tom. i. § 436, 1882.

³ See Gauss, *All. Theorie des Erdmagnetismus*, § 12; Maxwell, vol. i. § 113, vol. ii. § 468.

however many poles of the same kind there may be, true and false, the whole number must be odd. This of course disposes of the notion formerly held by some physicists that the earth actually had *two* north poles. As already indicated, Gauss concluded from his reduction of the magnetic observations at his disposal that, apart from purely local disturbances, the earth has, as a matter of fact, only one north and one south pole.

Local disturbances of magnetic parallels.

The effect of a deposit of magnetic ore, or other cause of the kind, might of course produce a disturbance, within a limited area, of the equipotential lines. It may assist the practical magnetist to indicate the nature of this disturbance in a particular case. Let us suppose that a magnet is placed some distance underground, vertical, with its north pole uppermost. Then, if its moment be sufficiently great, the equipotential lines will be as in fig. 27.¹ The upper side of the figure is supposed to be magnetic north, and it is supposed that the undisturbed

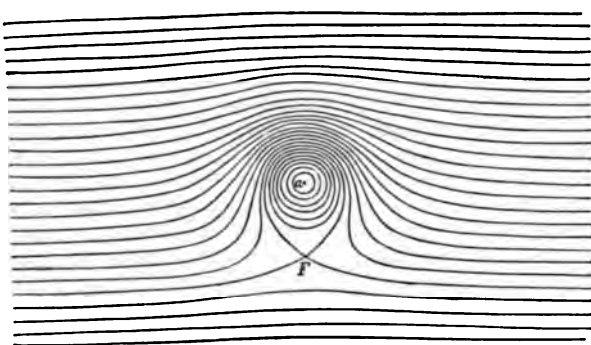


Fig. 27.

parallels would be straight lines running magnetic east and west, which is sufficiently near the truth in most cases. It should be observed that fig. 27 is in reality a transformation of figure 26, one of the poles being projected to infinity. The reader should notice that the double point F, due south of the point *a* vertically over the disturbing magnet, is a point of equilibrium at which the horizontal components of the forces of the earth and the magnet destroy each other; it will be a false pole, south or north according as the magnet or the earth prevails.

EXPERIMENTAL FOUNDATION FOR THE LAW OF THE INVERSE SQUARE.

Difficulties of the question as to the elementary law.

From what has already been laid down, it will be seen that the determination of the elementary law of magnetic action is a very complex problem. The action between two magnets depends, not only on their distance apart, but also on their relative angular position. Then we have to distinguish force of translation, which varies inversely as the fourth power of the distance, and directive couple, which varies inversely as the third power. It must also be remembered that the elementary law results in part from an hypothesis as to the nature and distribution of the cause of the magnetic action, for, until some such hypothesis is made, no clear conception is possible of what is to be understood by elementary action. Lastly, we have the disturbance which arises from magnetic induction, the consequence of which is that magnetically speaking two magnets are not the same at different distances apart. When all these circumstances are considered, it is not surprising to find considerable uncertainty and difference of opinion among the earlier magnetic philosophers. The truth is that the law as now established owes quite as

on and broad-side on.

much to the development of magnetic theory as to the work of magnetic experimenters.

The question attracted the notice of Huygens and Hooke, but Newton seems to have been one of the first who propounded any law on the subject. He says (*Principia*, lib. iii. prop. 6, cor. 5) that some rough experiments had led him to the conclusion that the magnetic force (*vis magnetica*) decreases according to the law of the inverse cube of the distance. No account of the experiments is extant, and it does not appear what he means exactly by *vis magnetica*. If the directive couple is meant, and the action of the *entire* magnet is intended, then, as we have seen, this is in agreement with modern theory. In a remarkable note in the annotated edition of the *Principia* by Le Sueur and Jacquier (assisted by Calandrini?) (1742) on the passage in question, a series of deflexion experiments are described, and an accurate discussion is given, from which results the law of the inverse cube for the deflecting couple. Hawksbee² made experiments with a view to determine the law of magnetic action, in which a deflecting magnet was moved at various distances round a compass, and the corresponding deflexions noted. A few years later Brook Taylor³ and the same experimenter made a series of observations in which the "end on" method of deflexion still in use was adopted. But in neither case was any definite result arrived at. A similar uncertainty appeared in the experiments of Whiston, who indicates the inverse $\frac{3}{2}$ th power of the distance as the law of decrease. Musschenbroek's experiments, which were extensive, also led to no final result. He used the method of Hooke, in which the attraction of a vertical bar magnet upon another suspended from one arm of a delicate balance is balanced by weights attached to the other arm. From some of his experiments he deduces as low a power as the inverse 1st, from others the $\frac{3}{2}$ th, and so on; but no attempt is made to analyse the phenomena. Michell, in his treatise on artificial magnets (1750), however, deduces the law of the inverse square from Musschenbroek's results. Although *Æpinus* does not arrive at any definite result as to the elementary law, there can be no doubt that his *Tentamen Theoriæ Electricitatis et Magnetismi* (1759) contributed powerfully towards the solution of the question. Tobias Mayer seems to have been the first to publish the law of the inverse square as the actual result of an experimental investigation. His paper was read before the Royal Society of Göttingen, and was referred to in the *Göttinger Gelehrter Anzeiger* for 1760, but never fully published; it is best known from the criticism of *Æpinus*, "*Examen Theoriæ Magneticæ a Tob. Mayero propositæ*" (*Nov. Comm. Acad. Petrop.*, 1768).⁴ The most important of the earlier contributions was undoubtedly that of Lambert.⁵ He seems to have been the first to analyse the physical circumstances of the problem in a thorough manner, and to point out the various elements of disturbance to be provided for. We regret that we are unable to devote space to an exhaustive account of his memoirs,⁶ which are most instructive reading even now. He showed that the effect of an oblique magnetic force on the needle varies as the sine of the inclination; and, making allowances for this, he deduced the law of the inverse square from deflexion experiments made at different distances. He also described the method of oscillations, but found difficulties in its practical application. It is upon his theoretical work, however, rather than upon his experiments, that his claim to be remembered rests. About the same time as Lambert,

² *Phil. Trans.*, 1712.

³ *Phil. Trans.*, 1715 and 1721.

⁴ *Comp. Hansteen, Mag. d. Erde*, p. 283, 1819.

⁵ *Hist. d. l'Acad. Roy. d. Sc. Berlin*, 1766.

⁶ An excellent one will be found in Hansteen, *Magnetismus der Erde*, pp. 295 sq.

¹ Gauss, *l.c.*, § 13.

we have Dalla Bella¹ and Robison,² the well-known professor of natural philosophy in the university of Edinburgh, working at the same subject. The former used the method of Hooke and Musschenbroek, but discussed more carefully the exact nature of the resultant action. His results indicated the law of the inverse square. Robison used both the method of deflexion and the method of oscillation, the peculiarity in his apparatus being the movable magnet, which was composed of two magnetized spheres connected by a slender rod, and suspended either in the field of the earth alone, or at different distances from a large magnet. He made several independent investigations, and seems to have arrived in each case at the law of the inverse square as his final result.

The researches of Coulomb,³ from which many date the commencement of the modern theory, present many features of great interest. He used the improved form of Michell's torsion balance, which had served him so well in his electrical experiments. In order to realize as nearly as possible the ideal case of a linear solenoid, whose action can be represented by positive and negative magnetism concentrated at its ends, he worked with magnets made of thin steel wire magnetized longitudinally. The circumstances of the experiment are thus considerably simplified, for the acting magnet may be so arranged that the action of one of the poles may be neglected, or, failing that, the action of both can be easily calculated.

In one of his experiments he took a magnetized steel wire 25 inches long, and $\frac{1}{4}$ lines thick, and placed it vertically in the magnetic meridian before a horizontal magnetic needle some 3 inches long, delicately suspended by a silk fibre. The rod was raised and lowered at a given distance from the needle until the attraction on the near pole of the needle, as tested by the rapidity of the vibrations, was a maximum; it was then found that the lower end of the bar was about 1 inch below the needle. Again, the rod being placed horizontal and perpendicular to the magnetic meridian on a level with the needle, it was displaced until the needle returned to the magnetic meridian; it was then found that the needle was directed to a point about 1 inch from the end of the bar. Both these experiments thus indicate that the magnetism at one end may be supposed concentrated at a point about an inch from the end of the bar. It is clear that, in these experiments, provided the rod is sufficiently long or the distance between it and the needle not too great, the action of the distant pole may be neglected, for the double reason that the pole is more distant and that the force exerted by it is nearly perpendicular to the direction in which it can be effective. Making this assumption, Coulomb observed the number of vibrations, when the vertical rod was absent, and when it was placed at various distances.

The forces thence deduced were found to vary very nearly as the inverse square of the distance. Statical experiments with the torsion balance led to a like result.

Later than Coulomb we have the experiments of Bidone,⁴ Hansteen,⁵ Steinhauser,⁶ and Scoresby.⁷ By far the most important among these is Hansteen, whose methods were a great step towards the more complete treatment finally adopted by Gauss. He uses Taylor's "end on" method of deflexion, and also the method of Hooke and Musschenbroek. The acting magnet was a bar magnet, the action of which he represents by a distribution of positive and negative magnetism on its two halves whose density at a distance x from the centre is λx^r . The force at distance D due to an element $d\mu$ of positive magnetism he assumes to be $d\mu/D^n$. He finds that in all his experiments the value $n=2$ best represents the results obtained; but that various values of r may be adopted with almost equal advantage; he inclines, however, to the value $r=2$.

In his classical memoir on the absolute measurement of the earth's magnetic force, Gauss took up the question in the most general manner yet attempted. Assuming that the force due to an element of positive magnetism varies as the inverse n th power of the distance, he showed that, when the distance between the magnets is sufficiently great compared with the greatest linear dimensions of either (more than four times as great in his own experiments), the deflexions ϕ and ϕ' for the "end on" and "broadside on" positions of the deflecting magnet are given by

$$\tan \phi = L_1 r^{-(n+1)} + L_2 r^{-(n+2)} + \&c.,$$

$$\tan \phi' = L_1' r^{-(n+1)} + L_2' r^{-(n+2)} + \&c.;$$

where $L_1/L_1' = n$. He made a series of deflexion experiments, and found that his results could be represented with sufficient accuracy by the formulæ⁸

$$\tan \phi = 0.086870r^{-3} - 0.002185r^{-5},$$

$$\tan \phi' = 0.043435r^{-3} + 0.002449r^{-5}.$$

The following table shows the closeness of the agreement between theory and experiment (r is measured in metres; Φ and Φ' denote observed and ϕ and ϕ' calculated values):—

r	Φ	$\Phi - \phi$	Φ'	$\Phi' - \phi'$
1.1			1 57 24.8	+2.8
1.2			1 29 40.5	-6.0
1.3	2 13 51.2	+ 0.8	1 10 19.3	+6.0
1.4	1 47 28.6	+ 4.5	0 55 58.9	+0.2
1.5	1 27 19.1	- 9.6	0 45 14.3	-6.6
1.6	1 12 7.6	- 3.3	0 37 12.2	-3.2
1.7	1 0 9.9	- 5.0	0 30 57.9	-1.2
1.8	0 50 52.5	+ 4.2	0 25 59.5	-3.4
1.9	0 43 21.8	+ 7.8	0 22 9.2	+2.6
2.0	0 37 16.2	+10.6	0 19 1.6	+5.9
2.1	0 32 4.6	+ 0.9	0 16 24.7	+4.9
2.5	0 18 51.9	-10.2	0 9 36.1	-2.5
3.0	0 11 0.7	- 1.1	0 5 33.7	-0.2
3.5	0 6 56.9	- 0.2	0 3 28.9	-1.0
4.0	0 4 35.9	- 3.7	0 2 22.2	+1.7

We have here a double proof of the law of the inverse square,—first, in the fact that $\tan \phi$ and $\tan \phi'$ can be expressed so accurately by two terms of a series, the first of which contains r^{-3} ; second, in the fact that the coefficient of the first term in $\tan \phi$ is exactly double that in $\tan \phi'$. These researches of Gauss are remarkable, not only for the great generality of the theory, but also for the novelty of the experimental method, and the exceeding accuracy and refinement of the observations. The law of the inverse square has in fact been regarded as settled ever since they were made. They are important from another point of view, to which we shall return presently.

MAGNETIC MEASUREMENTS, RELATIVE AND ABSOLUTE.

The most important magnetic determinations that have to be made are the direction of the axis of a magnet relatively to its mass, the magnetic moment of a magnet, the direction of a magnetic field, and the strength of a magnetic field, or its component in any given direction. In most of these cases the measurement may be either relative or absolute. For example, we may determine the moment of a magnet either relatively, in terms of the moment of some other magnet arbitrarily chosen, or absolutely, in terms of the fundamental units of space, mass, and time. The complete theory of measurements of the latter kind is due to Gauss, and the carrying of them into practice to him in conjunction with Weber and the *Magnetischer Verein*, of which these two German philosophers were the leading spirits. We shall discuss the

¹ *Mem. d. Acad. Real d. Sc. d. Lisboa.*

² See *Ency. Brit.*, supplement to 3d ed., 1801.

³ *Mém. de l'Inst.*, 1785, 1788.

⁴ *Gren's Journal*, 1811; *Gilb. Ann.*, 1820.

⁵ *Magnetismus der Erde*, 1819.

⁶ *De Magnetismo Telluris*, 1806-10.

⁷ *Jamieson's New Edinburgh Journal*, 1831.

⁸ *Intensitas Vis Magneticæ*, &c., § 21, 1833.

matter here only in so far as it concerns the work of a physical laboratory, the rest belonging more properly to the subject of Terrestrial Magnetism (see METEOROLOGY).

Determination of magnetic axis, and of the magnetic declination.

Axial Direction and Magnetic Declination.—The magnet is suspended, usually by means of one or more fibres of unspun silk, so as to be free to move about a vertical axis. We shall suppose, for simplicity, that the magnetic axis is in a horizontal plane. If this is not so, instead of determining the axial direction, we determine a vertical plane through it. In order to obtain a fixed line of reference in the magnet, two marks may be made on it as nearly in the direction of the axis as can be guessed to begin with; this arrangement is used with dipping needles and also for horizontal needles when no great accuracy is required. For declination needles two contrivances of greater refinement are used.

Mirror method.

1. A mirror is rigidly attached to the magnet, so that the normal to its surface is nearly parallel to the magnetic axis. The image of a fixed horizontal scale in this mirror is observed by means of a fixed telescope, and the angular motion of the magnet deduced from the motion of the scale divisions over the wires of the telescope. This is called the mirror method.¹

Collimator magnet.

2. A more compact arrangement is to attach to the magnet a small photographic scale and a lens, the former being placed at the principal focus of the latter, so that the line joining the middle division of the scale to the optical centre of the lens is nearly parallel to the axis of the magnet. The scale is viewed through the lens by means of a fixed telescope, and it is clear that the line just mentioned gives us a fixed direction in the magnet, and that the motion of the magnet can be followed by observing the apparent motion of the scale across the wires of the telescope. This may be called the collimator method.²

Unifilar magnetometer.

The apparatus usually employed in the United Kingdom for observing the magnetic declination,³ and also for other absolute magnetic measurements, is the portable unifilar magnetometer, the upper part of which is shown in figs. 28, 29. The lower part consists simply of a tripod stand supporting three V-shaped grooves, into which the points of the levelling screws attached to the fixed limb of the instrument are set. In making an observation of the declination the instrument is arranged as in fig. 28. The declination collimator magnet is suspended in the box A (the sides of which are removed to allow the interior to be seen) by means of the suspension fibre D, attached to the torsion head PH. The scale of the magnet is observed through the small telescope QBG. The first step is to remove the torsion as far as possible from the suspension fibre by hanging to it a brass plummet E of the same weight as the declination magnet. After this weight has come to rest, it is replaced by the declination magnet, so that the latter shall rest as nearly as possible in the magnetic meridian without introducing torsion of the fibre. The movable limb is now turned till some division of the magnet scale is on the cross wires of the telescope. It is then clamped. The magnet is now inverted, and the number of the scale division on the wires again read. The mean of these readings gives the point of the scale the line from which to the centre of the collimating lens is parallel to the axis of the magnet. This point of the scale (axial point) will remain the same so long as the magnetism of the magnet does not alter, or the adjustment of the scale and collimating lens is not interfered with. The tangent screw is now worked until the cross wires of the telescope are on the axial point of the scale. The verniers of the limb are then

read. The next step is to observe the azimuth of the sun or of some other heavenly body, by means of which we can refer the azimuth of the magnetic meridian to the true north. For this purpose the instrument is provided with a small transit mirror NO, which has a motion in altitude so as to bring any object into the field of the telescope. To use it, the limb is unclamped, and it and the

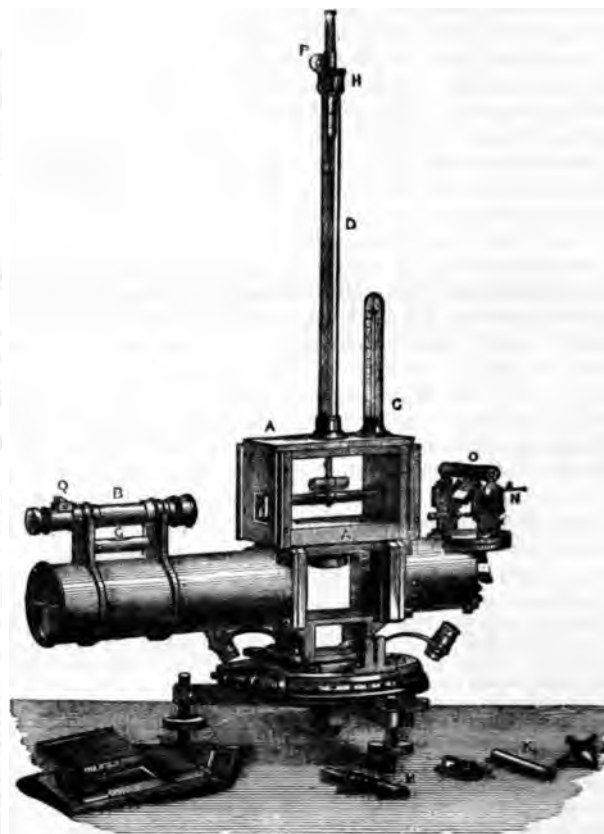


FIG. 28.—Unifilar Magnetometer, arranged to indicate declination.

mirror moved until the sun or star comes into the field of the telescope; the limb is then clamped and the time noted at which the heavenly body passes the intersection of the cross wires. The verniers are again read. The differences of the readings, added to the azimuth of the heavenly body found by means of the time from the *Nautical Almanac*, gives the declination at the time and place of observation.

There are several causes of error to be guarded against. (1) Torsion is reduced within as small limits as possible to begin with, and if there is reason to suspect any residual error we may test the apparatus by turning the torsion head of the suspension tube first 90° one way and then 90° the other. If the deflexion of the magnet is exactly the same and oppositely directed in the two cases, then we may conclude that the torsion is zero in the azimuth of equilibrium. If not, then we may turn the torsion head so as to reduce the error still further; or we may calculate its amount (assuming torsion to be proportional to twist) from the two observations, and allow for it. (2) If the axis of the magnet is not very nearly parallel to the line of collimation of the telescope to begin with, and consequently the two scale readings far apart, an error may arise⁴ from the vertical axis of suspension not being exactly reversed by the inversion. This error is reduced by repeating the observation, after adjusting the axis of the magnet and telescope so as to be more nearly parallel. (3) If mean declination for a given day be desired, correction must be made for the diurnal variation, and under certain circumstances this variation may even produce disturbances in the course of a single observation.

Magnetic Moment, Horizontal Intensity of the Earth's Magnetic Force.—If merely relative measurements of the magnetic moment K of a given magnet, or of the horizontal intensity H of the earth's force, are desired, there are two methods of obtaining them. The first is the method of vibrations. Having found the moment of inertia A of the magnet M about its vertical axis of suspension, and the

¹ The mirror method was first suggested by Poggendorff (*Pogg. Ann.*, vii., 1826). It was carried out in practice by Gauss.

² There seems to be some doubt to whom the collimator method is due. Airy, Lloyd, Lamont, and Weber all did something for it. See Lamont, *Handb. d. Magnetismus*, p. 154.

³ Want of space compels us to omit all but the leading points. Readers in search of full practical details must be referred to *The Admiralty Manual of Scientific Enquiry*, pp. 84 sq.; Maxwell, *Electricity and Magnetism*, §§ 449 sq.; Lamont, *Handbuch der Magnetismus*, and *Erd-Magnetismus*, where references to all the authorities up to his time will be found. They should also study the classical memoirs of Gauss to be found in the fifth volume of his collected works.

⁴ See Swan, *Trans. Roy. Soc. Edin.*, vol. xxi., 1855.

time T of its vibration under the earth's force, we obtain the product $KH = p$, say. Secondly, by the method of deflexion, of which two varieties, tangent deflexion and sine deflexion, are in use, the value of the quotient $K/H = q$, say, is found. In this method K is used as the *deflecting* magnet, and the moment K' of the *deflected* magnet does not appear in the result.¹ It is obvious that, if we know the value of H , or may assume it constant, either of these methods will enable us to express the moment of any magnet in terms of that of another arbitrarily chosen as unit; and, reciprocally, if we operate with a magnet of known or of constant moment, we can determine the values of H at different times and places in terms of its value at an arbitrarily chosen time and place.

By combining two observations, in one of which a magnet K is the vibrating and in the other the deflecting magnet, we can obtain both K and H in absolute measure, for we have two equations $KH = p$, and $K/H = q$, which give

$$K = \sqrt{pq}, \text{ and } H = \sqrt{p/q}.$$

Vibration Experiments.—If θ be the angle between the axis of the magnet and H at time t , γ the angle between the axis and H in the position of no torsion, τKH the coefficient of torsion, then the equation of motion of the magnet, when the arc of oscillation is very small, may be written

$$A\ddot{\theta} + KH\theta + \tau KH(\theta - \gamma) = 0. \quad (59)$$

This gives for the period of a complete vibration

$$T^2 = 4\pi^2 A / KH(1 + \tau). \quad (60)$$

The observations are made with the magnetometer arranged as for the declination experiment. The swinging magnet is brought to rest, and the circle so clamped that the axial point of the magnet scale is on the cross wire of the telescope; the magnet is then slightly disturbed so as to oscillate through a small arc (16' or so). The time of vibration is found first roughly, by taking the time of a single vibration, then more accurately by counting a large number of vibrations and timing the end of the last as accurately as possible. τ is found by observing the deflexion θ' and θ'' caused by turning the torsion head through an angle β in one direction and then through an angle β in the opposite direction; we thus get from equation (59)

$$KH\theta' + \tau KH(\theta' - \gamma - \beta) = 0,$$

$$KH\theta'' + \tau KH(\theta'' - \gamma + \beta) = 0;$$

and $\tau = (\theta' - \theta'') / (2\beta - \theta' + \theta'')$. From the same equation we may also determine γ when necessary.

The most troublesome part of the whole process still remains, viz., the determination of A . This is effected by attaching to the magnet a body whose moment of inertia B can be calculated from its dimensions. For this purpose Gauss fixed a cross bar of wood to the magnet, and attached to it at known equal distances from the axis of suspension two cylindrical weights of known mass and dimension. Sometimes a cylinder of gun metal is slung below the magnet by means of two loops. Perhaps the best method is to use a ring of gun metal attached to the magnet so that its plane is horizontal and its centre as nearly as possible in the line of suspension. The new time of vibration being T_1 , and the new coefficient of torsion (if different) τ' , we have the new equation

$$T_1^2 = 4\pi^2 (A + B) / KH(1 + \tau').$$

From this and (60) we get

$$A = B \left/ \left(\frac{1 + \tau}{1 + \tau'} \cdot \frac{T_1^2}{T^2} - 1 \right) \right.$$

¹ This important fact was first noticed by Lambert.

There are several corrections which, although in general negligible, may sometimes require to be considered. (1) H may vary so much during the experiment as to cause a sensible error; (2) if the arc of vibration be too large, it may be necessary to apply the reduction to infinitely small arcs; (3) if the amplitude of the vibrations decrease too rapidly, account must be taken of the resistance to the motion arising from the viscosity of the air, &c.;² (4) a correction has to be made for the alteration of the moment of the magnet by the earth's induction,³ and (5) a temperature correction for the magnetic moment and the moment of inertia.

Deflexion Experiments.—In Gauss's arrangement the deflecting magnet was placed in an east-west direction, i.e., end on to the original position of the deflected magnet. The equation of equilibrium in this case is [see equation (51)]

$$K'H(1 + \tau) \sin \theta = \cos \theta \left(\frac{2KK'}{r^3} - \frac{T_1}{r^4} + \frac{T_2}{r^5} + \dots \right)$$

or

$$\frac{r^3 H(1 + \tau)}{2K} \tan \theta = 1 - \frac{P_1}{r} + \frac{P_2}{r^2} + \dots \quad (61)$$

where $P_1 = T_1/2K'K$, $P_2 = T_2/2K'K$, &c.

In the method of sines the deflecting magnet is turned until it is perpendicular to the axis of the deflected magnet in its final position of equilibrium; the equation of equilibrium in this case is

$$\frac{r^3 H}{2K} \sin \theta = 1 - \frac{P_1}{r} + \frac{P_2}{r^2} + \&c. \quad (62)$$

The advantage of the method of tangents is that the moment of the deflector is not affected inductively by the earth's force. In the method of sines a correction has on this account to be made; but, on the other hand, there is no torsion, and, from the symmetry of the position of the two magnets, the approximate formulæ have a more exact application.

The new pattern of the unifilar magnetometer is adapted for the method of sines. The instrument arranged as in fig. 29 is first carefully levelled, and fitted with the graduated cross bar D , which

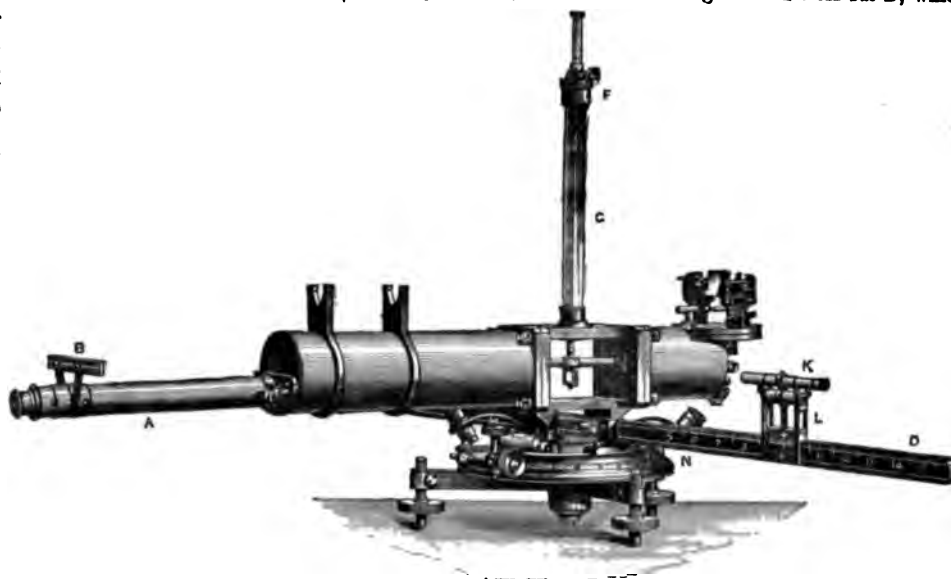


FIG. 29.—Unifilar Magnetometer, arranged to show deflexion.

is so set in its sockets as to be perpendicular to the line of collimation of the telescope A . The box is opened, and the torsion removed from the suspending fibre by means of a plummet as already explained. The deflected magnet is then suspended so as to be at the same height as the deflecting magnet when the latter is placed in its carriage on the cross bar. The sides of the box are now closed, and the circle of the instrument turned until the middle division of the scale B , seen by reflexion from a mirror attached to the deflected magnet, is on the cross wires of the telescope A ; the circle is then clamped, and the verniers read. The deflecting magnet K (the same as that used in the vibration experiments) is next placed in its carriage L on the cross bar at a distance r_1 (30 cm. or so) east; the circle is then turned until the middle division of the scale is again on the cross wires; the verniers are read once more. The difference between the two readings being θ_1 , we have

² In this connexion see more especially Lamont, *Handb. d. Magnetismus*, pp. 282 sq.

³ See Lamont, *Handb.*, p. 371; Maxwell, vol. iii. § 457.

$$\frac{r_1^3 H \sin \theta_1}{2K} = 1 - \frac{P_1}{r_1} + \frac{P_2}{r_1^2} + \dots \quad (63).$$

The deflecting magnet is reversed in its carriage, and the whole operation repeated. If the deflexion now be θ_2 , irrespective of sign, then

$$\frac{r_1^3 H}{2K} \sin \theta_2 = 1 + \frac{P_1}{r_1} + \frac{P_2}{r_1^2} + \dots \quad (64).$$

The mean of these gives

$$\frac{r_1^3 H}{4K} (\sin \theta_1 + \sin \theta_2) = 1 + \frac{P_2}{r_1^2} + \dots \quad (65).$$

The magnet is finally removed to a distance r_1 west, and the previous observations repeated; we thus get

$$\frac{r_1^3 H}{4K} (\sin \theta_3 + \sin \theta_4) = 1 + \frac{P_2}{r_1^2} + \dots \quad (66).$$

The mean of (65) and (66) is then taken, and we get

$$\frac{r_1^3 H}{2K} S_1 = 1 + \frac{P_2}{r_1^2} \quad (67),$$

where $S_1 = \frac{1}{2} (\sin \theta_1 + \sin \theta_2 + \sin \theta_3 + \sin \theta_4)$, or, what is practically the same, the sine of the mean of $\theta_1, \theta_2, \theta_3$, and θ_4 . The object in taking the mean of (65) and (66) is to eliminate any error arising from the non-coincidence of the middle point of the cross bar with the axis of suspension.

In order to eliminate P_2 , another set of observations are made with a new distance r_2 (26 cm. or so), giving the equation

$$\frac{r_2^3 H}{2K} S_2 = 1 + \frac{P_2}{r_2^2} \quad (68).$$

From (67) and (68) we have finally

$$\frac{K}{H} = \frac{r_1^2 S_1 - r_2^2 S_2}{2(r_1^2 - r_2^2)};$$

$$P_2 = \frac{r_1^2 r_2^2 (r_2^2 S_2 - r_1^2 S_1)}{r_1^2 S_1 - r_2^2 S_2}.$$

When great accuracy is required, several corrections have to be applied:—(1) the moment of the deflector must be corrected for induction; (2) the moment of the deflector must be corrected for temperature; (3) the lengths r_1 and r_2 on the cross bar must be corrected for temperature.

Statical instruments.

Statical Method.—There is another method by which we may determine the product KH , viz., we may oppose a statical couple to the couple exerted by the earth on the magnet in a given position, so that there may be equilibrium; the statical couple, which may arise from the torsion of a fibre, from a bifilar suspension, or other gravitational force, thus becomes the measure of the magnetic couple; and hence KH can be determined in absolute measure. Coulomb's torsion balance experiments are an example of this method. It finds numerous applications in the variation instruments of fixed magnetical observatories, and also in instruments for magnetic observations at sea, but it is very little used in the ordinary work of a physical laboratory.

Measurements by electro-magnetic induction.

Magnetic Measurement by Electromagnetic Induction.—It has been explained in the article ELECTRICITY that, if, either owing to the variation of the magnetic field, or owing to the motion of a closed linear conductor in it, the number of lines of magnetic force N passing in the positive direction through the conductor vary, this variation will cause an electromotive force $-dN/dt$ in the positive direction round the circuit. Let us suppose, to take a simple case, that we have a coil of wire made up of a number of parallel plane circular windings, and that the sum of all the areas of the separate windings is A . If we place this in a field of uniform intensity R , so that the normal to the windings makes an angle θ with R , the number of lines of force passing through the coil will be $N_1 = AR \cos \theta$. If we now suddenly reverse the coil, by turning it through 180° about an axis perpendicular to its normal, the value of N in the new position is $N_2 = -AR \cos \theta$. Hence the integral electromotive force during the motion is $-\int dt dN/dt = N_2 - N_1 = -2AR \cos \theta$, and the whole quantity Q of electricity which passes will be $Q = -2AR \cos \theta / S$, where S is the resistance of the

coil. If Q be found in absolute measure,¹ and A and S be known, we thus obtain the value of $R \cos \theta$. This is the principle of Weber's "earth inductor,"² by means of which the horizontal and vertical components of the earth's magnetic force can be measured, and in consequence the declination and inclination determined.

If the test coil be made very small, so that the portion of the field which it occupies may be supposed uniform, this method may be applied to measure the intensity at different parts of a non-uniform field.³ The small coil is placed with its windings perpendicular to the lines of force, and then suddenly reversed, or, if that be impossible, suddenly removed to a part of the field where the number of lines of force passing through it is zero. The integral electromotive force is of course in the latter case only half what it is in the former. This method is often of use where, owing to the great strength of the field and the consequent disturbances arising from induction, any other method would be utterly useless.

The method of electromagnetic induction may also be applied to measure the component of the magnetic moment of any body parallel to a given line.

Let $aa'bb'$ (fig. 30) be the section of a uniform cylindrical coil of length $2l$, made up of a single layer of flat circular windings of radius b , n to the centimetre. Let the axis of the coil be taken induct for x -axis, and let K be any magnet within the coil, placed with

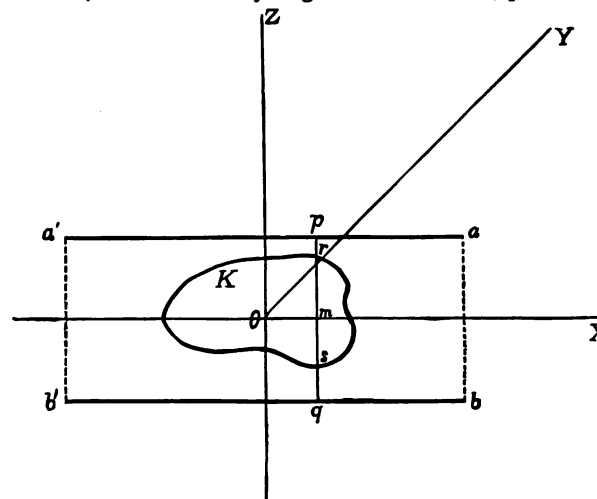


Fig. 30.

the given line parallel to the axis of the coil. Let pq be any single winding of the coil, then the surface integral of the magnetic induction for pq is given by $\iint adydz$; hence the whole number of lines of force through the coil is given by

$$N = \int ndx \iint adydz,$$

$$= n \iiint adxdydz,$$

the integration being extended all over the cylindrical space $abb'a'$. Now, since $a = a + 4\pi A = -dV/dx + 4\pi A$, we get

$$\left. \begin{aligned} N &= -n \iiint \frac{dV}{dx} adxdydz + 4\pi n \iiint A adxdydz, \\ &= -n \left(\iint V dydz - \iint V' dydz \right) + 4\pi n K, \\ &= +4\pi n K - n(S - S') \end{aligned} \right\} \quad (69).$$

where K is the component parallel to the axis of the coil of the moment of the magnet, and S and S' the values of the surface integral of the potential of the magnet (derived from Poisson's distribution) over the two ends of the coil. When there are more layers than one, we must of course sum the different parts of N arising from the different layers.

The formulæ are quite general, and some applications will be given later. Meantime we see that, if the coil be

¹ See arts. ELECTRICITY and GALVANOMETER.

² Pogg. Ann., xc., 1853.

³ Cf. Verdet, Ann. d. Chim. et d. Phys., xli., 1854.

so long that the magnetic potential of the body at its two ends may be neglected, then the integral electromotive force caused by the sudden removal of the body, or by the sudden destruction of its magnetism, is $4\pi n$ times the component of the magnetic moment parallel to the axis of the coil, n being the number of windings per unit of length of the coil.

Historical Remarks on the Progress of Magnetic Measurements.—The method of vibrations came very early into use in magnetic measurements. Whiston and Graham made vibration observations with a dipping needle. Musschenbroek and Mallet also used a horizontal needle. Lambert appears, however, to have been the first to thoroughly understand and appreciate the method. For long it was the only accurate process in use for obtaining relative measures of the earth's force. It was so used by Rossel, D'Entrecasteaux, and Humboldt. Coulomb, Hansteen, and Poisson, all contributed more or less to its improvement; and it finally reached perfection in the hands of Gauss,¹ who gave the experimental process for obtaining the moment of inertia, investigated the correction for resistance, and, by the introduction of the mirror and scale method, imparted astronomical accuracy to the determination of the period of vibration.

The method of deflexion, in one form or another, is very old. Its existence as a thoroughly scientific method, however, dates from Hansteen. The essential improvement of eliminating the constants depending on the magnetic distribution by observations at different distances is due to Gauss. The advantages of the sine method were first pointed out by Lamont in 1841.²

Poisson seems to have been the first to conceive the idea of absolute magnetic measurement. In a short but luminous article at the end of the *Connaissance des Temps* for 1828, he describes a method for obtaining the value of H in absolute measure. Horizontal vibration experiments are to be made with two magnets A and A' , whose moments of inertia A and A' are known. The times of vibration t and t' of A and A' , each suspended alone, are to be observed. Then both are to be placed in the magnetic meridian at a distance r apart in the same horizontal line, and the periods θ and θ' observed, of A when A' is fixed, and of A' when A is fixed. If r be very great compared with the linear dimensions of A and A' , then

$$H^2 = \frac{8\pi^2 \theta \theta' \sqrt{AA'}}{r^2 t t' \sqrt{(\theta^2 - \theta'^2)(\theta'^2 - \theta^2)}}.$$

He recommends, however, that comparatively small values of r be taken, and the constants of distribution eliminated by experimenting at different distances. His fundamental units are the gramme, metre, and second.

Nothing came of Poisson's proposal until Gauss took up the subject, both theoretically and experimentally, as above described. The first absolute measure of the earth's horizontal force was made by him at Göttingen on the 18th September 1832; the value found was 1.782³ in millimetre milligramme second units. The magnet he used (about a foot long and weighing about 1 lb) had for its moment 100877000⁴ in the same units.

The determination of the distribution of magnetism within a body, in other words, the determination of the magnetic moments of its individual elements, by observations of magnetic force at external points, is, as we have seen, an indeterminate problem. Nevertheless, a considerable part of the literature of magnetic science relates to it; and we must give some account of what has been done, although

the results obtained are of comparatively slight physical interest, and of small practical value.

Experimenters have been somewhat slow in recognizing the essential indeterminateness of the problem. This no doubt has arisen from their imperfect analysis of the phenomena. Thus, although we cannot determine the actual internal distribution, yet the problem to determine the Gaussian surface distribution which will represent the magnetic action at all external points, however difficult, is quite determinate. This surface distribution has been called by some the "free magnetism" of the body; and some, all the powerful contrary evidence notwithstanding, have imagined that this distribution has a physical existence, and have even spoken of the depth to which the free magnetism penetrates into the magnet. Others have confounded the free magnetism of Gauss's distribution with that of Poisson's; and in many cases it is impossible to gather what the experimenter meant to indicate exactly by the phrase.

The case in which, from the circumstances, the variation of the internal distribution is confined within the narrowest limits is that of bar magnets, whose length considerably exceeds their lateral dimensions; and this is practically the only case that has been much studied. The most natural way of attempting to represent the action of such a magnet would be to suppose it replaceable by a fixed ideal magnet, and then to determine by experiment the strength and position of the poles of this magnet. The earliest notion was that the poles were situated exactly at the ends of the bar. It was soon found, however, that, if the poles did exist, they were not in general exactly at the ends. Lambert and Kupfer⁵ concluded from their experiments that in many cases the poles lay outside the bar, while in weak magnets they lay inside. Coulomb, as we have seen, and also Dalla Bella, inferred from their results that the poles fell within. Recent experiments have been made by Pouillet,⁶ by Benoit,⁷ by Petruschewsky,⁸ and by others on the same subject; but it is needless to describe them here.

The word "pole," like the phrase "free magnetism," has been used by different writers in very different senses. Some have applied that name to the mass centres of the positive and negative magnetism of the actual molecules. But, although as a matter of convenience we have used these points in our theoretical development, they have, as far as physical observations are concerned, no existence. Others have defined the poles to be the mass centres of the positive and negative parts of Gauss's surface distribution. These might of course be determined, although the process would be extremely troublesome, and the result of no practical value whatever. In point of fact, if the magnet be in a uniform field, *i.e.*, at a very great distance from the system that acts on it, the action depends solely on the magnetic moment, and the magnetic distribution has nothing to do with it; the poles in this case are physically indeterminate. If, on the other hand, two magnets are within a moderate distance of each other, we may set to ourselves the problem to find two points in each of them such that the mutual action will be represented by quantities of positive and negative magnetism concentrated there. Then, in general, such points may or may not exist. Riecke has shown (see above, p. 233) that, if the distance between the magnets exceed a certain limit, then, as a matter of approximation, these equivalent poles, as he calls them, do exist. Except, however, in the case of magnets symmetrical about an axis, and also about an equatorial plane, they are not fixed in the magnets, but

¹ See his memoir, "Anleitung zur Bestimmung der Schwingungsdauer einer Magnetonadel," in *Res. d. Mag. Ver.*, 1837.

² *Handb. d. Magnetismus*, p. 309.

³ 17821, *C. G. S.*

⁴ 10087.7, *C. G. S.*

⁵ See Lamont, *Handb. d. Magnetismus*, p. 294 sq.

⁶ *Comptes Rend.*, 1868.

⁷ *Comptes Rend.*, 1875.

⁸ *Pogg. Ann.*, clii., 1874, and clx., 1877.

depend upon their relative position. Although his results are extremely interesting from a mathematical and theoretical point of view, we do not see that much practical advantage would attend the use of these equivalent poles; and we are inclined to think that, except in the popular usage for distinguishing one end of a magnet from the other, and in the case of ideal magnets, the word pole had better be abandoned altogether.

Distribution in a linear magnet. The idea of representing the action of a linear magnet by a continuous distribution of free magnetism, positive in one half and negative in the other, is very old. It appears in Bazin's work on the magnetic curves published in 1753; and Tobias Mayer, in his memoir above quoted, assumes that the density of the distribution is proportional to the distance from the middle of the bar. Four distinct methods have been used in attempting to determine the law of distribution.

1. The deflexions of a small needle in different positions near the magnet have been observed, and by means of these the constants in some formula assumed for the distribution have been calculated. This was the process adopted by Lambert and Hansteen, and, in some of his experiments, by Lamont.¹

2. Instead of measuring deflexion, we may count the oscillations of the needle, and proceed as before. This method was used by Coulomb, Becquerel, and Kupfer, but it led to no satisfactory results, partly owing to the disturbances arising from induction and the force of translation upon the needle, partly owing to the difficulty of putting a satisfactory theoretical interpretation upon the results.

Different methods employed. 3. Some observers have measured the force required to detach a small armature of soft iron or steel from different parts of the bar, thinking thereby to obtain a direct measure of the free magnetism. It is not very easy to say what is measured by this process, but it is obvious, on a little consideration, that the effect is complex, depending greatly on the nature and extent of the surfaces in contact, and also upon the mutual induction between the magnet and the armature. Experiments of this kind have been made by Dub, Lamont, and others.

4. Another method frequently employed is to slide along the bar a small ring-shaped coil embracing it as closely as possible, and to measure the induction currents for a given displacement. The assumption usually made is that the integral electromotive force is proportional to the free magnetism on the portion of the bar passed over, or, what amounts to the same thing, to the difference between the magnetic moments per unit of length of the sections of the bar on which the coil rests at the beginning and end of the motion. This is, however, only an approximation to the truth, and the accuracy of this approximation is very difficult to estimate in the practical case where the lateral dimensions of the bar are finite. The following investigation will show the nature of the difficulty.

The integral electromotive force is $-(N_1 - N_2)$, where N_1 and N_2 are the surface integrals of magnetic induction taken over the coil in its initial and final positions. Let us take first a linear solenoid SN (fig. 31) of length l , and magnetic moment m , and a coil of a single winding PQ, which moves so that its centre R is always in the line SN, and its plane always perpendicular to SN; then

$$N = \iint adydz = \iint adydz + 4\pi \iint Adydz,$$

the former integral extending all over PQ, the latter over the infinitely small section of the solenoid at R, a being the force due to the end distribution at N and S. We thus get

$$N = \frac{2\pi m}{l} (\cos \theta - \cos \theta') \quad \dots \quad (70),$$

where θ and θ' are the angles PSX and PNX. This shows, in the first place (see equation (25) above), that if the coil PQ were to expand and contract as it moves, so as always to remain a section of the same tube of force, there would be no variation of N , and no

electromotive force, which is as it should be. If we were at liberty to suppose PQ infinitely small, then, when R is between S and N, $\cos \theta - \cos \theta'$ would be the sum of two unities, and, when R is outside, the difference. In such a case, so long as PQ moved on the magnet, there would be no electromotive force, but if we suddenly move it over the end, there would be an electromotive force

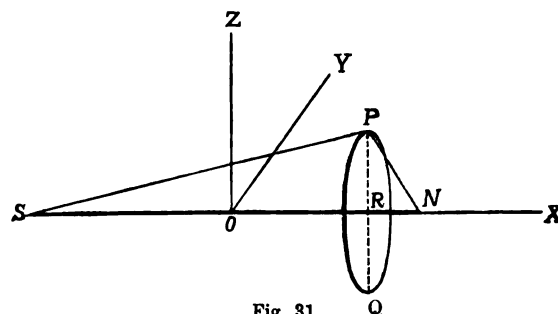


Fig. 31.

$-4\pi m/l$, which is proportional to the moment of the bar. When PQ is not infinitely small, there is a variable part of N , depending on the dimensions of PQ, which will give rise to an electromotive force, even when the coil is moved along a uniformly magnetized bar, where there is no free magnetism except at the ends.

It is now easy to form a conception of what happens in the case of an ordinary complex solenoidal bar. We may suppose such a bar made up of a number of simple linear solenoids. A certain number of these, corresponding to the end parts of Poisson's distribution, will have the same length as the bar; the others, corresponding to the lateral surface and volume parts of the distribution, will be of continuously diminishing lengths. If we were at liberty to suppose the lateral dimensions of the bar and the radius of the coil to be infinitely small, then, as the coil moves along the bar, we should have an electromotive force due to passage over the ends of the short solenoids, and, as it moves over the end, an electromotive force due to passage over the ends of the long solenoids. We might in this way by a sufficient number of observations determine the distribution of the free magnetism throughout the bar and at its ends; and in this case no distinction would be necessary between the volume and the surface distribution in any section.

If, however, the dimensions of the section of the bar, and consequently of the coil, be finite, a correction would have to be applied, depending, not only on the dimensions of the bar and coil, but also on the magnetic distribution. All that we can then do is to assume a formula for Gauss's surface distribution and determine its constants. We thus get Gauss's distribution, and a formula that will account for the electrical observations; but we obtain no information as to the actual internal distribution of the magnetism in the bar.

Lenz and Jacobi² appear to have been the first to apply the method of induction currents to the measurement of the magnetic distribution in bar magnets. They attempted no theoretical analysis of their results, although they assigned a law of distribution. Van Rees,³ who questioned their conclusions, gave an imperfect theory, and made some careful researches of his own. Rothlauf⁴ made further experiments, and entered more fully into the theory, though still with insufficient generality. The most recent experiments of the kind we are aware of are those of Schaper,⁵ who discusses the theory with complete generality, taking account of the ends of the bar.

After what has been said, the reader will scarcely be surprised to find that the different experimenters assigned very different formulæ for the distribution in bar magnets. Lambert deduces from his experiments a distribution whose density is Ax , A being a constant, and x the distance from the ends of the bar. Brugmans, V. Swinden, and Lenz and Jacobi adopt the law Ax^2 ; Hansteen, as we have seen, the law Ax^r , where $r = 2$ or 3 . Biot deduced from Coulomb's experiments the law $A(\mu^x - \mu^{-x})$ for the density of the free magnetism, which would give for the moment per unit of length of the bar the law $a - b(\mu^x + \mu^{-x})$, see above, p. 231. Becquerel,

² Pogg. Ann., lxi., 1844.

³ Over de Verdeeling van het Magnetismus in Magneten, Amst., 1847.

⁴ Pogg. Ann., cxvi., 1862.

⁵ Wied. Ann., ix., 1880.

¹ See also Airy and Stuart, Phil. Mag., 1873.

Van Rees, Lamont, and Rothlauf favour this last formula; but none of these experimenters give any proper account of the ends, which must be specially represented in all but those cases where the magnetic moment is zero there. Schaper finds that the results of experiment can be adequately represented by means of end distributions, and a lateral surface distribution following the law $Ax + Bx^2$. See his paper above quoted, p. 242.¹

Carrying Power of a Magnet.—It is obvious that the magnetization of a piece of iron must affect its force of cohesion. The most familiar case is that of a magnet to which an armature is fitted. If the surfaces of the pole and armature be carefully ground flat, so as to fit, we may regard the magnet and the armature as continuations of each other. The force of cohesion here is mainly due to the magnetism; and the force required to separate the two is called the "carrying power" of the magnet. To simplify the question, let us consider a cylindrical bar of section ω , uniformly magnetized in the direction of its length with intensity I . Suppose the bar cut so that the normal to the plane of section makes an angle θ with I , and let the surfaces of section be separated infinitely little, then the surface density of Poisson's distribution will be $I \cos \theta$ on each surface. Assuming that the cohesion is caused solely by the attraction of these surface layers, we get for the carrying power $P = 2\pi I \cos \theta \times I \cos \theta \omega \sec \theta$, i.e., $P = 2\pi I^2 \omega \cos \theta$. The carrying power is therefore greatest, viz., $2\pi \omega I^2$, when the surface of the pole is perpendicular to the lines of magnetization.

A great variety of experiments have been made on this subject by Joule, Dub, Tyndall, Lamont, and others, mostly, however, under circumstances that do not admit of the application of the above theory. For an account of what has been done, the reader should consult Wiedemann's *Galvanismus*, ii. § 425 sq. The most recent investigations on the subject will be found in the papers of Rowland, quoted below, p. 255, and in papers by Stefan and Wassmuth in the *Monatsberichte der Wiener Akademie* for 1880 and 1882.² The facts are not so simple as the above theory would indicate; but Wassmuth finds a modified form of it to agree sufficiently well with observation.

MATHEMATICAL THEORY OF MAGNETIC INDUCTION.

The two fundamental axioms of this theory are the following:—

1. The induced magnetism in any element of a body depends merely on the magnitude and direction of the resultant magnetic force (\mathfrak{H}) at the element.

2. The magnetic moment induced by any force \mathfrak{H} is the resultant of the magnetic moments induced separately by any forces of which \mathfrak{H} is the resultant.

With reference to axiom 1 it is to be remarked that account must be taken of the physical condition of the body as to temperature, and so forth; but it is implied that no account is to be taken of its magnetic state, except in so far as that affects the resultant magnetic force. In other words, it is asserted that the moment induced by any force does not depend upon any pre-existing magnetic moment in the element, and is the same whatever forces may have acted on the element previously. The full significance of these statements will be better appreciated when we come to consider the exceptions to them in case of strongly magnetic bodies. It should also be noticed that it is supposed

that the body has reached a state of magnetic equilibrium, and that by whole resultant magnetic force is understood, not only that arising from the given inducing system, including pre-existing magnetism in the body itself, but also that arising from induced magnetism.

In the mathematical theory no distinction is drawn between the part of the induced magnetism which disappears when the inducing force is removed, and that which remains. If anywhere we contemplate what happens after the removal of the force, it is assumed that *all* the induced magnetism disappears. This important restriction must be borne in mind in applying the results in practice.

Axiom 2 enables us to assign at once the law connecting the components of induced magnetization A_1, B_1, C_1 with the components α, β, γ of the resultant force. If r_1, q_3, p_2 be the components parallel to the three coordinate axes of the induced magnetization caused by a unit resultant force parallel to the axis of x , then, by the axiom, the components of magnetization induced by a force α in the same direction will be $r_1\alpha, q_3\alpha, p_2\alpha$; similarly, if p_3, r_2, q_1 be the components due to unit force parallel to the y axis, then the components due to β will be $p_3\beta, r_2\beta, q_1\beta$; and finally, if q_2, p_1, r_3 be components due to unit force parallel to z axis, the components due to γ will be $q_2\gamma, p_1\gamma, r_3\gamma$. Compounding all these, according to the axiom, we get

$$\left. \begin{aligned} A_1 &= r_1\alpha + p_3\beta + q_2\gamma \\ B_1 &= q_3\alpha + r_2\beta + p_1\gamma \\ C_1 &= p_2\alpha + q_1\beta + r_3\gamma \end{aligned} \right\} \dots \dots \dots (71).$$

General law of induction.

Hence the most general expressions for the components of magnetization compatible with our axioms are three linear functions of the components of the resultant force.

Here it is necessary to introduce a classification of bodies according to their magnetic properties.

If equal, similar, and *similarly situated* elements cut Homogeneity and heterogeneity. from different parts of a body have identical magnetic properties, it is said to be "magnetically homogeneous," if not, "heterogeneous."

If equal and similar elements cut around the same point in different directions be identical in their magnetic properties, the body is said to be magnetically "isotropic"; if not, "æolotropic."

These are not cross classifications; for a body (e.g., Iceland spar) may be æolotropic and yet homogeneous, and it might be heterogeneous and yet isotropic. We must regard the coefficients p, q, r of (71) as belonging to a point of the body; and we see that in a homogeneous body they will be the same for all points, whereas in a heterogeneous body they will vary from point to point, i.e., they will be functions continuous or discontinuous of the position of the point.

In the case of an isotropic body it is obvious *a priori* that the induced magnetization must be coincident in direction with the resultant force; the conditions for this are that the coefficients p and q should all vanish, and that $r_1 = r_2 = r_3 = \kappa$. The equations (71) thus reduce to

$$A_1 = \kappa\alpha, B_1 = \kappa\beta, C_1 = \kappa\gamma \dots \dots \dots (72).$$

In an æolotropic body, on the other hand, the coefficients may be all different from zero and from one another; but, as we shall see, at all events in the ideal case contemplated by the mathematical theory, the conservation of energy reduces the number of independent constants by three; while a proper choice of axes reduces it by three more; so that the magnetic properties of any element of an æolotropic body depend virtually on three independent constants.

The theory here given is the generalization of Poisson's theory due to Sir William Thomson. It aims at giving the simplest possible exposition of the results of experiment with the fewest assumptions as to the molecular structure of bodies. We first discuss specially a few of the cases

¹ Various experimenters have attempted to determine the "indifference zone" of magnets under different circumstances, i.e., the line separating the positive and negative parts of the surface distribution. For information as to this and other matters under the present head omitted for want of space, see Wiedemann, *Galvanismus*, ii. §§ 277, 356, 396, 401; and Lamont, *Handbuch*, §§ 6, 27, 63, 64, 65.

² Abstracted in *Wied. Beibl.* 1880 and 1882; see also Von Waltenhofen, *Wien. Ber.*, 1870, and Siemens, *Berl. Monatsber.*, 1881.

more important in practice, and then give a brief account of the general theory with a view to establish some general principles to guide us in the subsequent account of the (often very complex) phenomena observed by experimenters.

Synthetic solution for a sphere in uniform field. *Homogeneous Æolotropic Sphere in a Uniform Field of Inductive Force.*—We suppose that the sphere, to begin with, is not magnetized. If the sphere were uniformly magnetized,¹ with components A_1, B_1, C_1 , then (see above, p. 232) the force inside the sphere due to this magnetization would have for its components

$$\alpha_1 = -\frac{1}{2}\pi A_1, \beta_1 = -\frac{1}{2}\pi B_1, \gamma_1 = -\frac{1}{2}\pi C_1.$$

This uniform force combined with the given uniform force ($\alpha_0, \beta_0, \gamma_0$) of the inductive field would result in a uniform force

$$\alpha = \alpha_0 - \frac{1}{2}\pi A_1, \beta = \beta_0 - \frac{1}{2}\pi B_1, \gamma = \gamma_0 - \frac{1}{2}\pi C_1 \quad (73).$$

It is obvious therefore that the assumption of uniform magnetization will enable us to satisfy the law of induction.

In point of fact, substituting in (71) and transposing, we get three linear equations to determine A_1, B_1, C_1 in terms of $\alpha_0, \beta_0, \gamma_0$, viz.,

$$(1 + \frac{1}{2}\pi r_1)A_1 + \frac{1}{2}\pi r_2 B_1 + \frac{1}{2}\pi r_3 C_1 = r_1 \alpha_0 + p_2 \beta_0 + q_2 \gamma_0, \text{ \&c.} \quad (74).$$

Reduction in the number of induction coefficients. It is easy, by means of these and formulæ given above, to calculate the couple exerted on the inductively magnetized sphere. If we put $\alpha_0 = 0, \beta_0 = F \cos \theta, \gamma_0 = F \sin \theta$, we can calculate the work done on the sphere in turning through 180° about an axis perpendicular to the direction of the field. This, by the conservation of energy, ought to vanish, and we thus get the conditions $p_1 = q_1, p_2 = q_2, p_3 = q_3$. The equations (74) therefore reduce to

$$\left. \begin{aligned} A_1 - r_1 \alpha + p_3 \beta + p_2 \gamma \\ B_1 - p_3 \alpha + r_2 \beta + p_1 \gamma \\ C_1 - p_2 \alpha + p_1 \beta + r_3 \gamma \end{aligned} \right\} \quad (75).$$

Hence, if α, β, γ be parallel to a radius of the central quadric

$$r_1 x^2 + r_2 y^2 + r_3 z^2 + 2p_1 yz + 2p_2 zx + 2p_3 xy = 1,$$

A_1, B_1, C_1 will be normal to the diametral plane of that radius. We have, therefore, by the theory of surfaces of the second degree, the following conclusions.

Three principal magnetic axes. 1. The induced magnetization is not in general in the direction of the inducing force; but there are in general at every point three directions, called the three principal magnetic axes, mutually at right angles to each other, for which the directions of the induced magnetization and of the inducing force coincide. If the axes of coordinates be parallel to these principal axes, the equations (75) reduce to

$$A_1 = r_1 \alpha, B_1 = r_2 \beta, C_1 = r_3 \gamma \quad (76).$$

Principal magnetic inductive susceptibilities. The values of r_1, r_2, r_3 in this case are called the "principal magnetic inductive susceptibilities." Bodies for which these coefficients are all positive are called paramagnetic or ferromagnetic. Bodies for which they are all negative are called diamagnetic. No substance is known for which some are positive and others negative, although this is a mathematically possible case. Since intensity of magnetization and resultant magnetic force are of the same dimension [$L^{-1} M^{\frac{1}{2}} T^{-1}$], r_1, r_2, r_3 are pure numbers; for all substances except iron, nickel, and cobalt, they are extremely small. The value of the coefficients r and p for any other axes can be expressed in terms of the three principal susceptibilities by means of simple formulæ which we need not stop to deduce.

A physical meaning can be given to r_1 , as follows. Let the body be homogeneous, and let us cut from it a cylindrical piece whose axis is parallel to the principal axis of susceptibility r_1 . Place this cylinder in the direction of

¹ Here and in future the suffix 0 denotes components of magnetizing force, &c., due to given or pre-existent magnetization; while the suffix 1 denotes those due to induced magnetization. Letters without suffixes denote totals; e.g., $\alpha = \alpha_0 + \alpha_1, A = A_0 + A_1$, and so on.

the lines of force in a uniform field of unit strength, then, provided the cylinder be infinitely thin, and of longitudinal dimensions infinitely great compared with its lateral, the internal force due to the induced magnetization will be zero (see above, p. 229), and it will be magnetized inductively with a uniform intensity r_1 . Similarly for r_2, r_3 .

The three coefficients

$$\omega_1 = 1 + 4\pi r_1, \omega_2 = 1 + 4\pi r_2, \omega_3 = 1 + 4\pi r_3,$$

used later on, are called by Thomson the three principal permeabilities of the body at any point. These are of course pure numbers, and they are positive for all known substances.

2. If the susceptibilities for any two principal axes be equal, then every axis in the plane of these two is a principal axis.

3. If all three principal susceptibilities be equal at any point, then every axis through that point is a principal axis, and the susceptibility for every such axis is the same. The body is therefore isotropic at that point, and the direction of the induced magnetization coincides with the direction of the inductive force for every direction of the latter.

Returning to the problem of the æolotropic sphere, let us simplify our equations by taking the coordinate axes parallel to the common directions of the principal axes throughout the homogeneous sphere. We then get for the components of magnetization

$$A_1 = \frac{r_1}{1 + \frac{1}{2}\pi r_1} \alpha_0, B_1 = \frac{r_2}{1 + \frac{1}{2}\pi r_2} \beta_0, C_1 = \frac{r_3}{1 + \frac{1}{2}\pi r_3} \gamma_0 \quad (77).$$

Using these formulæ, we get, by means of (22), for the components of the couple acting on the sphere (of volume v),

$$\left. \begin{aligned} \mathfrak{X} &= v \frac{r_1 - r_2}{(1 + \frac{1}{2}\pi r_1)(1 + \frac{1}{2}\pi r_2)} \beta_0 \gamma_0 \\ \mathfrak{Y} &= v \frac{r_2 - r_1}{(1 + \frac{1}{2}\pi r_2)(1 + \frac{1}{2}\pi r_1)} \gamma_0 \alpha_0 \\ \mathfrak{Z} &= v \frac{r_1 - r_3}{(1 + \frac{1}{2}\pi r_1)(1 + \frac{1}{2}\pi r_3)} \alpha_0 \beta_0 \end{aligned} \right\} \quad (78).$$

There is of course no force of translation. As a special case let us suppose r_1, r_2 , and r_3 to be in descending order of algebraical magnitude, and suspend the sphere with the axis of r_1 perpendicular to the lines of force. We may put $\beta_0 = F \cos \theta, \gamma_0 = F \sin \theta$, where θ is the angle between the axis of γ (r_3) and the direction of the field, then we have

$$\mathfrak{X} = \frac{1}{2} v F^2 (r_2 - r_3) \sin 2\theta / (1 + \frac{1}{2}\pi r_2)(1 + \frac{1}{2}\pi r_3).$$

Hence the sphere tends to turn so as to place the axis of algebraically greatest susceptibility parallel to the lines of force. It will be in equilibrium when either principal axis is parallel to the lines of force; but in stable equilibrium only when the axis of greatest permeability is in that position. It is to be noticed that the couple is proportional to the square of the strength of the field.

There is another way of expressing these results more in accordance with the ideas of Faraday.

If N be the surface integral of magnetic induction taken over the meridian section (ω) of the sphere perpendicular to the direction of the vector \mathfrak{B} inside, or, as we may call it, the number of lines of force that pass through the sphere, then we have

$$N = 3F\omega \left\{ \left(\frac{\omega_2}{\omega_2 + 2} \right)^2 \cos^2 \theta + \left(\frac{\omega_3}{\omega_3 + 2} \right)^2 \sin^2 \theta \right\}^{\frac{1}{2}},$$

$$\mathfrak{X} = -\frac{1}{24\pi^2 R} \frac{(\omega_2 + 2)(\omega_3 + 2)}{\omega_2 + \omega_3 + \omega_2 \omega_3} \frac{d(N^2)}{d\theta},$$

R being the radius of the sphere.

From these formulæ we can draw the following conclusions:—

1. The number of lines of force that pass through the sphere is greatest, viz., $3F\omega\omega_2/(\omega_2 + 2)$, when the axis of greatest permeability is parallel to the direction of the undisturbed field, and least, viz., $3F\omega\omega_3/(\omega_3 + 2)$, when the axis of least permeability is in the same position.

2. In any position the number of lines passing through the spherical space is greater for a paramagnetic body, and

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less for a diamagnetic body, than it would be if the sphere were absent.

3. The sphere is in equilibrium when the number of lines of force passing through it is a maximum or a minimum, the equilibrium being stable in the former case, and unstable in the latter.

Homogeneous Isotropic Sphere in Uniform Field.—This case is obtained by putting $r_1=r_2=r_3=\kappa$ in the above formulæ. The magnetization is parallel to the undisturbed field; and the couple vanishes, so that the sphere is in equilibrium in all positions. If the strength of the field be F , we get for the intensity of magnetization

$$I = \frac{\kappa}{1 + \frac{4}{3}\pi\kappa} F = \frac{(\kappa-1)}{\kappa+2} F;$$

also

$$N = \frac{3\kappa}{\kappa+2} F\omega.$$

In order to familiarize the reader with this important case, we give two figures of the lines of force from Sir W. Thomson's *Reprint*, pp. 490, 491,—one for a paramagnetic

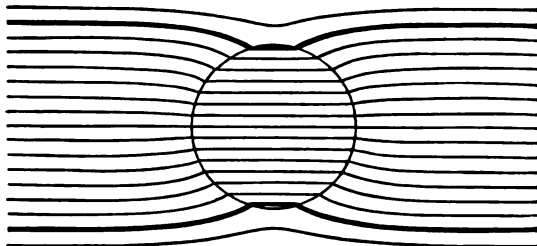


FIG. 32.—Lines of Force for a Paramagnetic Sphere.

(fig. 32) having $\kappa=2.8$, and another for a diamagnetic (fig. 33) having $\kappa=.48$. The former represents a paramagnetic whose susceptibility is something like $\frac{1}{400}$ th of the maximum observed for the best Norway iron. The latter corresponds to a diamagnetic having a susceptibility some 16,000 times that of bismuth, which is the most powerfully diamagnetic substance known.

The reader should observe that, although the field inside the isotropic sphere is uniform, this is not the case outside,

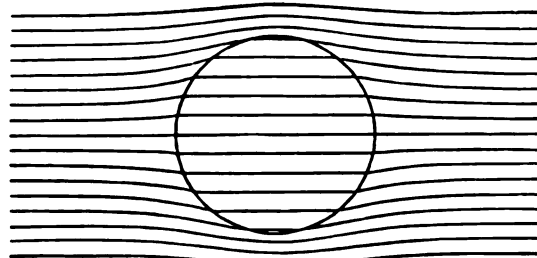


FIG. 33.—Lines of Force for a Diamagnetic Sphere.

a fact sometimes forgotten by experimenters. Of course the disturbance in the case of a bismuth sphere would be infinitesimal.

Homogeneous Anisotropic Ellipsoid in a Uniform Field.—In the case of a sphere the tendency to set in a uniform field is wholly dependent on the anisotropy of the sphere, and is independent of its form. It is important, in order to get a complete picture of the behaviour of inductively magnetized bodies, to obtain a solution for some case where the form has an effect upon the result. A solid bounded by a surface of the second degree affords such a case.

If an ellipsoid be uniformly magnetized so that the components of magnetization parallel to its three principal axes a, b, c be A_1, B_1, C_1 , this magnetization gives rise to a force

$$a_1 = -A_1L, \quad \beta_1 = -B_1M, \quad \gamma_1 = -C_1N;$$

when L, M, N have the values given above, p. 232. If we now

place this ellipsoid in a uniform field (a_0, β_0, γ_0), the force inside will be given by

$$a = a_0 - A_1L, \quad \beta = \beta_0 - B_1M, \quad \gamma = \gamma_0 - C_1N \quad (79).$$

It is obvious, therefore, that the equations (75) of induction can, as in the case of a sphere, be satisfied by the assumption of uniform magnetization.

There is no difficulty in dealing with the general case in which the principal magnetic axes are not parallel to the principal axes of figure; we shall content ourselves, however, with the case in which the principal magnetic axes r_1, r_2, r_3 are parallel respectively to a, b, c . Equations (76) then give at once

$$A_1 = r_1(a_0 - A_1L), \quad B_1 = r_2(\beta_0 - B_1M), \quad C_1 = r_3(\gamma_0 - C_1N) \quad (80);$$

whence

$$A_1 = \frac{r_1 a_0}{1 + r_1 L}, \quad B_1 = \frac{r_2 \beta_0}{1 + r_2 M}, \quad C_1 = \frac{r_3 \gamma_0}{1 + r_3 N} \quad (81).$$

The components of the magnetic moment are of course obtained at once from these by multiplying by the volume.

For the components of couple, $\mathfrak{X}, \mathfrak{Y}, \mathfrak{Z}$, tending to turn the ellipsoid about the axes a, b, c , we get

$$\left. \begin{aligned} \mathfrak{X} &= \frac{4}{3}\pi abc \frac{r_2 - r_3 + r_2 r_3 (N - M)}{(1 + r_2 M)(1 + r_3 N)} \epsilon_0 \gamma_0 \\ \mathfrak{Y} &= \&c., \quad \mathfrak{Z} = \&c. \end{aligned} \right\} \quad (82).$$

From these equations we can draw the following important conclusions,—first as to the magnetization of the ellipsoid.

1. When r_1, r_2, r_3 are so small that their squares may be neglected, as in fact is the case with all bodies except iron, nickel, and cobalt, the components of magnetization reduce to $r_1 a_0, r_2 \beta_0, r_3 \gamma_0$. A glance at equations (79) will show that what happens is simply that the part of the internal inducing force which depends on the squares of the susceptibilities is not sensible. In other words, the form of the body is without influence on the induced magnetization. Or, what is again equivalent to the same thing, the induced magnetism may be supposed to produce no disturbance in the inducing field.

These conclusions are of course not limited to the ellipsoidal form in particular; but we have the general result that, if the squares of the susceptibilities are negligible, then the form of the body has no effect on the induced magnetism.

2. On the other hand, when the susceptibilities (and consequently the permeabilities) are very great, since $A_1 = a_0/(1/r_1 + L)$, &c., it is clear that the influence of the form of the body predominates. The extreme case is that of a body of infinite permeability, in which the induced magnetism is wholly determined by the form.

3. If, however, the ellipsoid be very elongated in the direction of a , then L will be very small, and $r_1 L$ may be very small, notwithstanding the largeness of r_1 . In that case $A_1 = r_1 a_0$.

4. From 1, 2, and 3 we have the following most important results. In experimenting with weakly magnetic bodies in a uniform field—in order, say, to determine their susceptibility—the form of the body is indifferent. On the other hand, with strongly magnetic bodies an elongated form must be used, because in that case only does the induced magnetism depend mainly on the susceptibility of the material. With bodies approaching the spherical form differences in form produce far more effect on the experimental results than differences in the susceptibility of the material, so that in such cases the experimenter really measures the accuracy of his instrument maker¹ more than the magnetic susceptibility of his material.

5. For a flat disk (infinitely oblate ellipsoid), having its r_1 axis parallel to the lines of force, $L = 4\pi$, and $A_1 = r_1 a_0/(1 + 4\pi r_1) = a_0(\kappa_1 - 1)/4\pi\kappa_1$. If such a body were diamagnetic, and had $r_1 = -1/4\pi$, i.e., had zero

¹ As a matter of history, Riecke did unwittingly obtain in this way a tolerable approximation to the ratio of the circumference of a circle to the diameter. See Stoletow, *Phil. Mag.*, 1874, p. 202.

permeability, the normal magnetization would be infinite for any finite force.

Mag-
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Next, we have the following conclusions as to the magnetic couple. Let us suppose the ellipsoid free to move about its a axis, and let the direction of the field be perpendicular to a , so that $\beta_0 = F \cos \theta$, $\gamma_0 = F \sin \theta$; the couple tending to turn the b axis parallel to the undisturbed direction of the field is the sum of two parts:—

$$\mathfrak{X}_1 = \frac{3}{2} \frac{\pi abc F^2 (r_2 - r_3)}{(1 + r_2 M)(1 + r_3 N)} \sin 2\theta \quad \dots \quad (83),$$

and

$$\mathfrak{X}_2 = \frac{3}{2} \frac{\pi abc r_2 r_3 (N - M)}{(1 + r_2 M)(1 + r_3 N)} \sin 2\theta \quad \dots \quad (84).$$

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1. If the susceptibilities are so small that their squares and products are negligible, then \mathfrak{X} reduces to

$$\mathfrak{X}_1 = \frac{3}{2} \pi abc F^2 (r_2 - r_3) \sin 2\theta.$$

For
large
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In other words, the form of the body has no effect, and it behaves exactly like an aeolotropic sphere of the same volume; i.e., it will tend to turn its axis of greatest permeability parallel to the lines of force.

2. If the susceptibilities be very large, then the most important part of \mathfrak{X} will be \mathfrak{X}_2 . Now a glance at the values of M and N (34) shows that $N - M$ has the same sign as $b^2 - c^2$; hence the ellipsoid will tend to place its longest dimension parallel to the lines of force.¹ This is the general effect of the influence of form in the case of strongly magnetic bodies, or, if we choose to put it so, the effect of the disturbance of the field by the induced magnetism.

3. It is of course possible in the case of strongly magnetic bodies that both parts of \mathfrak{X} may be sensible, so that the resultant action would be affected both by form and by the magnetic structure, either predominating according to circumstances; for by properly shaping the ellipsoid we can give $N - M$ any value positive or negative from 0 to 2π . In this way, given an aeolotropic body for which $1/r_3 - 1/r_2$ is not greater than 2π , we might so shape it that it would turn its longest dimensions parallel to the lines of force, or so that it would turn its shortest dimensions parallel to the lines of force, the shortest axis in the second case being the axis of greatest permeability; or we might so shape it that the equilibrium would be neutral.

And, in general, given a body aeolotropic within certain limits, we might shape it in such a manner that the effect of its form would exactly neutralize the effect due to its structure, so that, as far as setting in a uniform field is concerned,² it would behave like an isotropic sphere.

Isotropic
ellipsoid.

Homogeneous Isotropic Ellipsoid in a Uniform Field.—The formulæ for this case are of course at once obtained by putting $r_1 = r_2 = r_3 = \kappa$ in the above formulæ. We thus get

$$A_1 = \frac{\kappa \alpha_0}{1 + \kappa L}, \quad B_1 = \frac{\kappa \beta_0}{1 + \kappa M}, \quad C_1 = \frac{\kappa \gamma_0}{1 + \kappa N} \quad \dots \quad (85);$$

$$\mathfrak{X} = \frac{3}{2} \frac{\pi abc \kappa^2 (N - M) \beta_0 \gamma_0}{(1 + \kappa M)(1 + \kappa N)}, \quad \mathfrak{Y} = \&c., \quad \mathfrak{Z} = \&c. \quad (86).$$

The following conclusions are worthy of notice:—

1. The resultant magnetization will not, as in the case of an isotropic sphere, be parallel to the resultant magnetic force.

Influence
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for a dia-
magnetic
as for a para-
magnetic
body.

2. The ellipsoid will tend to set its longest dimension parallel to the lines of force; and, since κ is involved in the numerator in the form κ^2 , this conclusion is the same for a diamagnetic as for a paramagnetic body. Of course we exclude the mathematically possible case of one of the factors $1 + \kappa M$ or $1 + \kappa N$ vanishing or becoming negative. For all weakly magnetic bodies, however, κ is so small

¹ Assuming, as is always the case in nature, that r_2 and r_3 have the same sign.

² In other respects it would not in general behave as if it were isotropic.

that the tendency of an elongated isotropic body to set in a uniform field is insensible.

Ring Electromagnet.—A simple case,³ which has recently acquired practical importance, is that of an electromagnet having a soft iron core shaped like an anchor ring, whose mean diameter is R , and radius of section a , wound uniformly with n turns of a primary coil in which flows a current i . The lines of force and the lines of magnetization will evidently be circles, and, since the Poisson's surface and volume distributions vanish, the whole magnetic force \mathfrak{H} will be simply that due to the current. At a distance ρ from the axis of the ring $\mathfrak{H} = 2\pi n i / \rho$; for the whole work done on a unit pole in passing round any coaxial circle of radius ρ is $\mathfrak{H} \times 2\pi \rho = 4\pi n i$.⁴ The intensity of magnetization is, therefore, $I = 2\pi \kappa i / \rho$, and $\mathfrak{H} = 2\pi i (4\pi \kappa + 1) / \rho = 2\pi \omega i / \rho$. Hence it appears that the total induction through a secondary coil of n' windings is $2\pi n' i (4\pi \kappa f dS / \rho + f dS' / \rho)$, where $f dS / \rho$ is taken over the section of the core, and $f dS' / \rho$ over the section of the coil. In the case of an anchor ring of circular section, if we neglect the difference between the radius of the primary coil and the radius of the core, the expression for the total induction through the secondary is $4\pi n n' \omega i (R - \sqrt{R^2 - a^2})$.

In a non-uniform field the problem of magnetic induction becomes very difficult for bodies of finite size. If, however, we deal with infinitely small bodies we may suppose the field uniform throughout the body, and apply the results already obtained to find the induced magnetism.

Small Aeolotropic Sphere in a Non-uniform Field.—Let A_1, B_1, C_1 be the components of the induced magnetization parallel to the principal magnetic axes of the sphere, $\alpha_0, \beta_0, \gamma_0$ the components of the strength of the undisturbed field at the centre of the sphere in the same direction; then, denoting $r_1/(1 + \frac{3}{2}\pi r_1)$ by s_1 , and so on, we have $A_1 = s_1 \alpha_0$, $B_1 = s_2 \beta_0$, $C_1 = s_3 \gamma_0$. If the magnetization of the small sphere (of volume v) were rigid, its potential energy W' would be $W' = -v(A_1 \alpha_0 + B_1 \beta_0 + C_1 \gamma_0)$. The actual potential energy, W , of the inductively magnetized sphere is different, because its magnetism varies as it passes from one part of the field to another. In any infinitely small displacement, however, we may calculate the work on the supposition that the magnetism is temporarily rigid. In other words, we may put $dW = dW'$, where the latter is taken on the supposition that A_1, B_1, C_1 do not vary, while on the other hand $\alpha_0, \beta_0, \gamma_0$ do vary, because the resultant force both alters its magnitude and its direction relative to the principal axes of the sphere. We thus get

$$dW = -v(A_1 d\alpha_0 + B_1 d\beta_0 + C_1 d\gamma_0).$$

In integrating we must take account of the fact that A_1, B_1, C_1 are variable. Substituting their values, we get

$$dW = -v(s_1 \alpha_0 d\alpha_0 + s_2 \beta_0 d\beta_0 + s_3 \gamma_0 d\gamma_0),$$

whence

$$W = -\frac{v}{2}(s_1 \alpha_0^2 + s_2 \beta_0^2 + s_3 \gamma_0^2) \quad \dots \quad (87).$$

This important formula contains the whole of the theory of the movement of small spherical masses of inductively magnetizable matter in any field of force. We can deduce from it at once the position of equilibrium of an aeolotropic sphere suspended in a uniform magnetic field, with freedom to rotate about a given diameter.

Let λ, μ, ν and l, m, n be the direction cosines of the given diameter from the axes of the sphere, and R the strength of the field; then $W = -\frac{1}{2} v R^2 (s_1 l^2 + s_2 m^2 + s_3 n^2)$. For stable equilibrium W must be a potential minimum, and for unstable equilibrium a maximum, i.e., there is a stable or unstable equilibrium according as $s_1 l^2 + s_2 m^2 + s_3 n^2$ is a maximum or a minimum under the given kinematical conditions, which will be expressed by a relation between λ, μ, ν and l, m, n . It is needless to work out the analytical solution; for it leads to results easily obtainable from formulæ already given. It is important, however, to show the identity of this method of treatment

³ See art. ELECTRICITY, vol. viii. p. 68.

⁴ Kirchhoff, *Pogg. Ann.*, Ergbd. v. 1870. In the same paper he discusses the effect of a rectilinear current in a cylindrical iron wire, and finds that the circular magnetization in a wire of length L gives rise to an apparent increase of the coefficient of self-induction equal to $2\pi \kappa L$.

with Faraday's view of the matter. If a, b, c be the components of the magnetic induction parallel to the principal axes of the sphere, then we get $a = (1 + \frac{1}{2}\pi s_1)Rl$, $b = \frac{1}{2}\pi s_2 Rm$, $c = \frac{1}{2}\pi s_3 Rn$. Whence if N denote the total induction through the sphere¹ in the direction of the undisturbed field, we have, ω being the area of its meridian section,

$$N = \omega(al + bm + cn) \\ = R + \frac{1}{2}\pi R(s_1 l^2 + s_2 m^2 + s_3 n^2).$$

N is thus a maximum or a minimum when $s_1 l^2 + s_2 m^2 + s_3 n^2$ is a maximum or a minimum.

We have therefore established quite generally Faraday's law that *an isotropic sphere suspended in a uniform field with freedom to rotate about any diameter will be in stable or unstable equilibrium according as the number of lines of force that pass through it is a maximum or a minimum. A particular case of this theorem has already been proved above (p. 245) for strongly magnetic bodies.*

We next apply the formula (87) to deduce the force of translation in a heterogeneous field.

1. We see that in a uniform field W is constant so long as there is translation merely without rotation, i.e., there is no tendency in an isotropic or isotropic sphere to move bodily in a uniform field.

2. If we suppose the sphere isotropic (i.e., $s_1 = s_2 = s_3 = \tau$), then $W = -\frac{1}{2}\tau\omega(\alpha_0^2 + \beta_0^2 + \gamma_0^2) = -\frac{1}{2}\tau\omega R^2$. Hence the force tending to move the sphere in the direction of ds is

$$-\frac{dW}{ds} = -\frac{1}{2}\tau\omega \frac{d(R^2)}{ds} = -\tau\omega R \frac{dR}{ds} \quad (88).$$

In other words, the small sphere is subject to a force of which the scalar potential is $\frac{1}{2}\tau\omega R^2$. If then we draw the isodynamic surfaces $R^2 = \text{const.}$, the force on an inductively magnetized isotropic sphere will be everywhere at right angles to these; that is, the direction of this force at every point will be tangential to the lines of slope of the resultant force, viz., in the direction in which that force varies most rapidly. In the case of paramagnetic bodies, for which τ is positive, the spheres will tend to move from places of weaker to places of stronger resultant force; in the case of diamagnetic bodies, for which τ is negative, from places of stronger to places of weaker force. This is the famous law found experimentally by Faraday, and afterwards theoretically established by Sir William Thomson.

It must be carefully borne in mind that the lines of slope of the field are not necessarily coincident with the lines of force, but may cross them at any degree of obliquity.

As strange mistakes have been made in this matter,² it may be well to illustrate this statement by a few examples.

In the case of an isolated north pole the lines of slope coincide with the lines of force, which are straight lines radiating from the pole. In this case a paramagnetic sphere would approach and a diamagnetic sphere recede from the pole along the lines of force.

The lines of force for a rectilinear electric current are circles of which it is the axis; the lines of slope are straight lines radiating from the current. A paramagnetic sphere would therefore move towards, a diamagnetic sphere away from the current in a direction perpendicular to the lines of force.

In the case of an infinitely small magnet, whose lines of force are given by $r = c \sin^2 \theta$, the lines of slope are given by $r = c' \sin^4 \theta / \cos \theta$; and the angle between the line of force and the line of slope at the point (r, θ) is $\tan^{-1} \{ \tan \theta (1 + \cos^2 \theta) / (3 + 5 \cos^2 \theta) \}$.

The theory of isodynamics and lines of slope in the case of plane fields of force, i.e., those for which the potential is given by the equation $d^2V/dx^2 + d^2V/dy^2 = 0$, is remarkably simple. If $\xi = x + iy$, $\eta = x - iy$, we know that $V = \phi(\xi) + \psi(\eta)$, ϕ and ψ being functions depending on the particular case. When these are known the isodynamics are given by

$$\phi'(\xi)\psi'(\eta) = \text{const.}$$

and the lines of slope by

$$\frac{\phi'(\xi)}{\psi'(\eta)} = \text{const.}$$

3. Next suppose an isotropic sphere allowed to move

¹ That is, the number of lines of force passing through the sphere (see above, p. 244).

² See Wiedemann, *Galvanismus* (ed. 1874), ii. 665; Todhunter, *Natural Philosophy for Beginners*, pt. i. § 387.

without rotation in any direction ds . Let the direction cosines of the field relative to its principal magnetic axes be l, m, n , then these are constant during the displacement; and, if R be the intensity of the field, $\alpha_0 = Rl$, $\beta_0 = Rm$, $\gamma_0 = Rn$; whence

$$-\frac{dW}{ds} = \frac{1}{2}\tau\omega(s_1 l^2 + s_2 m^2 + s_3 n^2) \frac{d(R^2)}{ds} \quad (89).$$

Hence, as before, the resultant force of translation on the sphere is along the line of slope, in the direction in which the force increases if the body be wholly paramagnetic, in the opposite direction if it be wholly diamagnetic.

Besides depending on the nature of the field, the force of translation, on account of the factor $s_1 l^2 + s_2 m^2 + s_3 n^2$, depends on the position of the body relative to the lines of force. Bearing in mind the theory of the radii of an ellipsoid, we have the following proposition:—

The force of translation on an isotropic sphere is greatest when its axis of (numerically) greatest magnetic susceptibility is parallel to the lines of force, and least when the axis of (numerically) least susceptibility is in the same position.

Or, using permeability instead of susceptibility,—

The force of translation is greatest for a paramagnetic sphere when its axis of greatest permeability is parallel to the lines of force, for a diamagnetic sphere when the axis of least permeability is parallel to the lines of force, and vice versa.

Or, yet again, in the words of Faraday:—

The force of translation exerted upon a paramagnetic sphere is greatest when it is so placed that the greatest number of lines of force pass through it, whereas in the case of a diamagnetic sphere the force is greatest when it is so placed that the least number of lines of force pass through it, and vice versa.

Approximate Theory of the Action on Bodies of Finite Size in a Non-Uniform Field.—We have seen that, if the square of the susceptibility be negligible, the effect of the form of the body and the disturbance of the field arising from the induced magnetism may be neglected. In that case we may replace the spheres of the foregoing discussion by cubes, and determine the action on a body of finite size by integrating the action on the elementary cubes of which it is composed. Thus the potential energy will be $-\frac{1}{2} \int (s_1 \alpha_0^2 + s_2 \beta_0^2 + s_3 \gamma_0^2) dv$, and the body need not necessarily be homogeneous. From this expression we can deduce the force under given circumstances.

It is quite easy to see, without any mathematical calculation, what will happen in a field of force which diminishes in intensity outwards from an axial line. If we suspend an elongated paramagnetic body with its centre in the axis of the field, it will evidently be in stable equilibrium with its longest dimension placed axially; for if it were slightly displaced every little cube of it would move into a place of weaker force, and would therefore tend to return. If, on the other hand, the body were diamagnetic, it would be in stable equilibrium in an equatorial position; for any displacement from that position would bring every little cube nearer the axis of the field, i.e., into a place of stronger force, and therefore each such cube would tend to return.

General Problem of Magnetic Induction.—It will be instructive to consider for a little the theory of induced magnetism in its most general form.

We shall suppose the induction to arise from given magnetic force ($\alpha_0, \beta_0, \gamma_0$), arising from pre-existing magnetism (A_0, B_0, C_0) or otherwise. Letters with suffix 1 denote components of induced magnetism, of force arising therefrom, and so on. Letters without suffix denote components of total force, total magnetization, &c. Thus V_0, V_1, V denote the potentials due to pre-existent, induced, and total magnetism respectively; and we have $V = V_0 + V_1$, and the like relation in other cases.

We suppose all the media within the field to have definite permeability; but there may be isotropy and heterogeneity to any extent, and discontinuity along given surfaces.

General problem of magnetic induction. Resolving along the principal magnetic axes at (x, y, z) , we get, by the law of induced magnetism, (l_1, m_1, n_1) , (l_2, m_2, n_2) , (l_3, m_3, n_3) being the direction cosines of the axes $\varpi_1, \varpi_2, \varpi_3$,

$$al_1 + bm_1 + cn_1 = \varpi_1(al_1 + \beta m_1 + \gamma n_1) + 4\pi(A_0l_1 + B_0m_1 + C_0n_1),$$
and two similar equations.

Multiplying these by l_1, l_2, l_3 , adding and so on, we get

$$\left. \begin{aligned} a &= s_1\alpha + t_2\beta + t_3\gamma + 4\pi A_0 \\ b &= t_3\alpha + s_2\beta + t_1\gamma + 4\pi B_0 \\ c &= t_2\alpha + t_1\beta + s_3\gamma + 4\pi C_0 \end{aligned} \right\} \dots \dots (90),$$

where

$$s_1 = \varpi_1 l_1^2 + \varpi_2 l_2^2 + \varpi_3 l_3^2, \quad t_1 = \varpi_1 m_1 n_1 + \varpi_2 m_2 n_2 + \varpi_3 m_3 n_3,$$

&c.;—that is to say, given functions of x, y, z .

Besides these, we have the conditions of normal continuity for \mathfrak{B} , viz.,

$$\frac{da}{dx} + \frac{db}{dy} + \frac{dc}{dz} = 0,$$

and at a surface of discontinuity, λ, μ, ν being the direction cosines of the normal at any point from the first medium to the second,

$$(a - a')\lambda + (b - b')\mu + (c - c')\nu = 0,$$

dashed letters referring to values on the first side of the surface, undashed letters to values on the second.

From these we get finally, for the determination of V ,

$$\frac{d}{dx} \left(s_1 \frac{dV}{dx} + t_2 \frac{dV}{dy} + t_3 \frac{dV}{dz} \right) + \&c. + \&c. + 4\pi\rho_0 = 0 \quad (91).$$

and

$$\left\{ s_1 \frac{dV'}{dx} - s_1 \frac{dV}{dx} + t_2 \frac{dV'}{dy} - t_2 \frac{dV}{dy} + t_3 \frac{dV'}{dz} - t_3 \frac{dV}{dz} \right\} \lambda + \left\{ \&c. \right\} \mu + \left\{ \&c. \right\} \nu + 4\pi\sigma_0 = 0 \quad (92).$$

Here

$$\rho_0 = - \left(\frac{dA_0}{dx} + \frac{dB_0}{dy} + \frac{dC_0}{dz} \right),$$

$$\sigma_0 = (A_0 - A_0')\lambda + (B_0 - B_0')\mu + (C_0 - C_0')\nu;$$

i.e., they are Poisson's volume and surface densities for the pre-existing magnetization.

It may be shown by a method¹ essentially the same as that used in the article ELECTRICITY, vol. viii. p. 27, that equations (91) and (92), with the condition that V be continuous everywhere and vanish at infinity, lead to a unique determination of V . When V is known, A_1, B_1, C_1 can be found at once from (90).

Case of homogeneous isotropic medium with no pre-existing magnetization.

In what follows we shall confine ourselves to homogeneous isotropic media, and we shall suppose that in parts of the medium inductively magnetizable there is no pre-existing magnetism. The equations (91) and (92) then reduce to

$$\frac{d^2 V}{dx^2} + \frac{d^2 V}{dy^2} + \frac{d^2 V}{dz^2} = 0 \quad (93),$$

and

$$\varpi \frac{dV}{d\nu} + \varpi' \frac{dV'}{d\nu'} = 0 \quad (94),$$

$d\nu, d\nu'$ being elements of normals drawn inwards in the two media. Equations (90) reduce to

$$a = \varpi\alpha, \quad b = \varpi\beta, \quad c = \varpi\gamma;$$

whence

$$A_1 = -\frac{\varpi-1}{4\pi} \frac{dV}{dx}, \quad B_1 = -\frac{\varpi-1}{4\pi} \frac{dV}{dy}, \quad C_1 = -\frac{\varpi-1}{4\pi} \frac{dV}{dz} \quad (95).$$

Induced magnetization in this case both solenoidal and lamellar.

From these last, combined with (93), we have the important consequence that the induced magnetization is both solenoidal and lamellar. This is true only for homogeneous isotropic media in which the pre-existing magnetism, if any there be, is solenoidal.

In all cases such as we are now considering, the part of the magnetic potential due to induced magnetism may be calculated wholly from surface distributions at the surfaces of discontinuity. If σ_1 be this surface density, we have

$$\frac{dV}{d\nu} + \frac{dV'}{d\nu'} + 4\pi\sigma_1 = 0 \quad (96).$$

From equations (94) and (96)

$$\sigma_1 = \kappa_1 \frac{dV}{d\nu} = \kappa_1' \frac{dV'}{d\nu'},$$

where

$$\kappa_1 = (\varpi - \varpi')/4\pi\varpi, \quad \kappa_1' = (\varpi' - \varpi)/4\pi\varpi'.$$

Let us suppose that a body A of permeability ϖ is sus-

pended in a medium of permeability ϖ' . If the susceptibilities be small, the forces arising from the induced magnetism will be so small that the direction of the normal force at the surface of A will be the same as if the field were undisturbed by its presence.

First, let the medium be vacuum, for which we suppose $\varpi' = 1$, then, if A be paramagnetic (i.e., $\varpi > 1$), κ_1 will be positive, and the surface magnetism will be positive where the lines of force leave the body, and negative where they enter it. If A be diamagnetic (i.e., $\varpi < 1$), κ_1 will be negative, and the magnetic polarity of the body as a whole will be opposite to what it was in the former case.

Secondly, let the surrounding medium have permeability ϖ' , then κ_1 is positive or negative according as $\varpi >$ or $<$ ϖ' ; in the former case A will behave like a paramagnetic body in vacuo, in the latter like a diamagnetic body in vacuo.

It appears then that, by virtue of differential action, a body may behave paramagnetically or diamagnetically according as it is placed in a less or in a more permeable medium than itself.

In practice it is most convenient in general to determine P_1 instead of V . The above equations can be easily modified equated to admit of this. In fact we get at once, remembering that $dV_0'/d\nu' = -dV_0/d\nu$, since V_0 has no discontinuity at the surface of the media,

$$\frac{d^2 V_1}{dx^2} + \frac{d^2 V_1}{dy^2} + \frac{d^2 V_1}{dz^2} = 0 \quad (97);$$

$$\varpi \frac{dV_1}{d\nu} + \varpi' \frac{dV_1'}{d\nu'} + (\varpi - \varpi') \frac{dV_0}{d\nu} = 0 \quad (98).$$

These equations, together with the condition that V_1 be finite and continuous and vanish at infinity, determine V_1 completely. Since the induced magnetization is lamellar, we may write $A_1 = d\phi_1/dx$ &c., we then get by (95)

$$\phi_1 = -\frac{\varpi-1}{4\pi} (V_0 + V_1) = -\kappa (V_0 + V_1) \quad (99),$$

which give the components of moment in terms of the known function V .

The number of cases in which the solution of the induction problem can be worked out is very small. Besides those already treated synthetically, one or two more, affording examples of the general method, will be mentioned in the historical summary below. Meantime, we must not omit to mention an extremely elegant transformation theorem, due to J. Neumann,² which enables us to deduce the magnetic moment of any body as a whole under the action of any forces whatever, when its magnetization in a uniform field is known.

Let A_1', B_1', C_1' be the components of induced magnetization produced in the body A by the uniform field whose components are $a_0', \beta_0', \gamma_0'$; A_1, B_1, C_1 the magnetization produced in A by any field whatever (a_0, β_0, γ_0). Let $\int dv$ denote volume integration throughout A ; and consider the function

$$U = \int dv (a_0 A_1' + \beta_0 B_1' + \gamma_0 C_1').$$

U is known, since we suppose $a_0, \beta_0, \gamma_0, A_1', B_1', C_1'$ to be known. But, since $A_1 = \kappa(a_0 + a_1)$, &c., $A_1' = \kappa(a_0' + a_1')$, &c., we have

$$U = \int dv \left\{ \left(\frac{A_1}{\kappa} - a_1 \right) \kappa(a_0' + a_1') + \&c. \dots \right\} \\ = \int dv (a_0' A_1 + \&c.) + \int dv (a_1' A_1 + \&c.) - \int dv (a_1 A_1' + \&c.).$$

Now the last two terms destroy each other; for they are simply different expressions for the mutual potential energies of the induced magnetism due to (a_0, β_0, γ_0) and to $(a_0', \beta_0', \gamma_0')$, regarded as separate rigid systems, although coincident in position. Hence we get

$$U = a_0' \int dv A_1 + \beta_0' \int dv B_1 + \gamma_0' \int dv C_1,$$

whence

$$\int dv A_1 = \frac{dU}{da_0'}, \quad \int dv B_1 = \frac{dU}{d\beta_0'}, \quad \int dv C_1 = \frac{dU}{d\gamma_0'}.$$

¹ See Thomson, *Reprint of Papers on Electrostatics and Magnetism*, pp. 548-552.

² *Crelle's Jour.*, xxxvii. 44 (1848). The proof given is a modification of Kirchhoff's, *Crelle*, xlviii. 386 (1854).

of For an ellipsoid this gives at once for the components of
solid, moment

$$P = \frac{\kappa}{1 + \kappa L} \int dv_0, \quad Q = \frac{\kappa}{1 + \kappa M} \int dv_{\beta_0}, \quad R = \frac{\kappa}{1 + \kappa N} \int dv_{\gamma_0}.$$

An interesting particular case is that of an infinite cylinder. If P be the component of the moment parallel to its axis, $P = -\kappa \iint dydz (V_{\infty} - V_{-\infty})$. If the inducing system be magnetic bodies at a finite distance, then $V_{\infty} = V_{-\infty} = 0$, and $P = 0$. If the cylinder be magnetized by a spiral current i , of n windings, of whatever form, then, r being the radius of the cylinder, $P = 4\pi^2 r^2 n i$.¹

Generalization of the Theory for Isotropic Media in which κ is not constant.—In the above theory we have supposed the magnetic susceptibility to be constant. This is by no means the case in nature, however. It is of importance therefore to consider how the theory must be modified when we assume κ to be a function of the magnetization already induced. Subject to the restriction (obviously necessary for isotropic media) that the resultant magnetization shall coincide in direction with the total resultant magnetic force (\mathfrak{F}), the most general assumption that can be made is

$$A = f(I)\alpha, \quad B = f(I)\beta, \quad C = f(I)\gamma. \quad (100),$$

where f is a function depending on the nature of the substance. From these equations, by squaring, adding, and extracting the square root, we get $f(I)/I = 1/\mathfrak{F}$, in other words, I , and therefore $f(I)$, are functions of \mathfrak{F} . Hence we may write the equations (95).

$$A_1 = F(\mathfrak{F})\alpha, \quad B_1 = F(\mathfrak{F})\beta, \quad C_1 = F(\mathfrak{F})\gamma. \quad (101).$$

It is easy by means of these to introduce the requisite modifications into the general equations of magnetic equilibrium. For the details we refer our readers to Kirchhoff's memoir in *Crelle's Journal*,² where the matter was first fully worked out. It will be seen at once that the induced magnetization is in general neither solenoidal nor lamellar.

There is one important class of cases in which the conclusions arrived at on the assumption that κ is constant still hold, viz., those in which the induced magnetization is uniform. In such cases I has the same value throughout the body, and κ is therefore constant throughout the body in any one case, although it differs from one case to another. For example, in the case of an ellipsoid the equations (85) above given for the components of magnetization still hold good, provided we understand κ to be defined by the equation

$$\kappa = F \left[\left\{ \left(\frac{\alpha_0}{1 + \kappa L} \right)^2 + \left(\frac{\beta_0}{1 + \kappa M} \right)^2 + \left(\frac{\gamma_0}{1 + \kappa N} \right)^2 \right\}^{\frac{1}{2}} \right]. \quad (102).$$

It is clear, therefore, that by experiments on an ellipsoid placed in a uniform field we could determine the function $F(\mathfrak{F})$, and also test the truth of the mathematical theory. For, A_1, B_1, C_1 being obtained by observation, one of the equations (85) will enable us to determine κ , and the argument \mathfrak{F} can be calculated from $\alpha_0, \beta_0, \gamma_0$ and A_1, B_1, C_1 ; the test of the truth of the theory would be the agreement of the three values of κ obtained from the three equations (85).

Historical Remarks on the History of the Mathematical Theory.—Although the *Tentamen* of Æpinus, published in 1759, and the discoveries of Mayer and Lambert did much to make clear the exact nature of the problems involved in the modern mathematical theory of magnetism, yet the origin of that theory is usually, and with justice, dated from Coulomb.³ Not only did the results of his careful and judicious experiments afford the means of bringing a mathematical theory to the test, but the marvellous sagacity he displayed in analysing the phenomena enabled him actually to lay the foundations upon which such a

theory could be constructed. After him, Biot⁴ and Han-Biot, ⁵ of whose services we have already spoken, are to be reckoned as pioneers. The theory as it now stands was virtually created by Poisson in four of the most admirable memoirs⁶ to be found in the whole literature of mathematical physics. In the first two he investigates expressions for the force due to bodies magnetized in any manner; he then applies his formulæ to the case of bodies inductively magnetized but having no coercive force. Although he confines his investigations to the case of isotropic bodies, he is quite aware of the general nature of the consequences of æolotropy, and in fact distinctly predicts as possible the magnetic phenomena afterwards discovered by Plücker and Faraday. The formulæ he gives are practically identical with those given above (p. 248). He works out in detail the solution for the case of a hollow or solid sphere exposed to any system of inducing forces having a potential,⁷ and in particular compares the results, when the inducing field is uniform, with the experiments of Barlow. In the second memoir he works out the solution of his equations for an ellipsoid in a uniform field, examining specially the case of an ellipsoid of revolution and its extreme cases (see above, p. 245). At the end of this memoir he discusses the disturbing forces on a compass, arising from the earth's induction on any distribution of soft iron, and shows that the given components of the disturbing force are expressed by linear functions of the components of the earth's force, involving nine constants which depend on the quantity and distribution of the iron. The third memoir, on magnetism in motion, is an attempt to explain the phenomena of the deviation of the magnetic needle caused by rotating metal spheres or disks. Although the physical interest of this memoir was in a great measure destroyed by the discoveries of Faraday as to the true nature of this action, yet, as a piece of profound mathematical investigation, this work of Poisson's is still worthy of study; nor is it perfectly certain that his theory will not after all be required to explain certain residual phenomena. The fourth memoir develops the mathematical theory of the deviation of the compass caused by the iron of ships. After Poisson the most important investigators are Green and Gauss. Green's services have already been alluded to in the article ELECTRICITY; we need only mention here his approximate solution of the problem of the magnetic distribution on cylindrical bars, which gives a formula agreeing with that of Biot. The all-important work of Gauss has already been detailed.

In *Crelle's Journal* for 1848 J. Neumann worked out the solution of the induction problem for an ellipsoid of revolution under the action of any conservative system; and six years later, in the same journal, Kirchhoff worked out the case of a circular cylinder of infinite length. We are not aware that the solution of Poisson's equations in particular cases has been carried any farther, unless we include as new the case of a hollow ellipsoid treated by A. Greenhill. G. Greenhill in the *Journal de Physique* for 1881.

The most important contributions to the general theory of magnetism since Poisson are to be found in a series of memoirs⁸ by Sir William Thomson, the first of which appeared in the *Philosophical Transactions* for 1851. He divests the theory of Poisson of all particular assumptions connected with the two-fluid theory, and bases it on a

⁴ *Traité de Physique*, 1816.

⁵ *Magnetismus der Erde*, 1819.

⁶ *Mém. de l'Inst.*, v., 1821 (two memoirs); vi., 1823; and xvi., 1838.

⁷ He did not use the word "potential," although he uses the corresponding function.

⁸ Reprinted in 1872 under the title of *Papers on Electrostatics and Magnetism*.

¹ Kirchhoff, *l. c.*

² *l. c.* 370, 1854.

³ *Mém. de l'Acad. de Paris*, 1780, 1785, &c.

small number of principles drawn from observation. He enters more fully than Poisson had done into the specification of magnetic distribution. He gives simple synthetic solutions of the induction problem for spheres and ellipsoids in a uniform field. He gives for the first time with full generality the theory of induction in æolotropic media, and shows that Poisson's theory thus fully developed leads to all the laws of paramagnetic and diamagnetic action discovered by Faraday, and also to the laws of magnecrystalline action discovered by Plücker and Faraday. The value of his theory was fully recognized by Plücker,¹ and apparently also by Faraday; indeed one of its ablest expositors was Beer² the friend and coadjutor of Plücker. The experimenters who followed these masters were less intelligent, and the theory of Thomson was for a number of years misunderstood or neglected, the result being much fruitless discussion in which the true issues were often confused. Of late the theory has obtained wide currency and the adhesion of every physicist worthy of the name. Quite recently Thomson's theory has been further developed in an interesting paper by Helmholtz,³ chiefly with a view to its application to the phenomena of dielectric polarization.

Plücker.

Beer.

Helmholtz.

For the benefit of the mathematical reader we append a list of the more important papers on the mathematical theory of magnetism that have appeared recently, and are not quoted above:—

Plana, "Mémoire sur la théorie du magnétisme," *Ast. Nach.*, xxxix., 1854; F. Neumann, *Vorlesungen über die Theorie des Magnetismus*, delivered 1857, edited by C. Neumann, 1881; Riemann, *Schwere, Electricität, und Magnetismus*, lectures delivered in 1861, edited by Hattendorf, 1876; Lamont, "Beitrag zu einer mathematischen Theorie des Magnetismus," *Sitzber. d. Bayer. Akad.*, 1862; L. Weber, *Zur Theorie der Magnetischen Induction*, Kiel, 1877, see *Wied. Beibl.*, 1878; Rowland, *Silliman's Jour.*, 1879 (calculation of couple on a body suspended in a heterogeneous magnetic field); Boltzmann, "Magnetisirung eines Eisenringes," *Wied. Beibl.*, 1879; Id., "Ueber die auf Diamagnete wirkende Kraft," *Wien. Ber.*, 1879; Riecke, *Wied. Ann.*, 1881 (approximative solutions of the problem of magnetic induction).

INDUCTION IN STRONGLY MAGNETIC BODIES.

Experiments of Barlow and Christie.

The earliest experiments bearing on the mathematical theory of magnetic induction are those of Barlow⁴ and Christie, who determined the deflexion of a compass needle placed in various positions relatively to spheres of cast iron inductively magnetized by the earth's force. They found that the deflexion α of the compass could be represented by $\tan \alpha = A \sin \theta \cos \theta \sin \phi / r^3$, where θ is the angle between the line of dip and the line joining the centres of the sphere and compass, and ϕ the angle between the plane of these two lines and the plane of the magnetic meridian. It was also found that the deflexion produced by a hollow sphere was as great as that produced by a solid sphere so long as the thickness of the former was not less than the $\frac{1}{12}$ th of its radius.

Magnetic screens.

All these results of Barlow and Christie are in agreement with the theory of Poisson.⁵ Another consequence of great practical importance follows from the mathematical theory, viz., that inside a hollow iron sphere of any considerable thickness the magnetic force is very small in comparison with the external inducing force. Sir William Thomson takes advantage of this principle to render his marine galvanometers independent of external magnetic force by surrounding them with a tube of soft iron.

Along with the experiments of Barlow we may rank

¹ See *Phil. Trans.*, 1858, p. 587.

² See his *Einleitung in die Electrostatik, die Lehre vom Magnetismus, und die Electrodynamik*, published after the death of its accomplished author, under the editorship of Plücker. This is one of the best works on the subject.

³ *Monatsber. d. Ber. Akad.*, 1881.

⁴ Barlow, *An Essay on Magnetic Attractions*, London, 1820.

⁵ See Poisson's first memoir, or Maxwell, *El. and Mag.*, vol. ii. § 433.

those of Plücker⁶ and Dronke⁷ as affording us the means of testing the general applicability of the mathematical theory to the magnetization of soft iron. In Plücker's experiments an ellipsoid of soft iron was fixed in a graduated brass ring with its longest and shortest axes (a and c) in the plane of the ring. When the ring was suspended with the longest axis a vertical in the nearly uniform field between the two flat vertical faces of the poles of an electromagnet, the mean axis b set itself parallel to the horizontal line of force; as the point of suspension was moved along the circumference of the ring a point was reached at which the plane of b and a ceased to set parallel to the lines of force, and the plane of a and c began to do so; ω , the number of degrees between this point and the end of the axis a , was observed. The times of vibration, T_a and T_c , of the ellipsoid, when suspended so that a and c were vertical, were then observed. By the theory we ought to have

$$\tan^2 \omega = T_c^2(b^2 + c^2)/T_a^2(a^2 + b^2).$$

The value of ω calculated by means of T_a and T_c from this formula was $30^\circ 13'$; the value observed was about 29° . The relation connecting T_a , T_b , T_c according to the theory is

$$(a^2 + b^2)/T_a^2 + (b^2 + c^2)/T_b^2 - (c^2 + a^2)/T_c^2 = 0;$$

and the observed values of T_a , T_b , T_c did, in fact, satisfy this equation very nearly. Dronke's experiments on ellipsoids of iron and nickel were of a similar character.

Deviation of the Compass.—One of the earliest and certainly the most important of the applications of the mathematical theory of magnetic induction was the discussion of the deviation of the compass caused by the magnetism of the iron in ships. This disturbance seems to have been first noticed by Wales the astronomer, who accompanied Cook on his voyages of discovery (1772 to 1779). The same thing was noticed during the voyage of D'Entrecasteaux in search of La Pérouse; and Beautemps-Beaupré, who accompanied him, calls attention to the errors thence arising in the surveying of coasts by means of the compass. Flinders,⁸ using the numerous observations made by Wales and by himself, endeavoured without success to construct empirical formulæ for correcting the errors of the compass. He also attempted to correct the errors partially by means of a vertical bar of soft iron placed near the binnacle. Barlow⁹ and Scoresby¹⁰ also occupied themselves with the problem.

The unusually great deviations observed during the Arctic voyage of the "Isabella" and "Alexander" in 1818 attracted the attention of Poisson, and gave rise to his memoir on the subject already alluded to. Important as the matter then appeared, it became still more so after the introduction of iron ships. Investigations both theoretical and experimental were made in England by Johnson,¹¹ Airy,¹² Evans,¹³ Smith,¹³ &c. It is to Smith that the mathematical theory as it now stands is mainly due.

The cause of the deviation of the compass is twofold; it arises partly from the permanent magnetism of the ship, partly from the temporary or induced magnetism. The permanent magnetism of the ship is acquired for the most part during the process of building. The earth's force acts on the iron, and the constant jarring in the process of construction enables it to induce a considerable permanent magnetization, which the ship carries with her to sea. The quantity and distribution of this magnetism will depend greatly on the build of the ship (whether of wood or of

⁶ *Phil. Trans.*, 1858, p. 555.

⁷ *Pogg. Ann.*, 1862.

⁸ *Phil. Trans.*, 1805.

⁹ *Ib.*, 1831.

¹⁰ *Ib.*, 1819 and 1832, &c.

¹¹ *Ib.*, 1836.

¹² *Ib.*, 1839, &c.

¹³ See *Admiralty Manual for the Deviation of the Compass*, 4th ed., 1874; also a very interesting obituary notice of Smith by Sir W. Thomson, *Proc. Roy. Soc. Lond.*, 1874.

iron), and on her position with respect to the magnetic meridian during building. A considerable portion of it is what Airy calls subpermanent; i.e., it diminishes gradually as the ship is worked. This magnetic settling down will take place more rapidly in a steamer which is constantly agitated by the jarring of machinery than in a sailing ship, unless the latter be subjected to shocks from the impact of waves in rough weather. After a time the ship reaches a more or less stationary condition as to permanent magnetism. Along with the phenomenon of subpermanent magnetism has to be classed what is sometimes called the sluggishness of ships' magnetism; this arises from the fact that all the temporary magnetism of a ship which has sailed for some time on any one magnetic course in any one latitude does not at once disappear when the course or the latitude is changed, so that to the permanent magnetism of the ship has to be added a subpermanent magnetism depending on her course and position several days before. It is evident that the cause of disturbance at present under discussion is somewhat capricious, and can only be controlled by constant attention on the part of the mariner.

The temporary induced magnetism depends on the ship's position on the earth, and on her angular position relative to the magnetic meridian; but, so long as the iron in the ship or the position of the compass is not altered, the constants which determine it remain the same; the disturbance can be foreseen, and either allowed for or mechanically corrected with much greater certainty than in the case of the permanent magnetism.

Mathematical Formulas for the Deviation.—Let the origin be at the centre of suspension of the compass card; and let the axes of x , y , and z be drawn in the direction from stern to head, in the perpendicular direction from port to starboard, and vertically downwards respectively, the ship for the present being supposed to be on even keel. Let P , Q , R be the components of the magnetic force parallel to these axes arising from the permanent magnetism of the ship; x , y , z the components of the earth's force; and x' , y' , z' the components of the whole force at the centre of the compass card. Then

$$\left. \begin{aligned} x' &= x + ax + by + cz + P \\ y' &= y + dx + ey + fz + Q \\ z' &= z + gx + hy + lz + R \end{aligned} \right\} \dots (103)$$

are, according to Poisson's general theory, the fundamental equations of the subject. P , Q , R are constants depending on the permanent, and a , b , c , d , e , f , g , h , k constants depending on the temporary induced magnetism.

By a synthetic process of great interest and importance we may show that the nine constants a , b , c , d , e , f , g , h , k are all independent. For example, if we place a rod of practically infinite length with its end before the binnacle, and stretching forward, or with its end abaft the binnacle and stretching aft, it will give rise to the term ax in x' . If a be negative the rod must be finite and it must run under the binnacle, ending a little fore and aft; again, to represent dx , we must have a pair of infinite rods with their ends to starboard and port of the binnacle, and running fore and aft or aft and fore respectively, according as d is positive or negative; finally, to represent gx , a pair of infinite rods with ends above and below the binnacle, running fore and aft or aft and fore respectively. The reader will have no difficulty in completing the scheme, the rule being that the ends lie in the direction of x' , y' , or z' , and the lengths in the direction of x , y , or z .

From equations (103) the deviation of the compass is expressed in terms of the magnetic or of the compass course as follows. Let H be the horizontal force of the earth; H' the horizontal force of the earth and ship; θ the dip; ζ the "magnetic course," i.e., the azimuth of the ship's head eastward from magnetic north; ζ' the "compass course," i.e., the azimuth of the ship's head eastward from the direction of the disturbed needle; $\delta = \zeta' - \zeta$ the easterly deviation of the compass. Then

$$\left. \begin{aligned} \frac{H'}{\lambda H} \sin \delta &= \mathfrak{A} + \mathfrak{B} \sin \zeta + \mathcal{C} \cos \zeta + \mathfrak{D} \sin 2\zeta + \mathcal{E} \cos 2\zeta \\ \frac{H'}{\lambda H} \cos \delta &= 1 + \mathfrak{B} \cos \zeta - \mathcal{C} \sin \zeta + \mathfrak{D} \cos 2\zeta - \mathcal{E} \sin 2\zeta \end{aligned} \right\} (104),$$

$$\text{where } \lambda = 1 + \frac{a+e}{2\lambda}, \quad \mathfrak{A} = \frac{d-b}{2\lambda}, \quad \mathfrak{D} = \frac{a-e}{2\lambda}, \quad \mathcal{E} = \frac{d+b}{2\lambda},$$

$$\mathfrak{B} = \frac{1}{\lambda} \left(\tan \theta + \frac{P}{H} \right), \quad \mathcal{C} = \frac{1}{\lambda} \left(f \tan \theta + \frac{Q}{H} \right).$$

From (104) we get

$$\tan \delta = \frac{\mathfrak{A} + \mathfrak{B} \sin \zeta + \mathcal{C} \cos \zeta + \mathfrak{D} \sin 2\zeta + \mathcal{E} \cos 2\zeta}{1 + \mathfrak{B} \cos \zeta - \mathcal{C} \sin \zeta + \mathfrak{D} \cos 2\zeta - \mathcal{E} \sin 2\zeta} \quad (105),$$

which gives the deviation on any given magnetic course.

From (105) we get by substitution

$$\sin \delta = \mathfrak{A} \cos \delta + \mathfrak{B} \sin \zeta' + \mathcal{C} \cos \zeta' + \mathfrak{D} \sin (2\zeta' + \delta) + \mathcal{E} \cos (2\zeta' + \delta) \quad (106),$$

an equation connecting the deviation with the compass course.

When the deviation is not greater than 20° or so, the (106) may be replaced with sufficient accuracy by

$$\delta = \mathfrak{A} + \mathfrak{B} \sin \zeta' + \mathcal{C} \cos \zeta' + \mathfrak{D} \sin 2\zeta' + \mathcal{E} \cos 2\zeta' \quad (107),$$

where \mathfrak{A} , \mathfrak{B} , \mathcal{C} , \mathfrak{D} , \mathcal{E} , are nearly the natural sines of A , B , C , D , E .

In the above it is supposed that the ship is on even keel. Strictly Effect of we ought to take into account both the pitch and the heel of the heeling ship; in practice the pitch is always so small as to be of no consequence, but the heel, especially in a ship under sail, may be very considerable. When the ship heels through an angle i , the deviation is obtained from the above formulæ by writing a_i , b_i , &c., in place of a , b , &c., where

$$\begin{aligned} a_i &= a, \quad b_i = b \cos i - c \sin i, \quad c_i = c \cos i + b \sin i, \\ d_i &= d \cos i - g \sin i, \quad e_i = e - (f+h) \cos i \sin i - (e-k) \sin^2 i, \\ f_i &= f + (e-k) \cos i \sin i - (f+h) \sin^2 i, \quad g_i = g \cos i + d \sin i, \\ h_i &= h + (e-k) \cos i \sin i - (f+h) \sin^2 i, \\ k_i &= k + (f+h) \cos i \sin i + (e-k) \sin^2 i, \\ P_i &= P, \quad Q_i = Q \cos i - R \sin i, \quad R_i = R \cos i + Q \sin i. \end{aligned}$$

If the soft iron be symmetrical with respect to the fore and aft central line, and if i be so small that its square may be neglected, then

$$\begin{aligned} \lambda_i &= \lambda, \quad \mathfrak{D}_i = \mathfrak{D}, \quad \mathfrak{A}_i = \frac{c-g}{2\lambda} i, \quad \mathcal{E}_i = -\frac{c+g}{2\lambda} i, \quad \mathfrak{B}_i = \mathfrak{B}, \\ \mathcal{C}_i &= \mathcal{C} + \frac{1}{\lambda} \left(c - k - \frac{R}{Z} \right) \tan \theta i = \mathcal{C} + J i; \end{aligned}$$

and if δ represent the deviation for the given compass course ζ' when the ship heels i to starboard, δ the deviation on the same course on even keel, then

$$\delta_i = \delta + \frac{c-g}{2\lambda} i + J i \cos \zeta' - \frac{c+g}{2\lambda} i \cos 2\zeta' \quad (108).$$

The part of the deviation which depends mainly on \mathfrak{A} is called Constant the "constant deviation"; it can only arise from horizontal induction on soft iron unsymmetrically placed.

The part depending mainly on \mathfrak{B} and \mathcal{C} , viz., $B \cos \zeta' + C \sin \zeta'$, Semicircular is called the "semicircular deviation" because it vanishes and changes sign on two diametrically opposite compass courses, or neutral points. The principal coefficient of the semicircular deviation is $\mathfrak{B} = (\tan \theta + P/H)/\lambda$; $\tan \theta/\lambda$ arises from vertical induction in soft iron before or abaft the compass; P/H arises from the permanent magnetism of the ship. The second coefficient $\mathcal{C} = (f \tan \theta + Q/H)/\lambda$ consists of $f \tan \theta/\lambda$, arising from soft iron unsymmetrically placed, and therefore in general very small, and Q/H arising from permanent magnetism. \mathfrak{B} can be reduced to zero by a magnet placed fore and aft with its centre in a transverse vertical plane passing through the compass, \mathcal{C} by means of a transverse magnet in a fore and aft plane through the compass.

In wooden ships the courses for which the semicircular deviation vanishes are nearly north and south; but in iron ships they approximate to those points of the compass towards which the stem and stern lay in building.

The terms $\mathfrak{D} \sin 2\zeta' + \mathcal{E} \cos 2\zeta'$, depending mainly on the constants \mathfrak{D} and \mathcal{E} , are called the "quadrantal deviation." This part is alternately easterly and westerly in the four quadrants, vanishing deviation on four compass courses. $\mathfrak{D} = (a-e)/2\lambda$ is the principal coefficient of the quadrantal deviation; it depends on horizontal induction in symmetrically placed fore and aft or transverse soft iron. It is in general positive, and in that case can be reduced to zero by two transverse rods with their ends symmetrically placed to starboard and port of the compass. In practice two hollow spheres an inch or so thick are used instead of the rods. The other coefficient $\mathcal{E} = (d+b)/2\lambda$ is in general small, as it depends on horizontal induction in soft iron unsymmetrically placed. It is only when the ship heels that this coefficient is in general of any importance.

Whereas the semicircular deviation depends both on the geographical position of the ship and on the state of its subpermanent magnetism, the quadrantal deviation is independent of both, and can be corrected mechanically once for all, or allowed for by means of tables constructed from observations made in any one place. The amount of the semicircular deviation in England does not exceed 10° for wooden ships of war, but in iron-built ships it frequently exceeds 30° even at the standard compass. The quadrantal deviation in wooden ships does not often exceed 1° or 2° ; in ordinary iron ships it ranges from 3° to 7° , but in some armour-plated iron ships of war it has reached as much as $8\frac{1}{2}^\circ$ at the standard compass, and 15° for compasses less favourably placed.

The chief part of the heeling deviation is the term $J\cos\zeta$, depending on the coefficient $J = (e - k - R/Z)\tan\theta/\lambda$. This coefficient may be reduced to zero by increasing or diminishing the earth's vertical force by means of a vertical magnet under the compass.

Deter-
mination
of devia-
tions by
"swing-
ing."

The usual way of ascertaining the deviations of a ship's compass is to "swing" the ship gently round so that her head comes into various positions, and to observe with the compass the magnetic bearing of some well-defined distant point (compass mark) on shore. The true magnetic bearing of this point is then ascertained, which may be done by taking the compass ashore, carefully placing it in a line joining the compass mark with the point on board at which the compass was formerly placed, and then taking the magnetic bearing of the mark once more. Care must of course be taken that there is no local magnetic disturbance at the shore station. The differences between the bearings on board and the bearing on shore give of course the deviations for the various positions of the ship's head.

When the deviations have thus been ascertained they may be either corrected by means of tables, by graphical methods, such as the steering diagram of Napier or the dygograms of Smith, or mechanically as we have partially explained. For full details on the subject the reader should consult the *Admiralty Manual on the Deviation of the Compass*.

Thom-
son's
compass.

Of late years Sir W. Thomson has devoted his great scientific knowledge and well-known practical sagacity and inventive skill to the improvement of the compass. By reducing the size of the magnets and increasing their number he has succeeded in reducing Airy's apparatus for the mechanical correction of the quadrantal deviation within convenient bulk, and by lightening the card and suspension of the magnets in a very ingenious manner (at the same time throwing all the remaining weight as much as possible to the circumference) he has reduced the friction on the pivot to a minimum while retaining a sufficiently long period of vibration to secure perfect steadiness. He has also contrived apparatus for facilitating the determinations of the deviation on different courses and of the heeling error.¹

Experi-
mental
difficul-
ties.

The experimental investigation of induced magnetism reduces itself mainly to the investigation of the dependence of the magnetic susceptibility κ ² (or the magnetic permeability μ) upon the magnetizing force \mathfrak{H} . Confining ourselves to the strongly magnetic metals, iron, nickel, and cobalt, it will be seen presently that κ depends, not only upon \mathfrak{H} , but also upon the magnetic condition of the body at the actual moment when \mathfrak{H} is in action, and upon its previous magnetic history. κ also depends greatly on the temperature, on the state of the body as to purity (notably in the case of iron and steel on the percentage of carbon present), and on the temper. Thus, if we make one experiment on a body by magnetizing it in any way, we permanently alter its magnetic properties, and can restore it to the magnetically virgin condition only by heating it to a high temperature; but in this process we are very apt to permanently alter its molecular condition, so that, although magnetically indifferent, it is physically changed. Owing to the fact, already insisted upon, that we cannot infer the magnetic distribution inside a heterogeneously magnetized body from its external magnetic action, and to the fact, presently to be established, that κ varies with \mathfrak{H} ,

¹ For a description of his compass see art. COMPASS, vol. vi. p. 228. Detailed descriptions of the compass with instructions for its adjustment are issued in the form of a small pamphlet (Maclehose, Glasgow, 1879).

² κ is sometimes called by Continental writers the magnetization function. They have also a habit of speaking of the ratio of whole magnetic moment of a body of any form divided by its volume to the strength of the field in which it is placed as the magnetization function for that particular form. This is a most inconvenient practice, and has led to considerable confusion.

it is of the last importance to choose the experimental circumstances so that both the magnetic field and the induced magnetization shall be uniform, or very approximately so. A further necessity for the fulfilment of these conditions arises from the fact that we must in all cases be able to render an account of the effect of the form of the magnetized body, because the true argument of κ is not the strength of the original field but the whole force \mathfrak{H} due to the original field and the induced magnetism together.

The simplest method for securing a uniform field whose strength can be controlled is to place the body inside a hollow cylindrical coil (usually called the magnetizing spiral), whose length so far exceeds that of the body that the disturbance arising from the ends of the coil may be neglected in the neighbourhood of the body. The results in all cases where the length of the body or core is nearly equal to or exceeds that of the coil are impure, and can only be used with the greatest caution in drawing general conclusions as to the value of κ . The core should always be either exactly or approximately one of the calculable forms, but preferably such that the dimension parallel to the axis of the spiral very much exceeds the others, because in this case the effect of the form is of secondary importance compared with the effect of the susceptibility (see above, p. 245). Thus a very thin cylindrical core is convenient, because the force inside it differs very little from that of the undisturbed field, and any small difference can be easily calculated by supposing the cylinder replaced by a very elongated ellipsoid. On the other hand, a thick cylindrical bar is a bad form of core for the determination of κ , both because the magnetizing force inside it is less than the intensity of the undisturbed field by a large quantity, which it is impossible to calculate, and because the magnetization at the end is not uniform, and the disturbance thereby arising is so great that it may mask the general character of the function κ altogether. A further question arises as to how far the time during which a magnetizing force acts affects the resulting magnetization, whether temporary or permanent. It is also important to consider the disturbances arising during the make and break of the current in the magnetizing spiral. As the resistance in the circuit is usually small, and the self-induction and capacity sensible, oscillatory currents may arise; to these will correspond oscillatory magnetizing forces, which may even vary in sign. When we consider that the permanent magnetization produced by any force may be very much weakened or even altogether destroyed by a smaller force in the opposite direction, it is evident that we have no right to conclude that these disturbances, especially at break, will be without effect upon the permanent magnetization. In order to elude these difficulties, some experimenters have followed the practice of first establishing the current, then gently introducing the core into its place, and finally removing it before breaking the circuit. In this way the disturbances just alluded to are avoided; but another difficulty is raised, for it is clear that in this operation the core passes through a heterogeneous field before it reaches the final position where the magnetizing force is uniform; different parts of it have therefore been subjected successively to different influences, and we are not at liberty *a priori* to conclude that this fact will not influence the results. Perhaps the best plan would be to place the core in its position, and allow the current to rise very slowly to the maximum value required, and then to fall slowly to zero. This, however, is not the place to dogmatize concerning the best method of experimenting; all that is necessary is to furnish the reader with points of

³ Carefully avoiding all shocks or tremors which exercise a very important influence on the induced magnetism, see below, p. 268.

view from which to criticize the experimental results now to be cited.

In the researches of Lenz and Jacobi¹ the magnetic moment of the core was measured by the induction current in a secondary coil placed upon the magnetizing spiral. A considerable portion of their work was directed to proving principles which we here take for granted, *e.g.*, that the magnetizing force is independent of the thickness of the wire of the magnetizing spiral, of the radius of its windings, and so on. They concluded from these experiments that the magnetization is proportional to the magnetizing force; *i.e.*, κ is constant for a given quality, &c., of metal. The experiments of Joule,² which were made independently about the same time, led in general to a similar result. His method consisted in measuring by means of a balance the attraction P between two electromagnets actuated by the same current C . If the magnetization of the core were strictly proportional to the magnetizing force, *i.e.*, to the current, then P would be proportional to C^2 , and P/C^2 would be constant. In most cases this was so; but in two cases, where the cores of the electromagnets were very thin and the windings more than usually numerous, the ratio P/C^2 was found to decrease as the current increased. This shows that the magnetization tends to a maximum value as the current increases, in other words, that, for very large values of \mathfrak{H} , κ decreases.

Müller,³ using the method of deflexions, arrived at a similar conclusion. His cores were 56 cm. long and from 9 mm. to 44 mm. thick, his magnetizing spirals from 48.2 cm. to 53.2 cm. long; his results are therefore impure, and the empirical formula by means of which he represents them of comparatively little importance; but the approach to a maximum of magnetization (saturation) is quite clearly demonstrated. He found, in accordance with theory, that if we increase the external magnetizing force (\mathfrak{H}_0) saturation is more quickly reached in thin than in thick bars. Somewhat similar experiments were made by Von Waltenhofen,⁴ who deduces⁵ from some of his own experiments with very thin cores, and from the experiments of Müller, Weber, and Dub, 1678 to 2125 mm. mg. sec. units of magnetic moment per mg. of iron as the maximum of magnetization. This would give from 1317 to 1668 C.G.S. units for the maximum magnetic intensity in iron. These numbers, derived from more or less impure results, are merely rough approximations, but they agree very well with those derived at a later date by methods less open to theoretical objections.

The approach to saturation may be very neatly demonstrated as follows.⁶ The same current is sent through a galvanometer and through the coil of an electromagnet with a thin core. The electromagnet is so placed that its action on the needle of the galvanometer just compensates the action of the galvanometer coil for a particular strength of current; the needle then points to zero. If now the current be increased, since the increase of magnetization does not keep up with the increase of the current, the action of the coil prevails, and the needle deviates accordingly.

The most extensive and important of the earlier researches into the general nature of magnetic induction are those of Wiedemann.⁷ An epitome⁸ of his results, with references to contemporary or preceding researches in the same direction, will put the reader in possession of almost all the more important general facts

known until the quantitative experiments of Stoletoew, Rowland, and their followers gave a complete account of the general characteristics of the function κ .

In these experiments the method of deflexion was used. The magnetizing spiral was placed magnetic east and west, and in the continuation of its axis was hung a magnetic steel mirror in a thick copper box to damp its oscillations. The deflexions of this mirror, read as usual with a scale and telescope when the core was not in, gave a measure of the current; and the increase of the deflexion on introducing the core gave a measure of the magnetic moment of the core. The cores were cylinders 22 cm. long, 1.35 cm. thick, and the length of the spiral was only 24 cm.,—so that perfectly pure results could not be obtained. To compensate to some extent for the shortness of the spiral, the bars were gently drawn to and fro several times before being placed in the final position for which the reading was taken. In order to measure the permanent magnetism the core was removed, the current broken, the core returned to its former position, and a reading again taken. The conclusions arrived at were as follows.

I. When a steel or iron bar is magnetized for the first time by a current C , the temporary moment K produced during the action of the current at first increases faster than the current, then more slowly, and finally tends to a maximum, as shown by Joule and Müller. The period of quicker increase is more marked in long than in short bars; it shows itself even on remagnetizing bars that have been several times magnetized and demagnetized. As C increases, the maximum of K is reached sooner in thin and long bars than in short and thick bars. Between the period of increase of K/C and its period of decrease there is no period of any considerable length for which it is constant. This last fact may be shown by means of the experiment of Koosen described above; *viz.*, if the compensation be made for very small currents, when the current is increased, at first the electromagnet prevails, and the needle goes to one side of zero, then the current in the coil prevails, and the needle returns towards zero, and finally deviates on the other side.

The point at which the ratio K/C has its maximum for any particular electromagnet is called by Wiedemann the "turning point" (Wendepunkt). The turning point relates to the body as a whole, and the value of the external magnetizing force \mathfrak{H}_0 for which it occurs depends both on the form of the body and on the nature of the metal. It has therefore no very definite physical meaning. It must be carefully distinguished from the "saturation point." Any element of a body is said to be magnetized to saturation when no increase of the magnetic force can increase its magnetization any farther. It may happen, however, that some parts of a body are magnetized to saturation while others are not. With regard to the turning point, Dub⁹ has shown that with similar and similarly wound cores the turning point occurs for the same value of the current. This is of course in agreement with an obvious corollary of the general theory of magnetic induction.¹⁰

II. In a freshly¹¹ magnetized bar the permanent moment which remains after the action of the current has ceased at first increases quicker than the producing current; but for stronger currents a turning point is reached; and then the moment increases more slowly than the current, and approaches a maximum.

III. In attempting to destroy the permanent magnetism of a bar by means of a demagnetizing current, it may happen that a current, which, during its action, already

Experimental method.

Maximum of magnetization and turning point.

¹ *Pogg. Ann.*, xlvii., 1839.

² *Sturgeon's Annals of Electricity*, vol. iv., 1839; *Phil. Mag.*, ser. 4, vol. ii.

³ *Sitzber. d. Wien. Akad.*, 1865.

⁴ *Pogg. Ann.*, lxxix., 1850.

⁵ *Pogg. Ann.*, cxxxvii., 1869.

⁶ Koosen, *Pogg. Ann.*, 1852; also Dub, *Ib.*, 1853.

⁷ *Pogg. Ann.*, c., 1857; *Ib.*, cvi., 1859; *Ib.*, cxvii., 1862.

⁸ Abridged from the author's own work, *Galvanismus*, Bd. ii.

§§ 309 sq.

⁹ *Pogg. Ann.*, 1868.

¹⁰ See Thomson, quoted by Joule, *Phil. Trans.*, vol. cxlvi., 1856.

¹¹ That is, after being heated white hot to destroy all pre-existing magnetism.

Demagnetization easier than magnetization.

produces a temporary magnetic moment of opposite sign, still leaves on ceasing to act a permanent magnetic moment of the same sign as before, although less in amount.¹ On increasing the demagnetizing current still farther the permanent moment is at last destroyed. In this process the permanent magnetism decreases faster than the demagnetizing current increases,—so that *the current required to destroy a given permanent magnetism is less than the current that originally produced it.*²

Unilateral property of demagnetized bar.

IV. When a fresh bar has been magnetized with any permanent moment, and then demagnetized by a current $-C'$, opposite to the magnetizing current C , a second application of $-C'$, or of any weaker current in the same direction, will not produce a reverse permanent moment, although a current C' in the same direction as C will magnetize the bar permanently in the original direction more or less strongly. It follows therefore that demagnetizing by an opposite magnetic force, although it may destroy the permanent magnetism of a body, does not render it magnetically indifferent, as heating to a white heat would do. The body remains in fact more easily magnetizable in one direction than in another.³

V. In certain cases a fresh bar was magnetized by a current C , and then partly demagnetized; it was then found that a current C was required to bring it back to its original permanent moment.

VI. In another case a fresh bar was magnetized by a current C to permanent moment K , then reduced by a demagnetizing current C' to permanent moment K' , then by a direct current C'' less than C brought to permanent moment K'' . It was then found that a current C' was necessary to bring it back to permanent moment K' ; and this held whether K' was positive, zero, or negative.

Repeated magnetization and demagnetization.

VII. When a bar is repeatedly magnetized and demagnetized by currents of the same intensity, the permanent magnetic moments corresponding to a given force become, to begin with, a little greater than at first; to begin with, they increase faster than the magnetizing force, though not so fast as at the first. The turning point, however, occurs for a weaker current than before. The magnetization obtained with the strongest current gradually decreases a little. The moments left by the demagnetizing current decrease less rapidly than before, so that a current at first capable of demagnetizing the bar altogether leaves after repeated magnetization and demagnetization a slowly increasing residual moment. After a large number of repetitions of the operation of magnetization by a current C and demagnetization by a current $-C'$, the bar finally reaches a constant state, so that each magnetization and demagnetization leaves a corresponding invariable permanent moment. When we pass beyond the limits C and $-C'$, these phenomena are repeated in the same order as before.⁴

Temporary and permanent magnetism in hard and soft steel and in soft iron.

VIII. All the above phenomena are most clearly seen in hard steel, less clearly in soft steel and iron. For small magnetizing forces the temporary moment in hard steel is less than in soft steel, and greatest of all in soft iron. The general rule is, the harder the material the less the temporary and the greater the permanent moment for a given magnetizing force.

IX. If, however, we consider the ratios of the temporary moments in soft steel and iron to the temporary moment in hard steel, all for the same force, then these ratios decrease gradually as the force increases; so that the tem-

porary moment in soft iron reaches its maximum sooner than in soft steel, and still sooner than in hard steel.⁵

The earliest experiments from which definite values of κ have been calculated are those of Weber.⁶ A cylindrical bar, 10.02 cm. long and .36 cm. thick, was placed inside a spiral so long that the magnetizing force throughout the length of the bar could be assumed to be uniform. The moment of the bar was measured by the method of deflexion, the action of the spiral on the deflected magnet being compensated by means of a part of its own circuit suitably arranged. The intensity of the current in the spiral was found in absolute measure by means of a tangent galvanometer. Assuming that the bar could be replaced by a very elongated ellipsoid, Kirchhoff calculated by means of the theory explained above (p. 249) the values of κ for values of \mathfrak{H} ranging from 29.6 to 248.4 (C.G.S. units), and found that it decreased steadily from 25.0 to 5.6. In the experiments of Von Quintus Icilius⁷ bars were used which had been reduced by filing as nearly as possible to the form of ellipsoids of revolution. The magnetic moments were measured partly by the deflexion method, partly by the method of electromagnetic induction. In this last method a secondary spiral is placed upon the magnetizing spiral, and the induced current in it, caused by reversing the magnetizing current, is observed first when the ellipsoid is in the magnetizing spiral, secondly when it is not. When these currents are known in absolute measure, the moment of the ellipsoid can be calculated. The experimenter did not himself reduce his results so as to obtain κ , but contented himself with remarking that the ratio of the whole moment of the ellipsoid K to the strength of the undisturbed field \mathfrak{H}_0 reached a maximum as \mathfrak{H}_0 was increased, this maximum occurring for smaller values of \mathfrak{H}_0 the more elongated the ellipsoid. The true meaning of his results was brought out by Stoletoew,⁸ who reduced them, and established the interesting fact that, as the magnetizing force⁹ \mathfrak{H} increases from very small values, κ at first increases rapidly, then reaches a maximum, and afterwards decreases more slowly. For one ellipsoid κ increased from 30.5, for $\mathfrak{H} = .24$, to a maximum 120.4, for $\mathfrak{H} = 4.56$, and then decreased to the value 39.4, for $\mathfrak{H} = 30.07$. In another, the initial value was 20.1 for $\mathfrak{H} = .518$, the maximum value 107.5 for $\mathfrak{H} = 4.92$, and the final value 2.86 for $\mathfrak{H} = 454.1$.

Thalén, adopting a method indicated by Weber,¹⁰ has determined the value of κ for small magnetizing forces. Long bars were placed in the axis of a cylindrical coil considerably exceeding them in length. This coil was caused to rotate 180° about a horizontal axis, so that the magnetization induced by the earth's vertical force was reversed relatively to the coil. The current thus caused was measured by means of the swing of a galvanometer in circuit with the coil; from this (see above, p. 240) the moment of the induced magnetism was calculated; and thence, assuming the bar to be replaceable by an ellipsoid, κ was calculated. From three bars of the same metal each 400.4 mm. long, having diameters of 36.4, 29.94, and 23.87 mm. respectively, the values of κ deduced were 32.32, 31.80, and 32.64. For other specimens of iron he found values of κ ranging from 27.24 to 44.23.

⁵ Similar results by Plücker, *Pogg. Ann.*, 1852 and 1855.

⁶ *Electrodynamische Maassbestimmungen*, Bd. iii. § 26.

⁷ *Pogg. Ann.*, cxxi., 1864. Similar results were obtained by Oberbeck, *Pogg. Ann.*, cxxxv., 1868.

⁸ *Pogg. Ann.*, cxlvi. p. 443, 1872.

⁹ \mathfrak{H} here means the whole magnetizing force, arising partly from the inducing field and partly from the induced magnetism. Experimenters have needlessly complicated the already complex problem of ferro-magnetic induction by neglecting the all-important distinction between \mathfrak{H} and \mathfrak{H}_0 .

¹⁰ *Abh. d. Gött. Gesellschaft*, Bd. 6.

¹ See Poggendorff, *Pogg. Ann.*, 1852.

² This result had also been arrived at by Abria, *Ann. d. Chim. et d. Phys.*, 1844; and by Joule, *Phil. Mag.*, 1847, *Phil. Trans.*, 1855.

³ Similar conclusions were arrived at by Ritchie, *Phil. Mag.*, 1833; Jacobi, *Pogg. Ann.*, 1834; Marianini, *Ann. Chim. et d. Phys.*, 1846.

⁴ On the same subject see Joule, *Phil. Trans.*, 1856; also Von Waltenhofen, *Pogg. Ann.*, 1864.

A set of observations on ellipsoids of revolution were made by Riecke,¹ by the method just described. The ellipsoids, seven in number, were all cut from the same piece of soft iron, but varied in volume and in eccentricity. The resulting values of κ were found to be independent of the volume of the ellipsoids and of the part of the iron from which they were cut; but, on the other hand, with one slight exception, they increased with the eccentricity of the ellipsoids. Kohlrausch, in communicating these results to *Poggendorff's Annalen*, remarked that they stand in contradiction with the theory of Poisson and Neumann; in so saying he probably considered the constant vertical force of the earth (\mathfrak{H}_0) to be the argument of the function κ ; but this is not so, as Stoletow points out in the paper already quoted. The actual magnetizing forces are greater in the more elongated ellipsoids; and Riecke's results simply prove that for values of \mathfrak{H} varying from .031 to .072 κ increases from 13.5 to 25.4.

In order to establish the initial increase of the magnetization function κ beyond all doubt, Stoletow (*l. c.*) made a new set of experiments on a carefully annealed iron ring² of rectangular section (exterior diameter 20 cm., in-

terior diameter 18 cm., height 1.47 cm.). The ring was carefully wound throughout with a primary coil of n ($= 800$) windings; over this, in one or more shorter or longer stretches, was wound a secondary coil of n' ($= 50$ to 750) windings. The induction current in the secondary, due to the reversal of a known current i in the primary, was sent through a galvanometer, and thus measured. If E be the electromotive force of this current, then (see above, p. 246) $E = 4\pi n n' i (4\pi \kappa M + P)$, where M and P can be calculated from the dimensions of the ring and its primary coil. All then that is necessary is to know E/i in absolute measure. We refer the reader to the original paper for the details of the measurements. The results are very interesting, and fully confirm the conclusions drawn from the results of Von Quintus Icilius and Riecke. The smallest value of \mathfrak{H} was .43, and the corresponding value of κ 21.5; the maximum value of κ was 174, for $\mathfrak{H} = 3.2$; the last value observed was $\kappa = 42.1$, for $\mathfrak{H} = 30.7$. The temperature varied from 15°C. to 20°C. , but it appeared from the experiments that κ did not alter much for moderate changes of temperature. In figure (34) is given a transcription of the curve that represents the results of

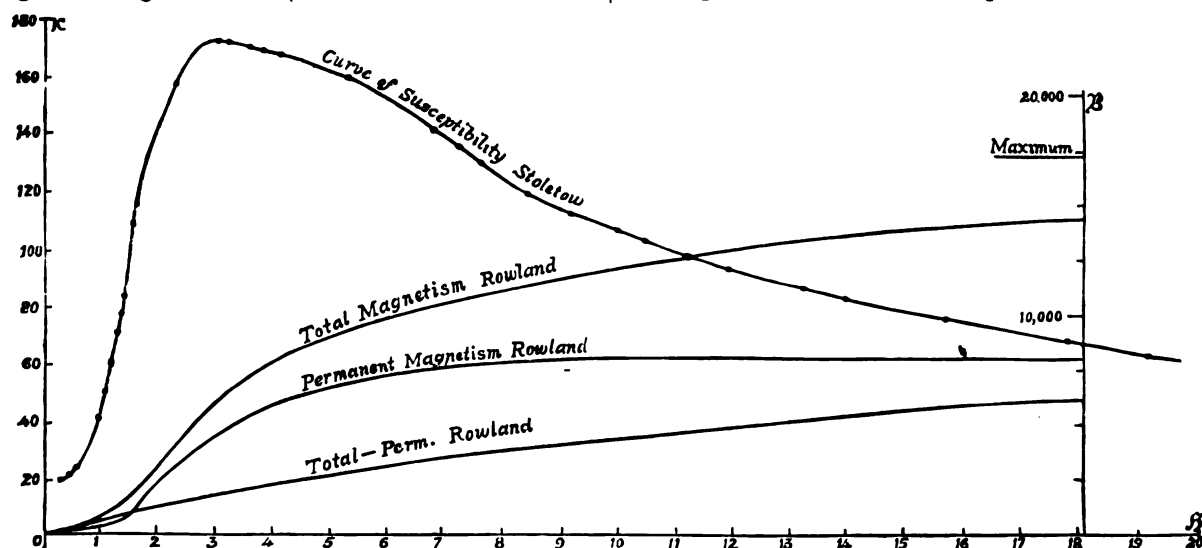


Fig. 34.

Stoletow's experiments; the abscissæ represent the values of \mathfrak{H} in C.G.S. units, and the ordinates the corresponding values of κ .

About the same time as Stoletow, and independently, Rowland³ made a much more extensive series of experiments, the results of which form one of the most important contributions yet made to our knowledge of magnetic induction. The experiments were made partly on very long bars; but the published results were mostly obtained from rings, it having been found that the effect of the ends of the bar was sensible even when the length was as much as 144 times the diameter. About a dozen rings of iron, nickel, and cobalt were used; the section was circular in all cases; and a primary and a secondary coil were used as in Stoletow's experiments. The primary current was measured by means of a tangent galvanometer in which 1, 3, 9, 27, or 48 coils could be brought into operation according to the sensibility required. The induction current in the secondary was measured by the swing of a Thomson's galvanometer fitted with a heavier needle than usual. The indications of this last were reduced to absolute measure by taking the swing caused by turning over a

horizontal coil of known area, inserted in its circuit, so as to produce the full induction current due to the earth's vertical force. In order to obtain the total induced magnetization the primary current was reversed. To obtain the permanent magnetism it was simply broken; this gives the part of the induced magnetism that disappears with the inducing force (temporary magnetism Rowland calls it); subtracting this from the total magnetization, we get the permanent magnetization. Care was taken in these experiments always to work with magnetizing forces of ascending magnitude, as it was found that the effect of any force is considerably modified if a greater force has previously acted on the body,—in other words, that the magnetic permeability of iron or steel is much affected by pre-existing permanent magnetism. This fact raises an objection to the ring method; for permanent magnetization in a ring is not easily discoverable, and would give it a one-sidedness, so that a magnetizing force would produce much more alteration when exerted in one direction than it would when exerted in the other.⁴ Rowland publishes about thirteen different tables, relating to rings of iron and steel in different states, and also to nickel and cobalt, under different conditions as to temper,

¹ *Pogg. Ann.*, cxli., 1870.

² A method suggested by Kirchhoff, *Pogg. Ann.*, Ergbd. v., 1870.

³ *Phil. Mag.*, 1873, 1874.

⁴ See Rowland, *Phil. Mag.* (4), 48, p. 336; also above, p. 254.

magnetization, and so on; the results for cobalt are, however, held to be less satisfactory than those for iron and nickel, for a variety of reasons which he assigns.

In treating his results graphically, two methods are followed. In the first the magnetic induction \mathfrak{B} is plotted against the magnetizing force \mathfrak{H} as abscissa. Figure 34 shows the curve obtained in this way from one of his tables. In the second method (1) the permeability μ is plotted against the magnetic induction \mathfrak{B} , or (2) the susceptibility κ is plotted against the intensity of magnetization \mathfrak{I} . Either variety of the second method leads to a curve having the general form shown in figures 35 and 36.

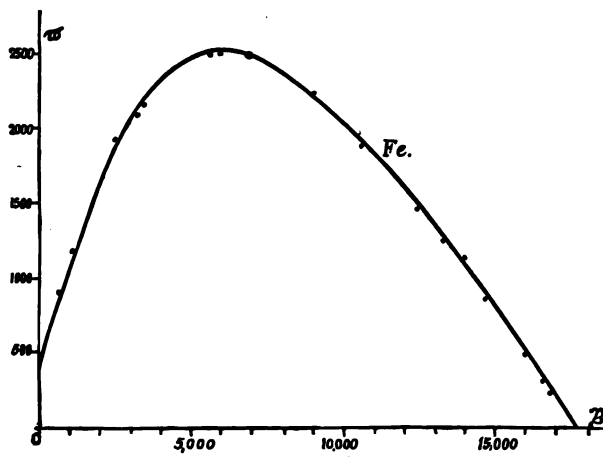


FIG. 35.—Curve $\mu\mathfrak{B}$ for Iron.

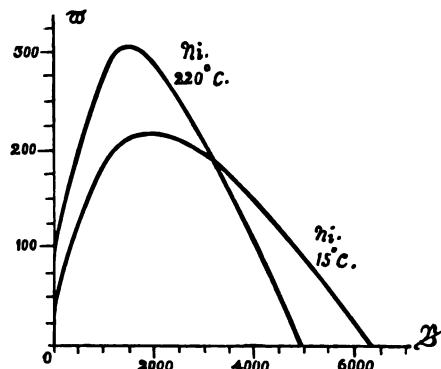


FIG. 36.—Curves $\mu\mathfrak{B}$ for Nickel at different temperatures.

The curves obtained, whether for μ and \mathfrak{B} , or for κ and \mathfrak{I} , fall very rapidly, and ultimately to all appearance almost straight, towards the axis of \mathfrak{B} or \mathfrak{I} . This suggests that \mathfrak{B} or \mathfrak{I} or both reach a maximum when \mathfrak{H} is increased indefinitely. Supposing such an increase of \mathfrak{H} possible, the question arises as to which it is that actually reaches a maximum. Most experimenters seem to assume that \mathfrak{I} does so, but it must be remarked that this is simply an assumption.¹ Several delicate points of great physical interest might be discussed here, but it will be sufficient to refer the reader to the introduction to Rowland's second paper.

The general conclusions to be drawn from these experiments are as follows:—

1. The magnetic properties of iron, nickel, and cobalt at ordinary temperatures differ in degree but not in quality.

2. As the magnetizing force \mathfrak{H} increases from 0 upwards, the permeability of iron, nickel, and cobalt increases until

¹ All the more so that it has been found by some experimenters that the curve ($\kappa\mathfrak{I}$) actually has a point of inflexion and becomes convex to the axis of \mathfrak{I} for very large values of \mathfrak{I} .

it reaches a maximum, and after that diminishes down to a very small value. The maximum value² is reached when the metal has attained a magnetization of from .24 to .38 of the maximum. The following table will give some idea of the order of the magnitudes involved; μ_0 denotes the permeability for $\mathfrak{H}=0$, μ' the maximum permeability, and \mathfrak{H}' the force for which it occurs. In some cases the actual maximum is given, in other cases simply the greatest recorded in the tables of experimental results, and the values of \mathfrak{H} are stated roughly; strict accuracy is of no consequence, owing to the great variability of all the magnitudes.

Shape.	Material.	Temper.	State.	μ_0	μ'	\mathfrak{H}'
Ring.	Very fibrous iron wire.	Annealed.	Burnt.	120	1106	...
Bar.	Soft wire.	"	"	180	2467	...
Ring.	Burden's best iron.	"	Normal.	352	2475	25
"	"	"	Magnetic.	216	2459	24
"	"	Carefully annealed.	Burnt.	544	3621	16
"	Norway iron.	"	"	720	5515	...
"	"	"	Magnetic.	440	4656	12
Bar.	Bessemer steel.	Natural.	"	200	1281	56
Ring.	Stubbs' steel.	"	Normal.	76	331	25.0
"	Cast nickel.	Annealed.	"	38	169	11.6
"	" at 15° C.	Natural.	"	...	222	9.1
"	" at 12° C.	"	Magnetic.	...	234	8.8
"	" at 220° C.	"	"	...	314	5.3
"	Cast cobalt at 5° C.	"	Normal.	...	142	18.9
"	" at -5° C.	"	Magnetic.	...	144	16.8
"	" at 230° C.	"	"	...	256	10.1

The smallest permeabilities (for large forces) observed were—for iron 258, for $\mathfrak{H}=64$; for steel 246, for $\mathfrak{H}=48$; for nickel 41, for $\mathfrak{H}=131$; for cobalt 55, for $\mathfrak{H}=147$.

3. The curve showing the relation between κ and \mathfrak{I} , or between μ and \mathfrak{B} , is of such a form that a diameter can be drawn bisecting chords parallel to the axis of \mathfrak{I} or \mathfrak{B} , and its equation is approximately

$$y = B \sin \left(\frac{x + by + d}{D} \right),$$

where $y = \kappa$ or μ , $x = \mathfrak{I}$ or \mathfrak{B} , and b, d, B, D are constants.

4. If a metal is permanently magnetized, its permeability is less for low magnetizing forces, but is unaltered for high magnetizing forces. This applies to the permanent state finally attained after several reversals of the magnetizing force; but if we strongly magnetize a bar in one direction, and apply a weak magnetizing force in the opposite direction, the change of magnetization will be very great.

5. Iron, nickel, and cobalt all probably have a maximum μ of magnetization, although its existence can never be entirely established by experiment, and must always be a matter of inference. If such a maximum exists, then at ordinary temperatures it will be roughly as follows:—

For iron when $\mathfrak{B}=17,500$, or when $\mathfrak{I}=1390$;
 For nickel when $\mathfrak{B}=6,340$, or when $\mathfrak{I}=494$;
 For cobalt when $\mathfrak{B}=10,000$, or when $\mathfrak{I}=800$.³

² Baur, *Wied. Ann.*, ii. p. 395, 1880, has remarked that the intensity of magnetization corresponding to the maximum permeability seems to be about the same for different sorts of soft iron; e.g., for two of the ellipsoids of Von Quintus Icilius it is 550 and 540; for Stoletow's ring, 550; for Baur's ring, 540. It would seem that it is much higher for steel, judging by Rowland's tables.

³ The maximum of magnetization for soft iron was calculated from the observations of various experimenters by Von Waltenhofen (*Wien. Ber.*, 1869; or *Pogg. Ann.*, cxxvi.). He finds 1670, or thereby, for the maximum intensity of magnetization. Stefan, using Rowland's graphical method (*Wien. Ber.*, 1874), had found 1400. Fromme (*Wied. Ann.*, xiii., 1881), who had himself actually observed an intensity of as much as 1531, examined the curve for κ and \mathfrak{I} , and found, in agreement with Haubner (*Wied. Beibl.*, v., 1881), that there is a point of inflexion about $\mathfrak{I}=1200$; taking this into account, he finds for the maximum value of \mathfrak{I} 1730, as a mean of results varying between 1720 and 1750. From a result of Weber's (*Elec. Maassbest.*, p. 573) he calculates the value 1737.

For the maximum permanent magnetization of steel, Weber (*Res. d. Mag. Ver.*, 1840) gives 314 (common steel magnet); Von Waltenhofen (*Pogg. Ann.*, 1871) 369 (glass hard wolfram steel); Schneebeli (*Wied. Galv.*, Bd. ii. § 308) 557 to 671 (sewing needles 25 to 66 mm. long and .6 mm. thick), and 765 to 832 (knitting needles 198 to 210 mm. long and .83 to 1.75 mm. thick). It must

6. The permeability of any metal depends on the quality of the metal, on the amount of permanent magnetization, on the total magnetization, and on the temperature.

7. The permeability of nickel and cobalt varies very much with temperature. In nickel for a moderate amount of magnetization the permeability increases with rise of temperature, but for high magnetization it decreases. This is very well shown in fig. 36, where the permeability curves for 15° C. and 220° C. intersect each other. In cobalt, on the other hand, the permeability appears to be always increased. The permeability of iron is not much affected by moderate changes of temperature.

8. The maximum of magnetization of iron and nickel decreases with rise of temperature, at least between 10° C. and 220° C., the first very slowly, the second very rapidly. At 220° C. the maximum for iron is $\mathfrak{B} = 17200$ or $\mathfrak{H} = 1360$, and for nickel $\mathfrak{B} = 4900$ or $\mathfrak{H} = 380$.

The researches of Stoletow and Rowland have undoubtedly made clear the main phenomena of magnetic induction; but in so doing they have raised a host of other questions which have not as yet been settled. There is no lack of recent work bearing on them, but it would be a difficult matter to give succinctly a complete account of the conclusions arrived at. The results of the different experimenters are not seldom contradictory, and the circumstances of experiment are often so complicated that criticism with the view of reconciling them seems hopeless in the meantime. While, therefore, we shall give a fairly complete list of the literature, the reader must not expect in this article an exhaustive analysis of the different memoirs that have recently appeared. Any remarks we shall make have chiefly for their object to call attention to the prominent questions that have been raised by the different workers.

1. Riecke¹ made a series of experiments on ellipsoids of soft iron; he expresses his results in terms of p the magnetization function for a sphere, and finds, as he ought to do, that, for a considerable range of values of the magnetizing force, p is approximately constant.² In point of fact this method of representation is bad, for the quality of the metal only begins to affect p about the fourth or fifth decimal place. Similar experiments on spheres and ellipsoids of soft iron were made by Fromme;³ and a very extensive series by A. L. Holz⁴ on ellipsoids of iron and steel, in which he gives tables and curves showing the values both of p (to a large number of decimals) and of κ ; and the values of the temporary, permanent, and vanishing magnetisms for a considerable range of magnetizing forces. The results, although wanting in regularity and smoothness for the harder kinds of steel, agree in the main with those of Stoletow and Rowland. Holz enters largely in this and in a former paper⁵ into speculations concerning the effect of the molecular structure of the metal upon its magnetic properties.

Relating more particularly to the phenomena of the permanent and temporary magnetization of steel we have important memoirs of recent date by Bouty, Fromme, and Auerbach. Bouty's papers,⁶ besides copious references to the general literature of the subject and interesting critical discussions of magnetic theory, contain the results of careful investigations as to the permanent magnetization attained by repeated applications of magnetic force under various circumstances, and verifications of the formulæ of

be remembered that the maximum of permanent magnetization which a body can attain is essentially conditioned by its form; since the more elongated the form the less the demagnetizing force arising from the existing magnetization.

¹ Pogg. Ann., cxlix., 1873.

² $= 3/4\pi = .2387$.

³ Pogg. Ann., clii., 1873. This paper contains also some results as to the permanent magnetism of soft iron.

⁴ Pogg. Ann., Ergbd. viii., 1877.

⁵ Pogg. Ann., cli., 1873.

⁶ Comptes Rend., 1875; Jour. d. l'Éc. Norm. Sup., 1875, 1876.

Green for the magnetic distribution in thin needles and cylindrical bars of steel. Two points as to his methods are worthy of notice. He employs a very simple method of measuring the magnetic moment of small pieces of steel: a small needle of moment m attached to a stiff stem, which carries a mirror, is freely suspended and allowed to come to rest in the magnetic meridian; the needle whose moment x is to be measured is then inserted into a tube fixed to the stem with its axis at right angles to the former needle. The deviation α of the compound system being measured by means of the mirror, we have $x = m \tan \alpha$. He studies the magnetic distribution in very thin hard needles by the method of rupture, finding that, if the needle be carefully broken, so that the distortion or shock caused by the bending does not extend far from the point of rupture, the magnetic moment of the different parts is little, if at all, affected. For thicker magnets he uses the ordinary method of deflexion.

Bouty found, in agreement with Hermann Scholz and Frankenheim,⁷ that, although the continued application of a magnetizing force does not increase the resulting permanent magnetization, the repetition⁸ of its application will. He finds for the magnetic moment y of a thin needle passed x times through a magnetizing spiral the formula $y = A - B/x$, where A and B are constants: e.g., in one case, $A = 57.78$, and $B = 6.32$. The ratio $A/(A - B)$, that is, the ratio of the moment attained by an infinite number of applications of the magnetizing force to that attained by one, decreases as the force increases; on the other hand, if R' be the force required to produce by a single application the same effect as R produces by an infinite number, he finds the ratio R'/R fairly constant⁹ (viz., from 1.060 to 1.065 in his best experiments) for values of R ranging from 10 to 42. In certain cases where the magnetization was effected by induced currents, he finds the formula $y = A + B(1 - e^{-ax})$ to represent the results better.¹⁰

He found that Green's formula,

$$y = Aa^2 \left(x - \frac{2}{\beta} \frac{e^{\frac{1}{2}\beta x} - e^{-\frac{1}{2}\beta x}}{e^{\frac{1}{2}\beta x} + e^{-\frac{1}{2}\beta x}} \right),$$

where

$$\beta = \frac{B}{a},$$

giving the moment of a cylinder of length x and diameter a , was sufficiently accurate both for temporary and for permanent magnetism, and for hard or soft tempered steel, whether saturated or not, provided the bars were in a virgin condition before magnetization. For example, in a saturated bar of soft steel ($a = 7$ mm.), for the temporary magnetism $A = 4.081$, $B = 1/7.142$; for the permanent magnetism $A = 2.34$, $B = 1/17.857$. In a non-saturated bar of soft steel ($a = 10$ mm.), for temporary magnetism $A = .9966$, $B = 1/7.142$; for permanent magnetism $A = .723$, $B = 1/17.857$; so that B is independent of the magnetic force. With hard tempered bars, A was less, both for temporary and permanent magnetism, than with soft bars; B was independent of the magnetizing force for temporary magnetism, but increased for permanent magnetism with large magnetizing forces. He calls the magnetic distribution long or short according as B is small or great, and

⁷ Pogg. Ann., cxliii., 1864.

⁸ In a very interesting paper (Phil. Mag., 1869 and 1870) dealing with certain phenomena of induced currents, Lord Rayleigh incidentally arrives at the conclusion that the magnetizing force of a current depends on its maximum intensity more than on its duration, or on the whole quantity of electricity that passes. This observation has an important bearing on certain experiments of Bouty as to the effect of the "extra current," which it does not seem necessary to mention here.

⁹ This conclusion is not in agreement with the results of Fromme.

¹⁰ The formula $y = R(1 - e^{-ax})$ was used by Quetelet for the moment induced in a steel bar by rubbing it x times with a magnet.

explains the phenomena of demagnetized or remagnetized bars by the superposition of long and short distributions. His final conclusion is that there is a greater independence between permanent and temporary magnetism than is usually admitted; and he starts a theory that magnetic bodies are composed of a mixture of two kinds of magnetic molecules, one kind retaining all the induced magnetism, the other wholly devoid of coercive force.

Magnetic after-effect.

It is obvious, from the results of Wiedemann, Frankenheim, and Bouty just alluded to, that the assumption made in the mathematical theory, that the effect of a magnetizing force is independent of the previous magnetic history of the body, is not even a first approximation to the actual truth. It becomes a matter of importance therefore to study the modification in the induced magnetism corresponding to any force produced by the forces that have preceded it. This effect has been called by German experimenters the *magnetic after-effect* (*Magnetische Nachwirkung*). Fromme and Auerbach have recently occupied themselves with this subject, and it may be of some interest to the reader to indicate a few of their conclusions.

Fromme. In his first paper¹ Fromme experiments with rotational ellipsoids of soft steel, using partly the method of Weber, Thalén, and Riecke, partly the ordinary method of deflexion. He found, in the first place, that the generalized theory of magnetic induction was applicable for values of \mathfrak{H} varying from '0061 to '132, κ decreasing between these limits from 23.5 to 8.68. He attempted to find the maximum force for which permanent magnetism first appears, and fixes it with some reserve at from .2 to .3.² The curve which he indicates for the temporary magnetization of soft steel has two points of inflexion, being first concave to the axis of \mathfrak{H} , then convex, and finally concave again.

He confirms the observation of Frankenheim that repeated applications of the magnetizing force increase the permanent magnetization up to a certain limit, and finds that when that limit is reached the body behaves towards all smaller forces having the same direction as if it were devoid of coercive force. Experimenting on ellipsoids permanently magnetized in this way, he found the mathematical theory of Kirchhoff to be inapplicable, it being impossible to fit the results obtained with the different ellipsoids together; and the discrepancy was greater with the softer than with the harder steel. For forces that are not sufficient to alter the permanent magnetization, κ decreases with decreasing force, as is the case with soft iron, so long at all events as the forces are not very great; and, again, for such forces the variation of κ is more regular the greater the permanent magnetization.

The number of impulses required to saturate with permanent magnetism was greater the greater the ratio of the moment of saturation to the initial moment, *e.g.*, greater for hard than for soft steel. It was found, in extension of a result of Frankenheim's, that, if U be the original moment, R_1 that produced by one and R that produced by an infinite number of impulses of the magnetizing force, then $(U + R_1)/(U + R)$ is tolerably constant; but R_1/R decreases with increasing magnetizing force.

With reference to the non-permanent magnetism of a bar repeatedly magnetized by the same constant current, he concludes from his researches that it diminishes, but in such a way that the total induced magnetism remains constant,—so that what is lost in non-permanent is gained in permanent magnetism.

In his second paper³ Fromme experimented both with

iron and with steel cylinders, pointed at the end, of lengths varying from 140 to 220 mm., and of thicknesses from 1.5 to 8 mm. The method of deflexion was used, the effect of the magnetizing spiral itself being compensated by an auxiliary spiral suitably placed. The cores were carefully introduced into the spiral after the current was established, removed before it was broken, and then replaced when the permanent magnetism was determined.

In the following extract from his conclusions T_n denotes the total induced magnetization, R_n the whole residual or permanent magnetization, V_n the non-permanent or vanishing magnetization, after n impulses of a given magnetizing force, the suffix being dropped when the number of impulses is not in question, and replaced by ∞ when the number is so great that by further increasing it no alteration in the effect is produced.

A constant force greater than all preceding induces a T which varies with successive impulses, sometimes increasing, sometimes decreasing. If a bar previously heated white hot be subjected to a large force, successive impulses usually give a decrease of T . If, however, the force is preceded by one somewhat smaller, successive impulses usually give an increase. It depends merely on the magnitude and the number of impulses of the preceding force P whether the repeated impulses of a force p will give an increasing or a decreasing T .

R always increases with successive impulses until the limit R_∞ is reached, and always faster than T ; hence increase of R and decrease of V go hand in hand; the magnitude of this increase depends on P and p , and approaches zero with $P - p$.

In order that the action of a force p may not be influenced by the after-effect of smaller forces preceding it, it must be applied so often that its further application ceases to increase R . When saturation for R is thus reached, then T , R , and V have the values corresponding to frequent impulses of p for a fresh bar.

R_1/R_∞ , R_2/R_∞ , &c., all starting from unity, decrease as the force p increases from zero, diverging more and more until they all reach minima for the same value of p ; they then converge again towards unity, which they all reach at the maximum of permanent magnetization. The values of p corresponding to the maxima of R_∞/p , R_2/p , R_1/p are in ascending order of magnitude, and the first of them is the value corresponding to the minima of R_1/R_∞ , R_2/R_∞ , &c.

What was stated for R_1/R_∞ , R_2/R_∞ , &c., holds word for word for T_∞/T_1 , T_∞/T_2 , &c. Hence the decrease of T is conditioned solely by the increase of R ; so that it would appear that the after-effect of a preceding force P depends on the R which it produces. It would therefore be more correct to say that the after-effect depends on $r - R$ than to say that it depends on $p - P$.

When a bar has been magnetized by any force P , all smaller succeeding forces leave R unaltered, yet by repeated impulses of p ($< P$) T decreases until it reaches a certain limit. We may repeat the process as often as we please by always beginning with a new application of a larger force P ; if we vary P , keeping p constant, T_1 , T_2 , &c., vary, but the limit T_∞ is always the same. In these experiments it was indifferent whether a few seconds or several hours elapsed between the applications of P and p ; time had no influence on the vanishing of this species of magnetic after-effect. On the other hand, several impulses of the greater force gave no more after-effect than a single impulse, of whatever duration. If N denote the after-effect of a greater force P upon the action of a smaller p , the law of the phenomenon is

$$N = cp^a(P - p)^b,$$

where c is a constant and a and b are constant positive

¹ *Pogg. Ann.*, Ergbd. vii., 1875.

² So far confirming Maxwell's conclusions from his modification of Weber's theory of molecular magnets, *El. and Mag.*, vol. ii. § 445.

³ *Wied. Ann.*, iv., 1878.

numbers, b being a proper fraction, and a possibly very near unity. This of course gives $N=0$ for $p=0$ and for $p=P$, and gives a maximum value of N for some value of p between 0 and P .

The interposition of a force P' between P and p increases the after-effect if $P' > P$, diminishes it if $P' < P$; and this holds irrespective of the sign of $P' - p$.

If we denote by k the susceptibility of a body for vanishing magnetism (V) induced by any force p , the question arises how far this is influenced by the permanent magnetism R induced by preceding greater forces. Jamin holds that k is approximately, and Chwolson that it is absolutely, independent of such permanent magnetism. Fromme finds that, when a force p , capable of itself producing a permanent magnetism r , acts on a bar already possessing a permanent magnetism $R > r$, then k is increased (by the presence of R) if $R - r$ is small, but diminished when $R - r$ is great.¹ The after-effect for small forces p may therefore be either increase or decrease of k ; but for large forces p it is always increase.

At the conclusion of his paper Fromme points out the contrast between magnetic and elastic after-effect, and dwells upon the analogy between his results and those of Thalén² concerning the limits of elasticity in solid bodies.

The experimental method followed by Auerbach³ was much the same as that of Fromme, except that the core was left in the magnetizing spiral during the make and break of the current. The core was generally a hollow cylinder of soft iron 148.1 mm. long, 17.8 mm. in diameter, 1.6 mm. thick, with end plates 1.5 mm. thick. He distinguishes two kinds of magnetic after-effect. The first kind consists in alteration of the magnetization of the body during the action of a constant force, or after it has ceased to act. The second kind is that already mentioned, in which the action of any force is influenced by preceding forces. It is this second kind of after-effect that is dealt with in the paper from which we are quoting.

The leading peculiarity of his view of the phenomenon is the introduction of the force zero, both as a preceding and as a final force. The fundamental principle laid down is the following:—

When the force p , which, following immediately after the force 0, would produce a magnetization T_0 , is preceded by a series of forces P_1, P_2, \dots, P_n , the magnetization which results is T , differing from T_0 by an amount N called the after-effect. N is wholly determined by the first of the preceding forces P_n which is such that all the forces that act between P_n and p lie in magnitude between P_n and p .

This general law is, however, subject to exceptions. For example, let the whole series of forces acting be P_{10}, p, P_0 , p (evidently an extreme case), then experience shows that neither T_{10} nor T_0 is the resulting magnetization, but something intermediate, much nearer to T_{10} , however, than to T_0 . In order to obtain T_0 a force $P_0 < p$ must be interposed before P_{10} ; even then the magnetization varies a little with P_0 , but, if the stationary condition for P_0, p be established by alternating P_0, p many times after applying P_0 , thus $P_{10}, p, P_0, p, P_0, p, P_0, p, P_0, p, \dots$ the limit is found to be independent of P_0 , and is held to be the true value of T_0 .

In this way, for a given p , T can be determined as a function of P . It is necessary, however, to attend to the following principle,—that, of two preceding forces lying in magnitude on different sides of p , the second determines

the after-effect exclusively only when it differs more from p than the first; in other cases both contribute to the after-effect; in no case does the first exclusively determine the after-effect. In the case where both preceding forces lie on the same side of p , the exceptions to the general law are far less marked; only where the second force is very nearly equal to p does it exercise a disturbing influence on the after-effect of the first.

The process used for obtaining T as a function of P , for a given p , say 10, is therefore to cause the influencing forces to alternate with the influenced, the succession of the former being such that the one preceding p always differs less from p than the one following. The stationary condition is supposed to be established for each pair as above explained; e.g., starting with $P=11$, the series might be 11, 10, 8, 10, 13, 10, 6, 10, 15, 10, 4, 10, &c. In this way $T_{11}, T_8, T_{13}, \&c.$, can be determined.

When the values of T are plotted against the values of P , the curves corresponding to different values of p have all a similar character (see figure 37). They consist of two congruent parts lying on the two sides of a point of inflexion, which is the only point that has any marked character. To the right of the inflexion the concavity is towards the axis of P , to the left in the opposite direction. The infinite branches appear to approach

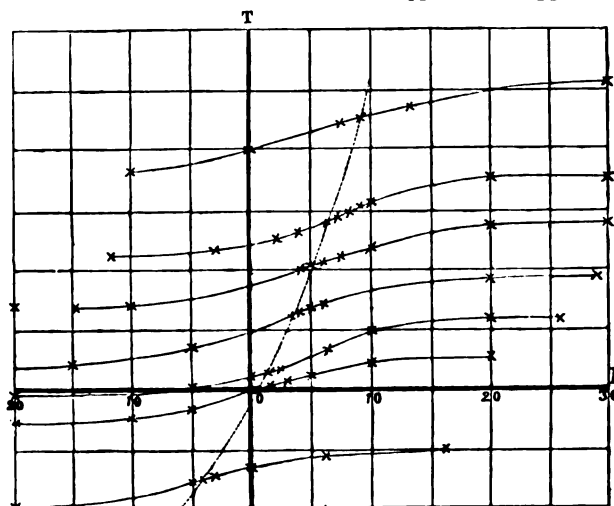


Fig. 37.

asymptotes parallel to the axis of P . The abscissa of the point of inflexion for any particular curve p is $P=p$; the ordinate is T_p , which may be called the normal magnetization corresponding to p when p alone has acted before. This of course is an ideal case; but a process is indicated for determining T_p directly.⁴ The dotted curve in the figure is the curve of normal magnetization, whose abscissa and ordinate are p and T_p .

From the symmetry of the curves representing the after-effect Auerbach concludes that the after-effect of forces on opposite sides of p as to magnitude, and equidistant from it, is equal and opposite, and ascribes the failure to observe the after-effect of forces smaller than p to the interposition of the force zero. He further concludes that the after-effect depends in the same way on $P-p$ as T depends on p .

There is one of the curves of after-effect, that, viz., for $p=0$, which has a special meaning. It is clearly the curve

¹ These conclusions are in agreement with the results of Herwig obtained from experiments on the longitudinal and circular magnetization of iron tubes, *Pogg. Ann.*, clvi., 1875.

² *Pogg. Ann.*, cxiv., 1865.

³ *Wied. Ann.*, xiv., 1881.

⁴ A particular case of this process is interesting and practically important; viz., in order to demagnetize a core (i.e., to find T_0) possessing a moment T . Apply in succession the forces $-P, +(P-\epsilon), -(P-2\epsilon), +(P-3\epsilon), \&c.$, down to 0, P being chosen of sufficient magnitude, rather too great than too small (the smaller ϵ the better).

of permanent or residual magnetism, which is thus in Auerbach's view a particular case of after-effect. To it we can apply the general rule given above, subject of course to like exceptions.

Effect of
duration
and re-
petition
of im-
pulse.

It would be premature to pronounce any opinion as to the ultimate value of Auerbach's results; but the elegance of his representation of the phenomena will scarcely be disputed. In the latter part of his paper he applies his views to explain the peculiarities in the curve of magnetization with forces of ascending magnitude obtained when the after-effect is neglected, and to the cyclical process discussed by Warburg.¹ He also discusses the influence of the duration of the impulse of the magnetizing force and of the sudden closing and opening of the current. His conclusions agree in the main with those of Fromme: in particular he inclines to Fromme's view² that there is a specific magnetic effect produced in certain cases by the breaking of the current while the core is in the spiral. This effect in certain cases (with short thick cores) is so great that a permanent magnetization of opposite sign to the total induced magnetism remains.³ This "anomalous magnetization" was first observed by Von Waltenhofen,⁴ who also establishes the more general result, of which this is an extreme case, viz., that the residual magnetism of the core depends upon the rapidity with which the magnetizing force is reduced to zero. Auerbach lays down as a general principle that when the variation of the magnetizing force is slow and continuous the velocity of the transition does not influence the final magnetization; but sudden transition causes the final magnetization to be less or greater than that obtained by gradual transition, according as the passage is from a greater to a less or from a less to a greater force.

Additional
literature.

The reader who wishes to pursue the present subject farther should consult the works of the following experimenters:—

Jamin,⁵ who holds what he apparently regards as a new theory of magnetization. It is in point of fact merely a modification of the theory of solenoids, somewhat restricted in its application to the phenomena of magnetic induction. His special point is that the lines of magnetization in a bar magnetized (say) by a magnetizing spiral only penetrate to a limited depth, which is greater the greater the current. The following experiments⁶ are adduced in confirmation of his views. The steel tube of a Chassepot rifle was plugged at both ends by screwing into it bolts of the same metal. Inside was placed a cylindrical rod. It was found that, so long as the current in the spiral was not very great, the rod was not sensibly magnetized; but, as the current increased, it became more and more affected, and by and by was as much permanently magnetized as if the enveloping tube had been absent. Again, the rod having been magnetized to saturation and inserted in the tube, a demagnetizing force was applied to the whole, and it was found possible to render the tube and core together seemingly neutral, or even oppositely magnetic, while the rod when taken out proved to be still powerfully magnetized in the original direction. Again, a bar was magnetized by a powerful current, and then magnetized in the opposite direction by another current. The surface of the bar was then eaten away to a certain depth; and it was found that the original magnetization reappeared. These experiments, although most interesting in themselves, do not appear to warrant the interpretation which their author puts upon them. Jamin has made extensive researches on the magnetic distribution in bars and ribbons of steel, partly with a view to obtain empirical rules for the construction of powerful permanent magnets, in which he has been very successful.

Gaugin, *Comptes Rendus*, passim; *Ann. d. Chim. et d. Phys.*, (5) xi.

¹ *Wied. Ann.*, xiii., 1881; cf. Fromme, *ib.*, xiii., 1881; also Himstedt, *ib.*, xiv., 1881. A similar phenomenon was observed by Meyer and Auerbach during their experiments on the gramme machine, *Wied. Ann.*, v., 1878.

² See an elaborate paper which we can only mention here, *Wied. Ann.*, v., 1878.

³ Experiments on the same subject have been made by Righi. *Comptes Rendus*, 1880, or *Wied. Beibl.*, iv., 1880; and by Bartoli and Alessandro, *N. Cim.*, 1880, or *Wied. Beibl.*, iv., 1880. Cf. Fromme, *Wied. Ann.*, xiii., 1881.

⁴ *Comptes Rendus*, passim.

⁵ *Comptes Rendus*, lxxx., 1875.

Christiansen, "Researches on the Magnetic Distribution in an Iron Bar, on one part of which is placed a Short Magnetizing Spiral," *Wied. Beibl.*, i., 1877.

Ruths, "Ueber den Magnetismus weicher Eisencylinder und verschieden harter Stahlsorten" (Dortmund, 1876), *Wied. Beibl.*, i., 1877.

Whipple, "Induction Constants of Permanent Magnets of various shapes, from the determination at Kew," *Proc. Roy. Soc. Lond.*, 1877.

Oberbeck, "Ueber die Fortpflanzung der magnetischen Induction im weichen Eisen" (Halle, 1878), *Wied. Beibl.*, ii., 1878.

Külp, "Experimentaluntersuchungen über magnetische Coërcitivkraft," *Carl. Rep.*, 1880.

Baur, "Experiments with an Iron Ring on the Magnetization Function for very small Forces," *Wied. Ann.*, xi., 1880.

Riecke, "On the Experimental Test of Poisson's Theory," *Wied. Ann.*, xiii. p. 485, 1881.

Siemens, a very interesting paper, "On the Effect of the Magnetization of Iron in any Direction upon its Permeability in the Perpendicular Direction," *Wied. Ann.*, xiv., 1881.

Righi, "Contributions to the Theory of the Magnetization of Steel," *Mem. d. Acc. d. Bologna*, 1880; *Wied. Beibl.*, v., 1882.

For a succinct account of several of the foregoing memoirs, see the "Nachträge" to Wiedemann's *Galvanismus*, and a paper by the same author in *Poggendorff's Annalen*, clvii. p. 257, 1876.

Influence of the Hardness and Structure of Iron and Steel on Permanent Magnetism.—Some information has already been given incidentally on this subject, and a lengthy discussion would be out of place here. The statements of the various authorities are very contradictory. This is not to be wondered at; for those best qualified to prepare the materials for experiment are generally deficient in the scientific knowledge requisite to enable them to form a sound judgment as to the result, while thoroughly trained scientific men have not as a rule acquired a command over the delicate manipulation of the forging and tempering of steel, an art which those who possess it usually find difficult to describe in words or reduce to rules. There is the further circumstance that many who have been successful in making good steel for magnetic or other purposes have found it for their interest not to publish the process by which success was attained.

Fineness of grain and uniformity of temper are the two greatest requisites in steel for permanent magnets. The latter in bars of any size is never attained in perfection, for the surface is always harder than the interior. The mischief which thereby arises may be understood by taking the extreme case of a thin steel tube magnetized to saturation, and then fitted with a perfectly soft iron core. It is clear that the core will act very much like the armature of a horse-shoe magnet; the lines of force will run back through it, and the external action will be in a great measure destroyed.

The different tempers of steel may be roughly classified as glass hard, straw colour, blue, and soft. The current statement is that the harder the steel the more difficult it is to magnetize, but the better it retains its magnetism. If this were so, provided sufficient magnetizing force to produce saturation were at command, the best temper for magnets would be glass hard. Lamont, however, whose experience was great, states that he found the loss after magnetization to be as great, and to continue as long, with glass hard as with blue tempered magnets. The same experimenter gives it as his opinion that great differences in the quality of magnets arise more from defects as to homogeneity, continuity, and uniformity of temper than from the quality of the steel in other respects; he inclines, however, to a preference for English cast steel.

Purity and homogeneity of structure are equally necessary in iron of high magnetic inductive susceptibility and small coercive force. Hammering, rolling, and drawing diminish the susceptibility and increase the coercive force. Rolling does so more in the direction of rolling than transversely, so that the iron becomes anisotropic. It is advisable in all cases where high susceptibility is wished to anneal the

body carefully after manufacture, by heating it in a wood fire and allowing it to cool very gradually; this process is still more effective when the iron is covered all over beforehand with half an inch or so of clay.

The reader who wishes for further details on this subject should consult Lamont's *Handbuch des Magnetismus*, chap. v. The following references to the literature may be useful.

Michell, *Treatise of Artificial Magnets*, 1750; Coulomb, *Mém. de l'Acad.*, 1784; Barlow, *Phil. Trans.*, 1822; Kater, *Phil. Trans.*, 1821; Sabine, *Phil. Trans.*, 1843; Hansteen, *Pogg. Ann.*, 1825; Häcker, *Pogg. Ann.*, 1848; Poggendorff, *ib.*, 1850; Müller, *ib.*, 1852; Matthiessen, *Phil. Mag.*, 1858; Airy, *ib.*, 1863; Von Waltenhofen, *Pogg. Ann.*, 1864; Trève, *Comptes Rendus*, 1869; A. L. Holz, *Wied. Ann.*, v., 1878; Ruths, *Wied. Beibl.* i. 1877; Cheesman, *Wied. Ann.* 1882.

Special Magnetic Character of Nickel and Cobalt.—Besides the results of Rowland above quoted, we have on record experiments by the following physicists:—Biot, *Traité de Phys.*, 1806; Gay Lussac, *Ann. d. Chim. et d. Phys.*, 1824; Lampadius, *Schweigger's Jour.*, 1814; E. Becquerel, *Comptes Rendus*, 1845; Plücker, *Pogg. Ann.*, 1854; Arndtsen, *ib.*, 1858; Hankel, *Wied. Ann.*, 1877; Becquerel, *Ann. d. Chim. et d. Phys.*, 1879; Gaiße, *Comptes Rendus*, 1881; Wild, *Wied. Beibl.*, 1877.

Experiments with Finely Divided Magnetic Metals and with Electrolytic Iron.—These have been made by various physicists, mostly to test the theory of molecular magnets. The earliest of the experiments with finely divided iron was made by Coulomb, who mixed iron filings with wax, and found that the magnetic moment was proportional to the mass of magnetic metal. Similar experiments were made by the elder Becquerel,¹ his result being that the magnetic moment was proportional to the weight of magnetic substance, so long as the filings were not too densely distributed; with increasing density the mixture acquires magnetic properties more like those of a continuous metallic mass. Several modern experimenters have gone into the matter with considerable care; but their results are not sufficiently concordant, or of sufficient general interest, to justify us in dwelling at length upon them here. A few references to recent memoirs will suffice.

Boernstein, *Pogg. Ann.*, 1875; Toepfer and Von Ettingshausen, *ib.*, 1877; Von Waltenhofen, *Wied. Ann.*, 1879; Auerbach, *ib.*, 1880; Baur, *ib.*, 1880.

Experiments on electrolytically deposited iron have been made by Beez, *Pogg. Ann.*, 1860; Jacobi, *ib.*, 1873; Beez, *ib.*, 1874; Holz, *ib.*, 1875; Baur, *Wied. Ann.*, 1880.

Using a fine scratch on a varnished silver wire as electrode, Beez deposited a thread of iron between the poles of an electromagnet, and thus obtained a permanent magnet of extreme tenuity. It was found that the inductive susceptibility of this linear magnet was very small, and that considerable magnetizing force produced no increase of its permanent magnetism. Thus in one case the original magnetism was 360, the total magnetism under the inducing force 370, the magnetism remaining after the force ceased to act 360. Broader, but equally thin, magnets deposited in a strong field in the same way gave more temporary magnetism than the linear magnets, but never more permanent magnetism than they possessed originally. Thicker plates exhibited greater temporary magnetism, and also an increase of the permanent magnetism acquired during deposition. With continued reversals of the magnetizing force electrolytic iron gave a continual decrease of the temporary magnetism down to a certain limit (as does steel); but the negative permanent magnetism never approaches so near the positive after many reversals as in the case of steel. On the other hand, Jacobi found that iron reduced electrolytically from ferrous sulphate and sulphate of magnesia, even after tempering, took a considerable temporary moment, but retained very little permanent magnetism. Holz found that the iron reduced from the solution of Jacobi and Klein was not sensibly hardened by heating and suddenly cooling, although its density was increased, and that its coercive force was diminished. On the other hand, it was found that hard tempering decreased the density of steel. He draws the conclusion that the coercive force is greater the farther apart the molecules. Baur's main result is that the maximum of magnetization with electrolytic iron occurs for much larger forces than with ordinary iron. These results are not wholly concordant; but the discrepancies may be reasonably assigned to differences in the preparation of the metal.

MAGNETIC PROPERTIES OF MATTER IN GENERAL

Among the earliest statements of the properties of the loadstone we find accounts of its action on other bodies; but it is clear from their surroundings that these statements are purely fabulous. Many experimenters at a later date

found indications of magnetic action in other metals besides iron; but with praiseworthy caution they ascribed them for the most part to the admixture of small quantities of iron.² There can be no doubt that the results of Cavallo³ obtained with brass (especially hammered brass) were due to impurity, for Bennet⁴ failed to obtain any indications of magnetism with pieces of brass made from pure zinc and copper, whereas he was immediately successful on adding small traces of iron to the metal.

Early observations.

It very soon appeared, however, that an independent magnetic property must be ascribed to nickel and cobalt, and to these were by and by added with more or less certainty manganese and chromium.⁵

Brugmans⁶ seems to have been the first to observe the repulsion by a magnet of a body not permanently magnetized. He found that a piece of bismuth floating upon mercury in a small paper boat was repelled by both poles of a magnet. Lebaillif⁷ confirmed the observation of Brugmans, and found that antimony possessed a like property. Saigey,⁸ who experimented on the same subject, concluded that all bodies when suspended in air behave like bismuth, unless they contain traces of iron.

Notwithstanding these results and others which we pass over,⁹ the whole matter remained in obscurity till the repulsion of neutral bodies was rediscovered by Faraday in 1845. He speedily unravelled the laws of the phenomenon, showing how much depends on the nature of the body, and how much upon the nature of the magnetic field. His observations enabled him in fact to comprehend under a few general principles the action of all magnetic bodies, whether of the nature of iron or of the nature of bismuth. The earlier observers had fallen into difficulties by neglecting the effects due to heterogeneity of field; these were pointed out for the first time by Faraday, and since then order reigns where there was formerly confusion.

Faraday's rediscovery and full investigation.

The best arrangement for testing the behaviour of weakly magnetic bodies is to suspend either a small sphere of the substance or else a small cylinder in a heterogeneous magnetic field. This field is usually produced by placing two pointed soft iron poles (fig. 38) on the arms of a powerful electromagnet. The line joining these poles is called the *axial* direction of the field; directions perpendicular to this line are called *equatorial*. The magnetic force varies

Experimental arrangements for testing weakly magnetic bodies.

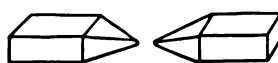


Fig. 38.

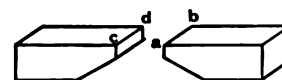


Fig. 39.

along the axial line, being less in the middle than at the poles; and it decreases everywhere from the axial line outwards. For some purposes poles of the shape shown in figure 39 are used; here the line along the upper edges of the poles are lines of greatest force, whereas the line in the plane of the upper faces equidistant from the upper edges is a line of weakest force; the force also decreases to the right of *ab* and to the left of *cd*.

In suspending small spheres the best plan is to hang them from one end of an arm of wood *db* (fig. 40). At the other end of this arm is placed a counterpoise *b*, and the whole is suspended by a fibre of unspun silk *a* from a torsion head *t*, by means of which the arm *db* can be brought into

² Cf. Lehmann, *Nov. Comm. Petrop.*, 1766; Brugmans, *Magnetismus seu de Affinitatibus Magneticis Observationes Academicæ*, Leyden, 1778; Coulomb, *Mém. de l'Inst.*, 1812; Biot, *Traité de Physique*, 1816, &c.

³ *Phil. Trans.*, 1786; or *Treatise on Magnetism*, 1787.

⁴ *Phil. Trans.*, 1792.

⁵ Ritter, *Gilb. Ann.*, 1800.

⁶ *Loc. cit.*

⁷ *Pogg. Ann.*, 1827.

⁸ *Bull. Univ. d. Sc.*, 1828.

⁹ See Von Feilitzsch, *Karsten's Ency.*, Bd. xvi.; Wiedemann's *Galvanismus*, Bd. ii. p. 546.

¹ *Traité Complet du Magnétisme*, chap. ii. p. 73.

any required position, and if necessary kept there by the exertion of a known torsional couple. The arm and suspension must be carefully guarded from draughts by enclosing it in a glass case, which fits over the poles of the electromagnet, and is provided with a door and with means for bringing the torsion head *t* over any given part of the magnetic field. When a cylindrical piece is to be tested it is suspended from the fibre *u* so as to hang horizontally. For this purpose Faraday was in the habit of using a stirrup of carefully selected writing paper attached to the lower end of the fibre. It is of the utmost importance to guard against magnetic action on the suspension; the least trace of iron in the arm *db* for instance, or in the paper stirrup, would in many cases be more than sufficient to mask the action proper to a weakly magnetic body.

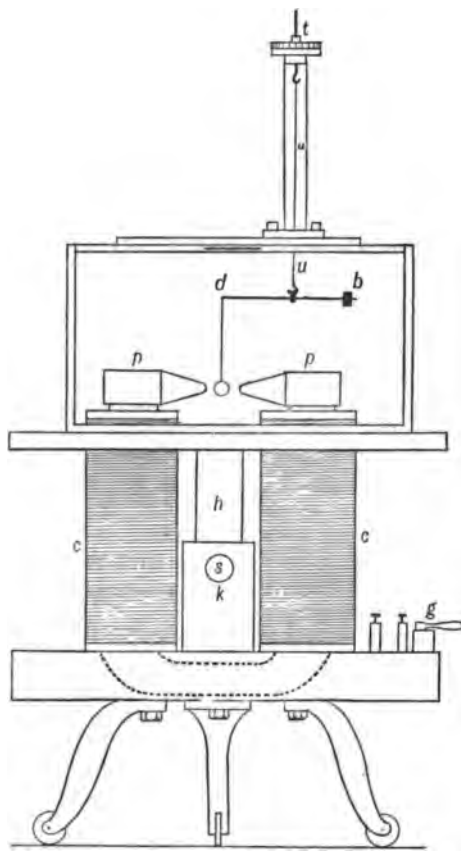


Fig. 40.

In every experiment the magnetic behaviour of the support should be tested by itself beforehand, so that if any residual effect be present it may be allowed for. The greatest caution is also requisite in choosing the material to be experimented upon. There must be no chemical impurity, especially no trace of iron; the spheres and cylinders must not be worked with iron tools or even with dirty hands. A source of error¹ to be specially guarded against in experiments with metals, or other good conductors, is the action arising from induced currents in the mass of the tested body caused by the increase and decrease of the strength of the magnetic field when the circuit of the electromagnet is made and broken. This error is wholly avoided by waiting till the suspended body has come to rest, and attending only to deflexions which are permanent after the intensity of the field has become steady.

Behaviour of heavy glass.

The first substance with which Faraday experimented was a bar of the heavy glass with which he had discovered the rotation of the plane of polarization of light. It took up the equatorial position between the poles of the electromagnet as soon as the current was established. There was no distinction between its ends, or according to the direction of the lines of force; the bar always took the shortest course to the equatorial position, and remained there in stable equilibrium. When placed in the axial position it was in unstable equilibrium, and on the slightest displacement either way it moved off in that direction to

the equatorial position. A further action was observed when the bar was placed with its centre of mass out of the centre of the field; it was then repelled as a whole away from the nearest pole (no matter which). On testing a small cube or sphere of the substance, no pointing tendency was observed, but the mass as a whole when it was placed unsymmetrically with respect to the poles tended to pass away from the poles towards the centre of the field, and from the axial line outwards.

Faraday sums up the matter by saying that every element of the heavy glass tends to move from places of stronger to places of weaker resultant magnetic force. This is exactly the opposite of the law for bodies like iron (see mathematical theory above, p. 247). All bodies that follow the same law as heavy glass he calls *diamagnetics*, all that follow the opposite law, like iron, *paramagnetics*. For the purposes of experimental demonstration it is better to take some weaker paramagnetic than iron, e.g., a tube filled with a solution of ferric chloride; for the order of magnitude of the effect obtained is then the same as with diamagnetics, and there is no danger of complications arising from the mutual action of the particles of the substance (see above, p. 245).

Faraday found the following substances to be diamagnetic; i.e., pieces of them tended to set their longest dimension equatorial between pointed poles, and spheres and cubes of them tended to pass from places of stronger to places of weaker force:—rock crystal, sulphate of lime, sulphate of baryta, sulphate of soda, sulphate of potash, sulphate of magnesia, alum, muriate of ammonia, chloride of lead, chloride of sodium, nitrate of potash, nitrate of lead, carbonate of soda, Iceland spar, acetate of lead, tartrate of potash and antimony, tartrate of potash and soda, tartaric acid, citric acid, water, alcohol, ether, nitric acid, sulphuric acid, muriatic acid, solutions of various alkaline and earthy salts, glass, litharge, white arsenic, iodine, phosphorus, sulphur, resin, spermaceti, caffeine, cinchonia, margarine acid, wax from shellac, sealing wax, olive oil, oil of turpentine, jet, caoutchouc, sugar, starch, gum arabic, wood, ivory, mutton (dried), beef (dried), blood (dried or fresh), leather, apple, bread.

In testing liquids Faraday used a very thin glass tube of the form shown in figure 41; the opening being very fine, there was no need for a cork or other stopper which might have caused disturbance; the slight diamagnetic effect arising from the glass was allowed for. Another way of testing a liquid² is to place it in the bottom of a watch glass which rests on the edges of the pole of the electromagnet. When the fluid is paramagnetic, it collects in the places of greater force, forming a depression in the centre of the field as in figure 42; when it is diamagnetic, it collects in the places of weaker force in the centre of the field, as in fig. 43. Yet another method³ is to put a small quantity of the fluid in a narrow tube, and place the tube horizontally in the equatorial line so that the end of the liquid column is just on the axial line. When the electromagnet is excited the liquid will be driven away from the axial line or drawn in according as it is diamagnetic or paramagnetic.



Fig. 41.



Fig. 42.



Fig. 43.

Faraday found that breaking a weakly magnetic body into pieces, or even reducing it to powder, produced no effect upon its magnetic behaviour provided its general form was unaltered. In order to avoid disturbance from

¹ See Faraday, *Exp. Res.*, 2309 sq.

² Plücker, *Pogg. Ann.*, 1848.

³ Quet, *Comptes Rendus*, 1854.

the magnecrystallic effect to be described presently, it is often advisable to reduce certain substances to powder before testing them; the powder is filled into a thin glass tube and then tested like a liquid. By means of powdered bismuth the tendency of a diamagnetic to pass from places of stronger to places of weaker force can be very prettily shown. If the powder be strewn upon the circular end of the core of an electromagnet, it will leave the edges and collect in the centre, whereas iron filings will leave the centre and arrange themselves round the edges, the fact being that at the edges the force is much more intense than in the centre.¹

y's Faraday arranges the metals in the following order of descending magnetic susceptibility:—

Paramagnetic.		
Iron.	Chromium.	Palladium.
Nickel.	Cerium.	Platinum.
Cobalt.	Titanium.	Osmium.
Manganese.		
Diamagnetic.		
Bismuth.	Mercury.	Arsenic.
Antimony.	Lead.	Uranium.
Zinc.	Silver.	Rhodium.
Tin.	Copper.	Iridium.
Cadmium.	Gold.	Tungsten.
Sodium.		

Silicium is given as strongly paramagnetic, and beryllium,² aluminium, potassium, and sodium³ as weakly magnetic; the last three were given as diamagnetic by Faraday; the magnetic character appears to depend on the method by which the material is prepared, being doubtless determined by the presence or absence of slight impurities. The copper of commerce is magnetic, owing to traces of iron; but when it is reduced by means of zinc from the chloride or sulphate it is diamagnetic. It would appear that the paramagnetism of titanium, palladium, platinum, and osmium is due to iron impurity.⁴ Platinum⁵ reduced from very pure chloroplatinate of ammonium by heating in a current of air is diamagnetic. According to Graham the magnetism of palladium when charged with hydrogen is due to the presence of hydrogenium; Blondlot,⁶ however, has recently found that palladium is less magnetic when charged with hydrogen than when uncharged, from which he concludes that condensed hydrogen is pretty powerfully diamagnetic. Tellurium, sulphur, selenium, and thallium are strongly, and niobium and tantalum weakly diamagnetic.

Magnetic Properties of Gases.—The earliest results of Faraday were of a negative description, but the discovery by Bancalari⁷ of the powerful diamagnetic action of flame again drew the attention of Faraday,⁸ Plücker,⁹ and y's Becquerel¹⁰ to the subject. Faraday caused the gas under examination to stream vertically upwards or downwards (according as it was lighter or heavier than the surrounding gas) between the poles of an electromagnet, and observed how the stream was deflected. In the case of colourless gases the deflexion was observed by allowing small traces of hydrochloric acid to mix with the gas, and then placing in different parts of the field small tubes containing pieces of filter paper moistened with ammonia; by noticing in which of these the white fumes of ammonium chloride were formed the course of the gaseous current could be determined. Another method employed was to fix two thin glass tubes containing gases to be tested to the ends of a cross piece on one end of the arm of a torsion

balance; the tube containing the most magnetic then moved towards the axial line. Another method, employed both by Plücker and by Faraday, is to blow soap bubbles with the gas to be tested, and observe their behaviour in the magnetic field, allowing of course for the feeble diamagnetism of the water film. Faraday's results are as follows:—

	In Air.	In Carbonic Acid.	In Hydrogen.	In Coal Gas.	List of gases.
Air	0	+	+ weak	+	
Nitrogen	—	—	— strong	—	
Oxygen	+	+	+ strong	+ strong	
Hydrogen	— strong	—	0	—	
Carbonic acid	—	0	—	— weak	
Carbonic oxide	—	—	—	— weak	
Nitrous oxide	—	— weak	—	—	
Nitric oxide	— ? weak	+	+	—	
Nitrous acid	+ ? weak	—	—	—	
Olefiant gas	—	—	—	— weak	
Coal gas	— strong	—	—	0	
Sulphuric acid	—	—	—	—	
Hydrochloric acid	—	—	— ? weak	—	
Hydriodic acid	—	—	—	—	
Fluosilicic acid	—	—	—	—	
Ammonia	—	—	—	—	
Chlorine	—	—	— weak	—	
Iodine	—	—	—	—	
Bromine	—	—	—	—	
Cyanogen	— strong	—	—	—	

+ means magnetic relatively to the surrounding gas, — diamagnetic; the epithets strong and weak relate of course to the apparent behaviour under the circumstances of the experiment.

It appears therefore that oxygen is the most paramagnetic of all the gases; on this account Faraday conceived that it probably played an important part as a cause of terrestrial magnetism.¹¹ Becquerel¹² has concluded from recent experiments of his own that the specific magnetism of ozone is still greater than that of oxygen. Faraday was able by filling thin glass bulbs with oxygen at different densities to show that the magnetic susceptibility decreased with the density, apparently in simple proportion. Some numbers giving an idea of the magnetic susceptibility of the various weakly magnetic bodies are given below.

In all experiments with gases or fluid media, and indeed Faraday's in every possible magnetic experiment more or less, it is important to notice that the resulting magnetic action is the difference between the action of the movable body and the action on the surrounding medium. This was first pointed out by Faraday.¹³ He prepared three solutions of ferrous sulphate. No. 1 contained 74 grains of the hydrated salt for every ounce of water; No. 2 was formed by diluting one volume of No. 1 with two volumes of water, No. 3 by diluting one volume of No. 1 with fifteen volumes of water. Three glasses g_1, g_2, g_3 and three tubes t_1, t_2, t_3 were filled with the respective solutions. The glasses were placed in succession between the pointed poles of the electromagnet, and the tubes tested in them with the following result:—

	In g_1 .	In g_2 .	In g_3 .
t_1	0	+	++
t_2	—	0	+
t_3	—	—	0

+ means pointed axially; ++ the same with greater decision; — pointed equatorially; 0 was indifferent.

We have here the experimental confirmation of the important theoretical conclusion (see above, p. 248) that any body will behave paramagnetically or diamagnetically according as it is surrounded by a medium less or more magnetic than itself. In cases where the square of the

¹ This fact explains the astonishing behaviour of a flat disk of thin iron when placed on the centre of the pole.

² See Wiedemann, *Galvanismus*, Bd. ii. § 552.

³ Laing, *Ann. d. Chim. et d. Phys.*, 1857.

⁴ Wiedemann, *l.c.*

⁵ Wiedemann.

⁶ *Comptes Rendus*, 1877. ⁷ Zantedeschi, *Pogg. Ann.*, 1848.

⁸ *Phil. Mag.*, 1847; or *Exp. Res.*, vol. iii. p. 467.

⁹ *Pogg. Ann.*, 1848, &c.

¹⁰ *Ann. d. Chim. et d. Phys.*, 1850.

¹¹ See *Exp. Res.*, 2847 *sq.*; also art. METEOROLOGY.

¹² *Comptes Rendus*, 1881.

¹³ *Exp. Res.*, 2362, 1845.

susceptibility may be neglected, it is clear that the resultant action on any body is the difference between the action upon it and the portion of the medium which it displaces. This principle, which is the analogue of the Archimedean law for floating bodies, is of great use in quantitative magnetic experiments. It was exemplified by Plücker,¹ and extensively applied in magnetic observations by Becquerel.² Becquerel found, for instance, that the differences between the couples tending to set a small rod of sulphur in water and in air, in magnesium chloride and in air, and in nickel sulphate and in air were very nearly the same as the corresponding differences for a rod of wax.

Plücker
and
Bec-
querel.

Different
liquids.

Constitu-
ents of a
thorough
mixture
not sepa-
rated.

Rarefac-
tion of
air
doubtful.

Magnetic
be-
haviour
of chemi-
cal com-
pounds.
Wiede-
mann.

Very curious qualitative illustrations of differential magnetic action are obtained by scattering drops of alcoholic solution of chloride of iron in olive oil;³ the drops of chloride collect and displace the olive oil in the places of stronger force. Another form⁴ of the same experiment consists in placing a layer of oil of violets over a layer of solution of chloride of iron. When a narrow cell filled in this way is placed equatorially with the interface of the two liquids in the axial line, on exciting the electromagnet the iron solution rises in the equatorial plane forming a disk-shaped mass around the axial line. Notwithstanding these results the general opinion of experimenters seems to be that no separation of the parts of a solution can be effected magnetically once the constituents have been thoroughly mixed. Thus Faraday⁵ could obtain no evidence of the concentration of an iron solution near the pole of a magnet, although it was exposed for days together in the magnetic field, and found no separation of the oxygen and nitrogen of atmospheric air, although they differ greatly in their magnetic character.

Plücker⁶ endeavoured to show that the air enclosed in a vessel placed between the poles of an electromagnet was rarefied by the magnetic action. Faraday, however, with almost identical experimental arrangements arrived at a negative result.

Elaborate investigations of the magnetism of chemical compounds have been made by G. Wiedemann with a view to connect their magnetic properties with their composition. A full account of these researches will be found in Wiedemann's *Galvanismus*, Bd. ii. § 590 sq.⁷ The following are some of the more important of his conclusions as to the effect of composition.

1. The magnetic susceptibility of the dissolved salt by itself is nearly independent of the solvent, being proportional to the concentration.

2. If the magnetic moment m induced by a field of unit intensity in a unit of weight of the salt dissolved in water be called the "specific susceptibility," and the product $\mu = Am$, where A is the molecular weight of the salt, the "molecular susceptibility," then the molecular susceptibility of the dissolved salt of the same metal with different acids is approximately the same. The mean molecular susceptibilities for nickelous, cobaltous, ferrous, and manganous salts are as 142 : 313 : 387 : 468.

3. The molecular susceptibility of cobaltous salts stands about midway between the molecular susceptibilities of nickelous and manganous salts; and the ferrous salts stand midway between cobaltous and manganous.

4. The molecular susceptibility of dry salts (combined with water of crystallization) is for the most part nearly the same as their molecular susceptibility in solution.

A similar law holds to a certain extent for insoluble

salts freshly precipitated; and generally, with like chemical properties of the metallic molecule, the molecular susceptibility remains the same.⁸

5. Two diamagnetic elements may give a magnetic compound; e.g., copper and bromine, both diamagnetic, give bromide of copper, which is paramagnetic.

6. When two solutions are mixed and the salts exchange their constituents by double decomposition, the specific magnetism of the solutions taken together is unchanged. Whence the conclusion is drawn that the susceptibility of a binary compound is made up by addition of the susceptibilities of its constituents, and that these constituents preserve their susceptibilities unaltered when their constitution or atomic arrangement in composition is unaltered.

Magnecrystallic Action.—In what precedes we supposed the inductively magnetized body, whether paramagnetic or diamagnetic, to be isotropic, and all experiments on its magnetic properties to be conducted in a heterogeneous magnetic field. In a uniform field such a body would be acted upon neither by force of translation nor by rotational couple. The case is otherwise if the body be magnetically æolotropic. In this case, according to the mathematical theory, (1) the body ought to set in a uniform field so as to place its axis of greatest magnetic permeability (i.e., of greatest paramagnetic and of least diamagnetic susceptibility) parallel to the lines of force, and (2) in a heterogeneous field Faraday's translational force from places of less to places of greater resultant force in the case of paramagnetic, and from places of greater to places of less in the case of diamagnetic bodies, ought to be greatest when the axis of greatest susceptibility is parallel to the lines of force, least when the axis of least susceptibility is in the same position, and intermediate for other positions of the body.

In observing the first class of phenomena above mentioned, poles with flat faces are placed on the electromagnet. Faraday recommends that the faces should be placed at a distance of about one-third of their breadth. He warns the experimenter, however, that the uniformity with this arrangement is by no means perfect, although in general sufficient. The best arrangement would be to use the magnetic field in the interior of a cylindrical coil of sufficient length were it not for the difficulty of attaining the requisite intensity in this way. In cases where there is any doubt it is well to give the body under examination a spherical or cubical shape, and so eliminate the tendency to set arising from heterogeneity of field.

The first observations of the magnecrystallic couple were made by Plücker,⁹ and elaborate investigations of the phenomenon were made by him in conjunction with Beer,¹⁰ in the course of which the magnetic properties of a large number of crystalline bodies were examined. Plücker also by detected the magnecrystallic property in a rapidly cooled cylinder of glass. Shortly after Plücker's first results were published, Faraday discovered the magnecrystallic action of crystallized bismuth. At first, misled no doubt by the language in which Plücker stated the newly discovered facts, he did not recognize the identity of the two phenomena; but on further investigation he was able to class all the observations under a few simple laws,¹¹ which in the mathematical form given to them by Thomson constitute the theory already given. To the observations of Plücker and Faraday Knoblauch and Tyndall added the important discovery that bodies in which the linear density in one direction is greater than in another, whether as a consequence of compression or of stratification artificial or natural, exhibit magnetic æolotropy.

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Mag
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given. To the observations
of Plücker and Faraday
Knoblauch and Tyndall
added the important
discovery that bodies
in which the linear
density in one direction
is greater than in another,
whether as a consequence
of compression or of
stratification artificial
or natural, exhibit
magnetic æolotropy.

¹ Pogg. Ann., 1849.

² Ann. d. Chim. et d. Phys., 1850.

³ Matteucci, Comptes Rendus, 1853.

⁴ Marangoni, Wied. Beibl., 1881.

⁵ Exp. Res., 2757; see also Righi, Wied. Beibl., 1878.

⁶ Pogg. Ann., 1848; see also Beer's treatise referred to above, p. 250.

⁷ See also Phil. Mag., 1877.

⁸ For qualifications see Wied., Galv., l.c.

⁹ Plücker, Pogg. Ann., 1847, 1848, 1849, 1852.

¹⁰ Plücker and Beer, Pogg. Ann., 1850, 1851.

¹¹ Exp. Res., 2797 sq., 1850.

It is convenient, following the analogy of physical optics, to divide magnetically æolotropic bodies into (a) "uniaxial" bodies, i.e., those that are symmetrical about one principal axis of magnetic susceptibility, or, in other words, have two of the principal coefficients of magnetic susceptibility equal ($\kappa_2 = \kappa_3$); and (b) "biaxial" bodies, i.e., those that have the three principal susceptibilities unequal.

Class (a) naturally divides itself into those in which the susceptibility parallel to the axis of symmetry is greater and those in which it is less than that in the plane perpendicular to it. The former (where $\kappa_1 > \kappa_2$) are said to be positive, the latter ($\kappa_1 < \kappa_2$) negative uniaxials. We have also to attend to the distinction which arises according as the mass of the crystal is paramagnetic or diamagnetic (κ_1 and κ_2 both +, or both -). We have then the following experimental behaviour in uniaxial bodies:—

		Sets the Axis of Symmetry
Positive	{ Paramagnetic Diamagnetic	Parallel to lines of force. Perpendicular to lines of force.
Negative	{ Paramagnetic Diamagnetic	Perpendicular to lines of force. Parallel to lines of force.

Faraday found for example that a crystal of bismuth suspended with its plane of smoothest cleavage vertical set with this plane perpendicular to the lines of force, but was indifferent when suspended with this plane horizontal. The axis of magnetic symmetry is therefore perpendicular to the plane of smoothest cleavage, and, since the substance is diamagnetic, it is a negative uniaxial. When suspended in any other way the crystal set so that the magnetic axis rested in a vertical plane through the direction of the lines of force. The difference between the behaviour of magnetic and diamagnetic uniaxials is beautifully illustrated by the behaviour of pure Iceland spar, which is a positive diamagnetic uniaxial, and sets the optic axis, which is the magnetic axis, equatorially; when, however, as sometimes happens, the calcium is partly replaced by iron, its physical properties (optical included) being thereby unchanged, except that the mass of the crystal becomes magnetic, it sets the optic axis axial.¹

All these cases are of course included in the single rule that the body sets the axis of greatest permeability parallel to the lines of force. Since the setting of weakly magnetic æolotropic bodies depends merely on the differences between the principal magnetic susceptibilities, it follows that it is independent of the medium in which the body is placed. This was established experimentally by Faraday,² who found the magnecrystalline couple exerted on a crystal of bismuth to be the same whether it was surrounded by air, by water, or by a saturated solution of protosulphate of iron.

Plücker gives the following list of uniaxial æolotropic bodies:—

MAGNETIC.	
Positive.	Negative.
Spathic iron ore.	Tourmaline.
Scapolite.	Beryl.
Green uranite.	Diopside.
Ferruginous sulphate of magnesia.	Vesuvian.
	Sulphate of nickel.
	Ammoniochloride of copper.
DIAMAGNETIC.	
Positive.	Negative.
Calc-spar.	Bismuth.
Antimony.	Arsenic.
Molybdate of lead.	Ice.
Arsenide of lead.	Zircon.
Sulphate of potash.	Mellite.
Nitrate of potash.	Cyanide of mercury.
	Arseniate of ammonium.

¹ Tyndall and Knoblauch, *Phil. Mag.*, 1850.

² *Exp. Res.*, 2498 sq., 1848. A later but much more extensive series of experiments led to the same result (*Exp. Res.*, ser. xxx., 1855).

The phenomena in the case of biaxial æolotropics are naturally more complicated; but they are all comprehended in the simple rule given above (page 244) that the axis of greatest paramagnetic or of least diamagnetic resultant susceptibility in the horizontal plane tends to set itself parallel to the lines of force, or, in the words of Faraday, the body tends to set so as to allow the greatest number of lines of magnetic induction to pass through it. In the azimuth just mentioned the body is in stable equilibrium, in the perpendicular azimuth in unstable equilibrium. There are two axes of suspension in the plane of the axes of least and greatest susceptibility, viz., the normals to the circular sections of the ellipsoid $\kappa_1 x^2 + \kappa_2 y^2 + \kappa_3 z^2 = \text{constant}$, such that the body behaves indifferently; these axes were called by Plücker the "magnetic axes" of the body. If we observe the times of vibration T_1, T_2, T_3 of a sphere of the substance, when the axes $\kappa_1, \kappa_2, \kappa_3$ respectively are vertical, then we have at once by the theory already given

$$T_1 : T_2 : T_3 :: 1/\sqrt{\kappa_2 - \kappa_3} : 1/\sqrt{\kappa_1 - \kappa_3} : 1/\sqrt{\kappa_1 - \kappa_2};$$

whence $1/T_1^2 + 1/T_2^2 = 1/T_3^2$, and $\tan \omega = T_3/T_1$, ω being the angle between either magnetic axis and the axis of greatest magnetic susceptibility. These results were verified experimentally by Plücker.³

Magnecrystalline phenomena of the second kind were looked for by Faraday very early in the history of the subject, but at first he was unsuccessful in detecting them. He seems, however, to have understood and clearly represented to himself in his own way their close connexion with the phenomena of the first kind, for he alluded to the subject more than once, and finally in the twenty-sixth series of his experimental researches, where he explained at length his magnetic theory, he showed that such an effect ought to exist, and actually succeeded in observing it in the case of a crystal of bismuth, which he found to be less repelled from places of stronger to places of weaker force when its axis was parallel to the lines of force than when it was perpendicular to them. He concluded that with Iceland spar the translational force ought to be greatest when the axis is parallel to the lines of force, and least when it is perpendicular to them; but his apparatus was not sufficiently delicate to show the effect.

Unlike the magnecrystalline phenomena of the first kind, those of the second kind depend on the difference between the susceptibilities of the body and of the surrounding medium. Faraday⁵ demonstrated this conclusion experimentally by covering crystals of the red prussiate of potash with a thin layer of wax to prevent dissolution, and immersing them in solution of sulphate of iron of various strengths between the pointed poles of an electromagnet. In water the crystal was attracted to places of stronger force in all positions, in concentrated solution of sulphate of iron repelled in all positions, while in a solution of 14 or 15 volumes of the concentrated solution to 6 volumes of water it was attracted when the axis of symmetry was parallel to the lines of force and repelled about as strongly when the axis was perpendicular to them. Here then the crystal actually behaved paramagnetically in one direction and diamagnetically in the other. Similar results can be obtained with Iceland spar in a mixture of alcohol and water.

The prediction of the second class of magnecrystalline phenomena is one of the most extraordinary instances of the theoretic insight which formed so large a part of the genius of Faraday. The laws of the phenomena of the first class might be regarded as merely a skilful classification of observed facts, but the passage therefrom to the second class was a step of the first magnitude; it constitutes in fact the root of the whole matter. To Sir William

³ *Phil. Trans.*, 1858.

⁴ *Exp. Res.*, 2552, October 1848.

⁵ *Exp. Res.*, ser. xxx., 1855.

Thomson belongs the credit of throwing the laws of magnecrystalline action into the appropriate mathematical form, and of showing that they range themselves quite naturally under the theory of Poisson.¹

Tyndall verifies the prediction of Faraday and Thomson for Iceland spar.

Tyndall succeeded, where Faraday had failed, in showing the magnecrystalline phenomenon of the second kind in Iceland spar. It is particularly instructive to compare his results for carbonate of iron and carbonate of lime, both positive uniaxials, but the one magnetic and the other diamagnetic. His results are as follows:—

Substance.	Axis Axial.	Axis Equatorial.	Magnetic Character.
Carbonate of iron	100	71	+
Carbonate of lime	100	90	—
Sulphate of iron.....	100	85	+
Bismuth	71	100	—

Hankel.

Hankel² measured the repulsion on a cylinder of bismuth placed with its magnetic axis inclined at angles of 15°, 45°, and 75° with the lines of force, and compared his results with the theoretical formula $90.7 + 45.3 \sin^2 \phi$; the result was as follows:—

	± 15°.	± 45°.	± 75°.
Observed repulsion	94.1	113.3	132.4
Calculated repulsion	93.7	113.3	133.0

Relation of magnetic æolotropy to crystalline form.

Influence of Crystalline Form, Compression, &c., in producing Magnetic Æolotropy.—In general the magnetic æolotropy stands in close relation to the crystalline form, and consequently to a considerable extent also to the optical properties. Thus crystals of the regular system exhibit as a rule no magnecrystalline properties, but there appear to be exceptions in the case of certain pyroelectric crystals such as boracite. Again, crystals that have one crystallographic axis of symmetry are usually magnetically uniaxial, but the optical distinction of positive and negative does not involve the corresponding magnetic distinction, as is shown by the results of Tyndall and Knoblauch with pure Iceland spar and Iceland spar in which part of the calcium is replaced by iron. Crystals that are optically biaxial are as a rule magnetically biaxial, but the magnetic properties cannot be deduced immediately from the optical.

Effect of compression. Tyndall and Knoblauch.

If a small cylinder be made of a paste formed with finely ground bismuth and gum water it will point equatorially in a heterogeneous field, but if the roll be squeezed flat the plate thus formed will point axially, although its length be ten times its breadth. A roll of paste of powdered carbonate of iron, again, will point axially, the plate formed by squeezing it flat equatorially.³ From these results Tyndall and Knoblauch concluded that, if the arrangement of the particles of any body be such as to present different degrees of proximity in different directions, then the line of closest proximity, other circumstances being equal, will stand axial if the mass be magnetic, equatorial if the mass be diamagnetic. They constructed parallelepipeds (1 in. × $\frac{1}{4}$ in. × $\frac{1}{4}$ in.), first, by gumming together rectangular slips of sandpaper (1 in. × $\frac{1}{4}$ in.), secondly, by gumming together squares of the same ($\frac{1}{4}$ in. × $\frac{1}{4}$ in.). The paper was comparatively indifferent, while the sand by itself was magnetic; and it was found that the first model set its longest dimension axially, while the second set its longest dimension equatorially; i.e., the layers of sand set in both cases axially. Tyndall⁴ has observed similar magnecrystalline actions with naturally stratified bodies such as shale, and in fibrous bodies such as wood. He was even able by

Effect of laminar structure.

Stratified and fibrous bodies.

squeezing plates of bismuth to apparently reverse the magnetic character of the substance; for the compression rendered the plates æolotropic with an axis perpendicular to their longest dimension, and in consequence they set axially like plates of a paramagnetic substance. A crystal of bismuth compressed in a direction perpendicular to the ordinary magnetic axis, i.e., parallel to the planes of principal cleavage, had its behaviour reversed as to the second class of magnecrystalline effects, the ratio of the repulsions when the crystal was set with its original axis axial and with its original axis equatorial having been changed from 71:100 to 112:100. It was also found possible by squeezing a ball of bismuth dough unequally in two perpendicular directions to imitate a biaxial magnetic crystal such as heavy spar. Tyndall and Knoblauch attempt to explain the magnetic phenomena exhibited by crystals proper by means of these results. They assume that the planes of cleavage are directions of closer aggregation, and therefore tend to point axially in magnetic and equatorially in diamagnetic crystals. For example, the first of the above-mentioned sandpaper models would represent magnetic crystals that cleave parallel to their axis, the second magnetic crystals which cleave perpendicular to their axis. If we regard this theory merely as a way of representing the facts of observation, even if we allow it to be sufficient, it is far inferior in simplicity to the theory of Faraday and Thomson, the sufficiency of which is not disputed. Regarded as an attempt to penetrate a little farther into the relation between molecular structure and magnetic properties, it is of great interest and importance, even if we admit that like most other speculations of the kind it leads us but a little way; for the question arises immediately, How does proximity of the molecules increase specific inductive capacity? This last question is all the more difficult to answer that no experiment has ever yet been adduced wherein the effect of the mutual induction of the parts of a diamagnetic or weak paramagnetic body plays an undoubted part.⁵

Discussion as to the Existence of Diamagnetic Polarity.—Soon after Faraday's first discovery of diamagnetism, an animated discussion arose as to the proper way of stating the facts involved in the new phenomenon. Faraday himself inclined in the first instance to put the matter by saying that under the action of an inducing force a diamagnetic body is magnetized in a direction opposite to that of soft iron; at a later period he abandoned this form of statement in favour of what he called the theory of magnetic conduction, which fitted better with his ideas as to the part played by the surrounding medium by means of which magnetic action is transmitted from one body to another. Faraday's first theory under the name of the theory of diamagnetic polarity was immediately adopted by the Continental physicists, such as Weber, Reich, and Pogendorff, who naturally found it consonant with their favourite views as to action at a distance. It was also supported in England by Tyndall and others. Many experiments were advanced on both sides of the question, and the result was much instructive illustration of the laws of magnetic action. But the controversy settled nothing, because in point of fact there was nothing to settle. Either theory was perfectly sufficient, when properly applied, to represent the phenomena, and each left the question of the ultimate nature of paramagnetic and diamagnetic action where it found it. This ought to have been evident after Thomson had shown that the phenomena were included in a perfectly natural generalization of Poisson's theory, indicated in fact by

¹ *Phil. Mag.*, March 1851; or *Reprint*, chap. x.

² In 1851; see *Wied., Galv.*, ii. § 639.

³ Tyndall and Knoblauch, *Phil. Mag.*, 1850.

⁴ *Phil. Mag.*, 1851.

⁵ Faraday, *Exp. Res.*, 2825, &c. There is a further difficulty in the case of diamagnetic bodies. See Thomson's letter to Tyndall, *Reprint of Papers on Elec. and Mag.*, p. 536.

Poisson himself, and demonstrated that Faraday's conception of the phenomena was only another method of viewing the facts leading to identical conclusions. Faraday himself¹ seems in the end to have considered that the difference was a matter of phrases. Since Clerk Maxwell's elaborate mathematical reconstruction of the theories of Faraday this seems to be universally recognized, and the discussion has subsided. For a full account of the various interesting experiments that were made during the controversy the reader may be referred to Wiedemann's *Galvanismus*, § 558 sq., and to the reprint of Tyndall's *Papers on Diamagnetism and Magnecrystalline Action*, pp. 76 sq.

Numerical Data respecting the Susceptibility of Weakly Magnetic Bodies.—The earlier experimenters arrived for the most part at the conclusion that the susceptibility κ of weakly magnetic bodies is constant. Among these may be mentioned Weber, who experimented with bismuth, E. Becquerel,² Tyndall,³ Joule,⁴ Reich,⁵ and Matteucci,⁶ who experimented on various substances by means of the torsion balance; Christie,⁷ who worked with bismuth, and Arndtsen,⁸ who worked with ferric sulphate and ferric chloride, both using Weber's diamagnetometer; and Wiedemann,⁹ who experimented with solutions of a variety of salts. E. Becquerel, however, in some of his experiments, e.g., with sulphate of nickel, found that κ showed a tendency to decrease for very large values of the magnetizing force; Plücker,¹⁰ who tested a great variety of substances (powdered or in solution) by measuring with a delicate balance the attraction or repulsion exerted upon them by an electromagnet, arrived at a similar conclusion; but the methods of both these experimenters are open to suspicion.

A large number of relative results were obtained by the earlier experimenters,¹¹ but in some cases the methods employed were not satisfactory, and in others the results so evidently depend on the state of aggregation of the material that they are of little importance. The following tables will give the reader some idea of the relative magnitudes of the susceptibilities of different substances:—

Plücker's Table for Magnetics.

Iron	100,000	Ferric chloride, conc. soln.	98
Magnetic iron ore	40,227	Ferric sulphate ..	58
Ferric oxide	286	Ferrous chloride ..	84
Hematite	134	Ferrous sulphate ..	126
Specular iron ore	533	Nickelous oxide	35
Hydrated ferric oxide	156	Hydrate of do.	106
Ferric sulphate	111	Hydrated manganic oxide	70
Green vitriol	78	Mangano-manganic oxide.	167
Nitrate of iron, conc. soln.	34		

The numbers here denote specific magnetic susceptibility; i.e., equal weights of the substances are compared.

Results of Faraday and Becquerel.

Ferrous chloride, conc. solution	+ 655	Hydrogen	- 0.1
Ammoniacal solution of cuprous oxide	+ 134	Ammonia gas	- 0.5
Do. of cupric oxide	+ 120	Cyanogen	- 0.9
Oxygen	+ 17.5	Glass	- 18.2
Air	+ 3.4	Pure zinc	- 75
Olefiant gas	+ 0.6	Ether	- 75
Nitrogen	+ 0.3	Alcohol absolute	- 79
Vacuum	0.0	Oil of lemons	- 80
Carbonic acid	0.0	Camphor	- 82
		Camphire	- 83
		Linseed oil	- 85

¹ See a letter to Matteucci dated November 2, 1855, published in Bence Jones's *Life and Letters of Faraday*, reprinted in Tyndall's *Diamagnetism and Magnecrystalline Action*, p. 180.

² *Ann. d. Chim. et d. Phys.*

³ *Phil. Mag.*, 1851.

⁴ *Pogg. Ann.*, 1856.

⁵ *Pogg. Ann.*, 1858.

⁶ *Pogg. Ann.*, 1865.

⁷ E.g., Plücker, *Pogg. Ann.*, 1848, 1851, &c.; E. Becquerel, *Ann. d. Chim. et d. Phys.*, 1850, 1851, 1855, &c.; Matteucci, *Comptes Rendus*, 1853, and *Cours d'Induction*, 1854; Wiedemann, *Pogg. Ann.*, 1865, 1868, &c.

⁸ *Phil. Mag.*, 1852.

⁹ *Ann. d. Chim. et d. Phys.*, 1859.

¹⁰ *Pogg. Ann.*, 1858.

¹¹ *Pogg. Ann.*, 1854.

Olive oil	- 86	Chloride of arsenic	- 122
Wax	- 87	Fused borate of lead	- 137
Nitric acid	- 88	Phosphorus*	- 167
Water	- 96.6	Selenium*	- 168
Ammonia solution	- 98	Pure copper*	- 171
Bisulphide of carbon	- 100	Pure silver*	- 235
Sat. solution of nitre	- 100	Pure gold*	- 350
Sulphuric acid	- 104	Bismuth	- 1967
Sulphur	- 118		

The results marked with an asterisk are taken from Becquerel; the rest are from Faraday. The numbers relate to equal volumes of the substances, and the medium is supposed to be vacuum; so that water in air would be represented by 100.

The numbers of Faraday, Becquerel, and Matteucci agree very fairly; e.g., according to Faraday the susceptibilities of water, oxygen, and air are as -100 : +1.8 : +352, according to Becquerel as -100 : +1.82 : +382. Plücker's results do not agree so well with those of Faraday and Becquerel; but his method was faulty.

Within the last five years a large number of absolute determinations of κ have been made, chiefly for bismuth and ferric chloride. Toepler and Von Ettingshausen¹² in their experiments on bismuth used with some alteration the method of induced currents employed by Weber¹³ in the earliest attempts that were made to determine the susceptibility of bismuth. Like Weber they compare bismuth with iron, an unsatisfactory procedure on account of the great variability of the susceptibility of iron for different magnetizing forces, and for different samples with the same magnetizing force.

Silow worked with ferric chloride. In his first set of experiments¹⁴ he observed the time of vibration of an astatic needle suspended over a cylindrical vessel filled with the solution; in his second investigation¹⁵ the solution was placed in a glass globe, on the outside of which insulated wire was wound so that a given current in it produced a uniform magnetic field whose strength could be calculated; the deflexion of a properly astaticized needle suspended inside the globe, was observed when the globe was empty and when it was full, and thence κ was calculated; in his third determination¹⁷ he used the method of Toepler and Von Ettingshausen so improved as to allow an absolute determination of κ to be obtained directly.

Borgmann¹⁸ enclosed one coil within another, and filled Borgmann the hollow cylindrical space between them with the solution of ferric chloride to be tested; he also used the ring method of Stoletoew and Rowland.

Jacques¹⁹ following a method elaborated by Rowland, Jacques measured the repulsion of crystals of bismuth and Iceland spar placed with their magnetic axes axially and equatorially between the poles of a Ruhmkorff's electromagnet, the field of which was carefully explored after the manner of Verdet by means of a small coil moved through a known distance in different parts of it; from these observations the two principal magnetic susceptibilities were calculated.

Schuhmeister²⁰ experimented with ferric chloride, using the same method as Rowland and Jacques.

In the experiments of Eaton²¹ the method formerly employed by Wiedemann was adopted; the data in his paper are, however, insufficient for an absolute determination either of the magnetizing force or of κ ; in fact he determines merely the force with which the magnetic body is attracted and the magnetic moment of the electromagnet, assuming that the strength of the magnetic field at a given point is proportional to the latter,—which is not necessarily true, for the magnetic distribution in the core of the electromagnet may alter with increasing current.

¹² *Pogg. Ann.*, 1877.

¹³ *Wied. Ann.*, 1877.

¹⁴ See Maxwell, *El. and Mag.*, vol. ii. § 672.

¹⁵ *Wied. Ann.*, 1880.

¹⁶ Silliman's *Jour.*, 1879. The published results are vitiated by some error of calculation; but the experiments are to be repeated.

¹⁷ *Wien. Ber.*, 1882.

¹⁸ *El. Maasbest.*, Thl. iii.

¹⁹ *Wied. Beibl.*, 1879.

²⁰ *Wied. Beibl.*, 1879.

²¹ *Wied. Beibl.*, 1879.

Von
Ettings-
hausen.

Von Ettingshausen¹ in the most recent research on the subject with which we are acquainted has made determination of the susceptibility of bismuth by four different methods. The first of these was that formerly used by Toepler and himself, with the addition that the action of the bismuth bar was compared with that of a solenoid of as nearly the same dimensions as possible through which flowed a current of given strength (an artifice previously used by Christie). The second method consisted in measuring the force with which a portion of the diamagnetic substance hung in the axis of a coil and near one of its ends was repelled out of the coil when a known current passed through it. The third method was that of Rowland and Jacques. The fourth consisted in measuring the deflexion of a magnetometer needle produced by placing a piece of the diamagnetic substance between the poles of a powerful magnet under whose action the magnetometer needle had come to rest in the first instance.² The agreement between the results obtained by all the different methods was very fair considering the smallness of the effects to be measured in some of them. The second method is pronounced to be the best, and by means of it he gives also a determination of κ for ferric chloride.

Results
for
bismuth.

Some of the results of the different experimenters for bismuth are given in the following table:—

Magnetizing Force.	$-10^6\kappa$.	Authority, &c.
63	14.6	{ Calculated by Stoletow from certain results of Weber's, see Silow, <i>Wied. Ann.</i> , 1882.
301	14.9	
...	16.4	{ Calculated by Von Ettingshausen, <i>i.e.</i> , from Weber.
...	14.6	
25.8 to 128	13.99	{ Calculated by Von Ettingshausen, <i>i.e.</i> , from Christie.
71.4 to 110.2	14.54	
39.2 to 82.2	13.48	
		{ Three different samples by Von Ettingshausen's second method.

Most of the specimens contained slight traces of iron. Although the range of the magnetizing force in Von Ettingshausen's experiments was considerable, κ was very nearly constant; if there was any tendency to variation, it was decrease with the large magnetizing forces.

Results
for
ferric
chloride.

The results for ferric chloride are not so concordant. Silow, after comparing his own earliest result ($10^6\kappa = 81$ for a solution of density 1.475, magnetized by the earth's horizontal force) with those of Borgmann ($10^6\kappa = 48.8$, density 1.87, magnetic force 40 to 59), concluded that the susceptibility of ferric chloride probably follows the same law as that of iron; *i.e.*, it first increases, then reaches a maximum, and afterwards decreases more or less slowly. His later experiments confirm this conjecture, and he finds that κ has a maximum value for a magnetizing force of about .4 C.G.S. The smallest force used was about .08 C.G.S. and the corresponding value of $10^6\kappa$ was 34; the largest value of $10^6\kappa$ occurring in his tables is 179. The values obtained in his last investigation are smaller than those given in his first table, but there is the same increase and decrease. The following are his latest results:—

Silow.

\mathfrak{H}	$10^6\kappa$.	\mathfrak{H}	$10^6\kappa$.
1.15	96	2.45	104
1.35	104	3.73	70
1.60	131	5.33	69
1.70	131	5.35	68
1.81	142	6.54	65
1.90	141	7.00	62
1.96	131	10.00	60
2.13	111	12.60	55
2.40	99		

¹ *Wien. Ber.*, 1882.

² This was one of the experiments adduced by Weber in the controversy regarding diamagnetic polarity; see *Pogg. Ann.*, 1848.

The unit of \mathfrak{H} is the earth's horizontal force, presumably at Moscow.

From the observations of Arndtsen on a solution of density 1.495 Silow³ calculates $10^6\kappa = 57.5$ (magnetic force 20.3). For a solution of density 1.395, with magnetizing forces from 38 to 252, Schuhmeister gets $10^6\kappa = 30$ to 39. Von Ettingshausen, for a solution of density 1.48, with magnetizing force 14 to 20, gets $10^6\kappa = 59$ to 56.

The following are the values of $10^6\kappa$ obtained by Schuhmeister for various substances.

\mathfrak{H}	61.5	130.8	252.7	Schulmeister
Water	-.55	-.45	-.44	
Alcohol	-.45	-.42	-.38	
Bisulphide of carbon	-.46	-.39	-.37	
Ether	-.40	-.29		

\mathfrak{H}	66.8	141.8	272.2
Oxygen from chlorate of potash046	.059	.122
.....	.056	.067	.128
Oxygen from electrolysis117	.181	...
ozonized103	.177	...
Nitrogen0278	.0377	.0496
.....	.0232	.0380	.0437

RELATION OF MAGNETISM TO OTHER PHYSICAL PROPERTIES.

Shocks, Jarring, or Vibration.—The effect of these in aiding the action of an inductive magnetic force was known to Gilbert; and it was also known to the earlier experimenters that the permanent magnetism of a body not subject to external magnetizing force was destroyed by like causes. The action is precisely similar to that found in the case of bodies temporarily or permanently deformed by mechanical stress, and, again, to the first effects of temperature on bodies temporarily or permanently strained, or temporarily or permanently magnetized.⁴ The effect may be conceived as consisting of a loosening of the molecules for the moment, so that they follow more easily any force acting on them whether mechanical or magnetic. The following parallel statements, taken from the results of Wiedemann, who has devoted much careful study to these phenomena, will sufficiently illustrate the matter:—

1. Jarring a body under twisting stress causes increase of twist.
2. Permanent twist in a wire is diminished by jarring.
3. A wire permanently twisted and then partly untwisted loses or gains twist when jarred according as the untwisting is small or great.⁵

- I. Jarring a bar under magnetizing force causes increase of magnetization.
- II. Permanent magnetization in a bar is diminished by jarring.
- III. A bar permanently magnetized and then partly demagnetized loses or gains magnetization according as the demagnetization is small or great.⁶

Minuter details regarding the effects of jarring will be found in memoirs by Wiedemann, Fromme, Auerbach, and others already quoted. The reader may also consult Warburg, *Pogg. Ann.*, 1870, and Villari, *Pogg. Ann.*, 1869.

Mechanical Strain produced by Magnetization.—The starting point of accurate research on this subject was the discovery made by Joule⁶ in 1842 that a bar of soft iron lengthened when it was temporarily magnetized in the longitudinal direction. When the magnetizing force was removed the bar shortened, but in general not quite to its

³ *Wied. Ann.*, 1882.

⁴ Compare also the effect of the same causes on the temporary and residual charge of Leyden jars, art. ELECTRICITY, vol. viii. p. 40.

⁵ It is possible in this way even to cause a wire to reverse its twist and a bar to reverse its magnetization by jarring.

⁶ His attention had been drawn to the subject in 1841 by a Mr Arstall, who had suspected the existence of some such effect. Joule's papers on the subject are in Sturgeon, *Ann. of El.*, 1842, and *Phil. Mag.*, 1847.

original length. This residual extension was due in part to permanent magnetism, but he found that the permanent magnetization due to a current 1088 was reversed from -1.3 to $+2.5$ by a current 175, while two-thirds of the permanent extension was still left. The actual elongation of an iron bar magnetized to saturation was found to be from $\frac{1}{100000}$ th to $\frac{1}{200000}$ th of its whole length. The extension varied approximately as the square of the intensity of magnetization (temporary or permanent). The general character of the phenomena is the same in soft or hard iron, and in soft or hard steel;¹ but the effects are smaller with hard than with soft bars.

It was found that longitudinal compression of the bar influenced the magnetic extension little if at all; on the other hand, longitudinal traction was found to diminish it, and in the case of thin wires under considerable tension the magnetization caused a contraction. Thus in the case of a bar 1 foot long, $\frac{1}{4}$ inch in diameter, with a weight of 600 lb, there was neither extension nor contraction, even with a current of 1600; with weights of 1040 lb and 1680 lb, and a current of 1804, there was a contraction of $.00002$ inch and $.000032$ inch respectively. The contraction under tension was found to vary approximately as the product of the magnetizing current and the intensity of magnetization. After the magnetizing force was withdrawn the wire regained its original length, permanent magnetization notwithstanding.

Joule made careful experiments to determine whether the magnetization of an iron bar produced any alteration of its volume, but could find none. He therefore concluded that the longitudinal extension of a magnetized bar is accompanied by an equal lateral contraction; and, in accordance with this conclusion, he found that when an iron tube is circularly magnetized, perpendicular to its length, by passing a current along its axis, it contracts longitudinally.

The results of Joule have been verified and to some extent added to by Wertheim,² Buff,³ Beez,⁴ Tyndall,⁵ Mayer,⁶ Righi,⁷ and Ader.⁸ The three first experimented with magnetizing coils shorter than the bar, and found that the extension was much greater when the coil was near the free end than when it was near the fixed end of the bar. This of course raises the question how far the extension is due to electromagnetic action between the coil and the bar, and how far to internal molecular disturbance.⁹ Mayer's results are in agreement with Joule's except in the case of bars of soft steel, which (not under traction) when the magnetizing current was first established, elongated in some cases and retracted in others,—at the first break elongated, and subsequently retracted at make and elongated at break. Righi's results for longitudinal magnetization are in agreement with those of Joule; he also gives a variety of interesting results regarding the effects of circular and longitudinal magnetization on the length of iron wires. Barrett¹⁰ has recently arrived at the interesting result that nickel behaves oppositely to iron,—retracting about $\frac{1}{100000}$ th when magnetized to saturation; he gives for the elongation of iron and cobalt under like circumstances $\frac{1}{100000}$ th and $\frac{1}{200000}$ th respectively.

Effect upon Magnetization of Traction along the Lines of Magnetization.—Matteucci¹¹ seems to have been the first to discover that when a bar subject to a magnetizing force in the direction of its length is stretched in the same direction its temporary magnetization increases. When the stretching force is removed the magnetization again diminishes. Wertheim¹² confirmed Matteucci's observation. Villari,¹³ however, found that, after the first effect, which

¹ In the case of a bar of hard steel he found a considerable increase in length every time the magnetizing current was interrupted. This he attributes to a state of "tension" in the hardened steel.

² *Ann. d. Chim. et d. Phys.*, 1848.

³ Cited by Wiedemann, *Galv.*, ii. § 504. ⁴ *Pogg. Ann.*, 1866.

⁵ *Diamagnetism and Magnecrystalline Action*, 1870.

⁶ *Phil. Mag.*, 1873. ⁷ *Nuov. Cim.*, 1880.

⁸ *Comptes Rendus*, 1880.

⁹ See Wiedemann's remarks, *Galv.*, ii. § 503.

¹⁰ *Nature*, vol. xxvi., 1882.

¹¹ *Comptes Rendus*, 1847; *Ann. d. Chim. et d. Phys.*, 1858.

¹² *Ann. d. Chim. et d. Phys.*, 1857, 1858.

¹³ *Pogg. Ann.*, 1868, also 1865, 1869.

is always increase, the application of the traction will cause increase if the intensity of magnetization is not beyond a certain critical value, but decrease if that value is surpassed; the removal of the traction causes in each case the opposite effect to the application.

The effect of the first traction on the permanent magnetization, whether of iron or steel, is a diminution; the effect of subsequent tractions in steel is a diminution on application, with increase on removal; in soft iron an increase on application, a diminution on removal. Partial demagnetization of a steel bar by an opposite magnetic force causes it to behave like soft iron; when the demagnetizing force is sufficient to reverse its polarity, the effect of even the first traction may be to increase the magnetization.

Sir W. Thomson¹⁴ has carefully studied the phenomena in question, as exhibited in a very soft iron wire $.075$ cm. in diameter permanently stretched by a weight of 1 lb, and alternately stretched by weights of 7 lb, 14 lb, or 21 lb, and unstretched (so that there was no permanent elongation). As the magnetizing force was increased, the increase of magnetization caused by the application of traction increased to a maximum, then diminished, and became zero for a certain critical value of the magnetizing force; after the critical value was passed, the traction caused a diminution of the magnetization, which increased asymptotically towards a fixed limit as the magnetizing force was increased more and more. The following table will give an idea of the results.

I denotes the maximum increase, and D the limit of the decrease, roughly estimated in the same arbitrary unit; \mathfrak{M}' is the force corresponding to I, and \mathfrak{M}_0 the critical force, each expressed in terms of the earth's vertical force at Glasgow as unit; T is the traction, ℓ the temperature.

T	ℓ	I	D	\mathfrak{M}'	\mathfrak{M}_0
7	Ord.	+31	-6	5.9	34
...	100°	+26	-3	6.4	35
1	Ord.	+35	-14	4.8	25
...	100°	+32	-9	4.5	26
21	Ord.	+54	-21	4.5	26
...	100°	+51	-15	5.0	29

Bars of nickel and cobalt were also examined; and it was found that after the first effect the result of applying traction in the direction of magnetization was in both cases to diminish the magnetization. The effect appeared to increase up to a maximum, and then to diminish as the magnetizing force increased; but the critical value was not reached with the largest forces employed.

Traction perpendicular to the lines of magnetization was found by Thomson to diminish the magnetic susceptibility. The experiment was made by means of a gun barrel magnetized longitudinally, and subjected to internal hydrostatic pressure.

The effect of pressure along or perpendicular to the magnetization would in all probability be opposite (and equal?) to that of an equal amount of traction; but no experiments have as yet been made on the subject.¹⁵ The effect of traction is therefore to produce magnetic anisotropy, the susceptibility being increased in the direction of the stress and diminished in the perpendicular direction so long as the intensity of magnetization is not above a certain critical value; above that value the effects are reversed. The effect of pressure would be opposite in every particular. Hence the effect of a shearing stress would be increase of magnetic susceptibility along the principal axis of elongation, and decrease (to an equal extent?) along the principal axis of compression.

¹⁴ *Phil. Trans.*, 1876 and 1879, p. 55.

¹⁵ Pressure applied outside Thomson's gun barrel would enable us to observe the effect of transverse pressure; and by magnetizing the barrel circularly the effect of pressure along the lines of force could be determined.

Wiedemann. Torsion and magnetization.

Relations between Torsion and Magnetization.—These were investigated by Matteucci,¹ and after him by Becquerel and Wertheim.² The whole subject was carefully studied by G. W. Wiedemann,³ who has done more than any living physicist both in discovering new facts in this interesting field and in coordinating those formerly known. We extract from his *Galvanismus*⁴ the following series of parallel statements, which will serve the double purpose of making the reader acquainted with the principal facts, and of drawing his attention to the close analogy between the mechanical and magnetic properties of bodies, and to the almost perfect reciprocity of their experimental laws.

Parallel statements for strain and magnetization.

1. The permanent torsion of iron wires is diminished by magnetization in a proportion decreasing with increasing magnetization.

2. Repetition of magnetization in the same direction diminishes permanent torsion very little farther; but magnetization in the opposite direction causes a fresh and considerable diminution.

3. When the permanent torsion of a wire has been removed as far as it can be by magnetizations within certain limits repeated alternately in opposite directions, it takes a maximum of torsion when magnetized in one direction, a minimum when magnetized in the other direction.

4. A permanently twisted wire partially untwisted loses less of its twist when magnetized than an ordinary permanently twisted wire. If the untwisting has been considerable, feeble magnetization causes an increase of torsion, which rises to a maximum and then decreases as the magnetization is increased. The greater the untwisting the stronger the magnetization corresponding to this maximum, and, when the untwisting is very great, the maximum may not be reached at all.

5. If a wire under the influence of a twisting stress is magnetized, the twist increases with weak but decreases again with strong magnetization. The first effect of magnetization is usually to increase the twist; but, if the wire be jarred beforehand, the magnetization at once causes untwisting, which disappears when the magnetization ceases.

6. If we magnetize an iron wire so that its free end has north polarity, and then pass a current from the fixed to the free end, or first pass the current and then magnetize, the free end of the wire as seen from the fixed end twists in the direction of the hands of a watch. The reversion of current or of magnetization reverses the twist; reversion of both leaves it unaltered.

[It would appear that when the magnetizing force and the current are both in action the twist tends to a maximum when either is increased, the other remaining constant.]

The alterations of the longitudinal and circular magnetization of

I. The permanent magnetization of steel bars is diminished by torsion in a proportion decreasing with increasing torsion.

II. Repetition of torsion in the same direction diminishes permanent magnetization very little farther; but torsion in the opposite direction causes a fresh and considerable diminution.

III. When the permanent magnetization of a bar has been removed as far as it can be by twisting within certain limits repeated alternately in opposite directions, it takes a maximum of magnetization when twisted in one direction, a minimum when twisted in the other direction.

IV. A permanently magnetized bar partially demagnetized loses less of its magnetization when twisted than an ordinary permanently magnetized bar. If the demagnetization has been considerable, feeble twist causes an increase of magnetization, which rises to a maximum and then decreases as the twist is increased. The greater the demagnetization the greater the twist corresponding to this maximum, and, when the demagnetization is very great, the maximum may not be reached at all.

V. If a bar under the influence of a longitudinal magnetizing force is twisted, the magnetization increases with small twists but decreases again with large twists. The first effect of twisting is usually to increase the magnetization; but, if the bar be jarred beforehand, the twist at once causes a decrease, which disappears when the twist ceases.

VI. If we twist the free end of a wire in the direction of the hands of a watch as seen from the fixed end, while a current from fixed end to free end either is passing through it or has passed through it, the wire becomes longitudinally magnetized so that its free end has north polarity. The reversion of current or of twist reverses the magnetization; reversion of both leaves it unaltered.

iron wires may be shown by the induced currents thereby caused in a coil surrounding the wire or in the wire itself. For example, if an iron wire be circularly magnetized by passing a current through it, and then twisted in either direction, an induction current flows through the wire in the same direction as the original current; and an opposite current is observed when the wire is untwisted again. This shows that twisting the wire diminishes the permanent circular magnetization, while untwisting partially restores it.⁵

The relation between bending stress and magnetization has been studied by Guillemin,⁶ Wertheim,⁷ Ader,⁸ and Kimball;⁹ but the results are not of sufficient interest to be cited here. The question has also been raised whether magnetization affects the elasticity of bodies, and has been answered by Wertheim and Wartmann in the negative. Both Kimball⁹ and Piazzoli¹⁰ find that the breaking tension of iron wires is increased by longitudinal magnetization; the former puts the increase at 0.9 per cent. when the wire is saturated.

This is the place to mention the so-called "magnetic Magna sounds" which accompany the magnetization and demagnetization of the strongly magnetic metals. It is now established beyond all doubt that these sounds have their origin partly at least in the mechanical strains accompanying magnetization. In many cases direct magnetic or electromagnetic action, and even electrostatic and thermal actions, concur in producing them, and it is often difficult to say how much is due to each of these several causes. This is especially to be observed where the sounds are produced by the passage of interrupted or undulatory currents through wires of the strongly magnetic metals. A full discussion of the matter belongs more properly to the subject of electric telephony; but a few notes on the history and literature of the subject may be given here.

Page¹¹ seems to have been one of the first to notice phenomena of the kind; but Joule¹² appears to have first stated clearly that magnetic-mechanical strain was a specific cause. He says that the magnetic extension in the core of an electromagnet takes place so suddenly that the shock is sensible to the touch, and is accompanied by a musical note arising from vibration in the metal. Marrian,¹³ Matteucci,¹⁴ Beatson, and Wertheim¹⁵ all took up the matter; and De la Rive¹⁶ published many investigations concerning it. In 1861 Reiss published the invention of an electric telephone for the transmission of music and speech, which depended essentially on the magnetic sounds produced by a varying current in an iron core. This instrument was the prototype of the telephone of Gray, and of the still more famous instrument of Bell, whose action, although often described as purely electromagnetic, is no doubt in part due to the magnetic strains. Among the more recent investigations on this subject may be mentioned Ferguson, *Proc. Roy. Soc. Edin.*, 1878 and 1880; Ader, *Comptes Rendus*, 1879; Du Moncel, *ib.*; Chrystal, *Nature*, vol. xxii., 1880; Hughes, *Proc. Roy. Soc. Lond.*, xxxi. and xxxii., 1881.

General Remarks.—Wiedemann has remarked with justice that most of the effects of strain upon magnetization and *vice versa* are complex. Apart from the possible admixture of direct magnetic action, we must distinguish (1) the mere disturbing effect of jarring: thus the first effect application of a mechanical stress has the same effect as a compound shock, *i.e.*, it loosens the molecules of the body, as it were, and renders them more ready to follow any inductive magnetic force, while the first effect of magnetization upon a body under stress is precisely similar, and may in fact be imitated by mechanical jarring pure and simple; (2) after-effect, whether mechanical or magnetic, the consequence of which is that the effect due to any mechanical stress or magnetizing force is affected by pre-existing stress and magnetization; (3) the proper effect of mechanical stress or magnetic force, which appears at once where one or the other is applied, and disappears when it is removed.

⁵ See Wiedemann and Villari, *l.c.*; also Gore, *Phil. Trans.*, 1874; H. and F. Strenitz, *Wien. Ber.*, 1877; Hughes, *Proc. Roy. Soc. Lond.*, 1881.

⁶ *Comptes Rendus*, 1846.

⁷ *ib.*, 1846, &c.

⁸ *ib.*, 1879.

⁹ *Sill. Jour.*, 1879.

¹⁰ *Wied. Beibl.*, 1880; see also Hoffmann, *ib.*

¹¹ *Pogg. Ann.*, 1838.

¹² Sturgeon, *Ann. El.*, 1842; *Phil. Mag.*, 1847.

¹³ *Phil. Mag.*, 1844.

¹⁴ *Wied., Galv.*, ii. § 515.

¹⁵ *A.*

¹⁶ *Comptes Rendus*, 1845; *Phil. Trans.*, 1847, &c.

¹ *Comptes Rendus*, 1847.

² *Comptes Rendus*, 1852.

³ *Pogg. Ann.*, 1858, 1859, 1860.

⁴ *Bd. ii.* § 492.

In his excellent analysis of the phenomena, Wiedemann coordinates them throughout by means of an extension of Weber's theory of "molecular magnets" (*Drehbare Molecularmagnete*). This of course involves an attempt to pass beyond the mere results of experience; and there can be no question that, on the whole, this theory explains the facts in a highly instructive and suggestive manner. The main defect in it is the multitude of assumptions and the want of clearness and definiteness in its conclusions. Thus it is sometimes not easy to see why exactly the opposite conclusion should not be drawn; and it appears hopeless to bring it to the test of a quantitative comparison with experiment.

Without entering into the ultimate causes of magnetism, we might endeavour to reduce the phenomena to the smallest number of experimental facts. Thus, assuming merely the effects of longitudinal and transverse traction upon magnetization and the magnetic extension and compression along and perpendicular to the lines of magnetization, we might explain many of the results concerning the relation between torsion and magnetization.

Let us take for example No. VI. of Wiedemann's parallel statements. In fig. 44 let the upper end of the wire be the fixed end, and let P be a point in any of the thin coaxial cylindrical shells into which the wire may be supposed divided. First suppose the wire to be circularly magnetized by the action of a downward current, the resultant magnetic force at P being in the horizontal direction PB. If now the wire be twisted in the direction of the arrow T, it acquires two axes of greatest and least magnetic susceptibility P_e and P_r . The resultant magnetic force PB being resolved along these axes will induce more magnetism along P_e than along P_r ; hence the anisotropy will cause the resultant magnetization to take the direction PB' ; it will therefore have a positive vertical component downwards, which agrees with statement VI. In fact the twisting converts the circular lines of magnetization into right-handed helices.

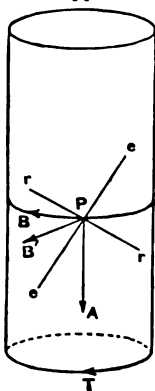


Fig. 44.

Next let us suppose the wire untwisted to begin with, but magnetized both circularly and longitudinally, the components being PB and PA. The resultant magnetization will then have some direction such as P_e , but, by Joule's principle, this will cause extension along P_e and compression along the perpendicular direction P_r ; consequently the wire will twist in the direction of the arrow T, which agrees with statement 6. Moreover, since magnetization along PB alone would simply cause the tube to expand along a horizontal section, and magnetization along PA alone would simply cause longitudinal extension, it is clear that when either A or B is given the twisting reaches a maximum and then diminishes when the other is increased.¹

It does not seem unreasonable to expect that a general mechanical theory of this kind will yet be found to co-ordinate all the facts; although there are difficulties in its way at present.² The phenomena will then be reduced to two or three experimental facts at the utmost, which it will be the business of some ultimate dynamical theory of magnetism to explain.

Effect of Temperature.—Some information on this subject has been given incidentally above, p. 256. We collect here a few additional facts; but a complete account of all that has been done could not be compressed within our available space, owing to the great diversity of opinion upon the subject. That the question is a very difficult one will appear at once, if we reflect that variations of temperature influence the density and molecular structure of magnetic bodies to a remarkable degree, and that thus secondary

influences arise in addition to the proper effect of temperature.

That very high temperatures destroy both the magnetic susceptibility and the power of retaining magnetism altogether has been known since the infancy of magnetic science. Thus Gilbert found that a loadstone and a piece of iron equally lost their power of affecting the magnetic needle when heated very hot, and remarks that the magnetic property returns to the iron after it has cooled a little, but that the magnetic virtue of the loadstone is altogether destroyed.³ Similar results were obtained by Brugmans, Boyle, Cavallo, Barlow and Bonnycastle, Christie, Ritchie, Erman, Scoresby, Seebeck, and others. Faraday⁴ found that a steel magnet lost its permanent magnetism rather suddenly at a temperature a little under the boiling point of almond oil; it behaved like soft iron till it was raised to an orange-red heat, and then it lost its magnetic susceptibility and became indifferent. The temperature at which retentive power for permanent magnetism was lost appeared to vary in steel with the hardness and structure; in fragments of loadstone it was very high: they retained their permanent magnetism until just below visible ignition in the dark, but, on the other hand, they lost their susceptibility at dull ignition, i.e., at a much lower temperature than iron. Nickel was found to lose its magnetic susceptibility at a much lower temperature than iron, viz., about 330° to 340° C.⁵ Cobalt is much more refractory, for it retains its susceptibility, according to Faraday, nearly up to the melting point of copper, i.e., to a white heat. The writer had occasion to verify these results in the course of some experiments on the magnetic sounds in wires of iron, nickel, and cobalt traversed by an interrupted current of electricity.⁶

Destruction of magnetism by very high temperature.

The effect of extreme cold, produced in the ordinary way by means of solid carbonic acid and ether, was, according to Trowbridge,⁷ to diminish the moment of a steel magnet (magnetized at 20° C.) by about 60 per cent.

Effect of extreme cold.

The effect of moderate alteration of temperature varies greatly according to circumstances. We shall consider separately the effect upon the magnetic susceptibility and upon the permanent magnetism; but it must be noticed that no such separation is possible in actual experiment.

Mode-rate variations of temperature.

The temporary magnetism of bars of cast iron, smithy iron, soft iron, soft steel, and hard steel magnetized by the earth's vertical force was found by Scoresby⁸ to be insensible at a white heat, but to be much greater at a dark red heat than at the temperature of the air. The difference was most marked in the case of hard steel, no doubt partly because of the softening of the bar. Similar experiments were made by Barlow, Seebeck, and others. Kupfer⁹ experimented on the subject using variations of temperature between 0° and 100° C., and found the susceptibility of soft iron to increase with the temperature. Wiedemann's conclusion is that the first alteration of temperature, whether increase or decrease, increases the temporary magnetism of iron or steel, whatever the temperature at starting. If the temperature be repeatedly altered and brought back to its initial value, the magnetization continues to increase, but after a time becomes more and more nearly constant at the initial temperature. After this state has been reached, an increase of temperature causes increase of magnetization in very hard steel bars, a decrease of temperature a decrease of magnetization; the behaviour of soft steel bars is exactly opposite.

Effect on magnetic susceptibility.

Wiedemann's results.

¹ According to the results of Villari and Thomson, if the magnetization were beyond a certain critical value, P_e would become an axis of compression and P_r an axis of extension, in which case the wire would twist in the opposite direction.

² See Thomson, *Phil. Trans.*, 1879, p. 73. Not the least of these arise from gaps in our experimental knowledge; e.g., regarding the effects of permanent set caused by traction and compression.

³ *De Magnete*, lib. ii. cap. 3.

⁴ *Exp. Res.*, vol. ii. p. 220, 1836.

⁵ According to Becquerel about 400° C., according to Pouillet about 350° C.

⁶ *Nature*, vol. xxii., 1880.

⁷ *Sill. Jour.*, 1881.

⁸ *Phil. Trans. Roy. Soc. Edin.*, vol. ix.

⁹ *Wied. Galv.*, ii. § 521.

Baur's
results.

Baur¹ and Wassmuth² have recently taken up the matter with all the advantages of modern experience. The former concludes from his experiments on iron by the ring method, at temperatures between 0° and 150° C., that the magnetic susceptibility for a given magnetizing force increases with the temperature if the force be below a certain critical value (3.6 or so), but decreases as the temperature increases if the force be above that value.³ The smaller the magnetizing force the greater the influence of temperature on the magnetic susceptibility. The result of his experiments at very high temperatures is that, for small magnetizing forces, the susceptibility at first increases rapidly as the temperature increases, reaches a maximum at red heat, and then falls suddenly to zero. For large forces, the susceptibility decreases gradually until red heat, and then falls suddenly to a very small value. According to him, if a bar be cooled from white heat the first traces of susceptibility are observed at a very bright red, the brighter the greater the magnetizing force. He gives a variety of interesting results concerning the phenomenon of Gore,⁴ all in accordance with what we have just stated.

Diamagnetism
of flame.

Bancalari.

Faraday's
experiments
on
heated
gases.

In his earlier researches Faraday was unsuccessful in obtaining any evidence of the influence of temperature on the susceptibility of weakly magnetic bodies, such as the chlorides of the magnetic metals or of diamagnetic bodies.⁵ His earliest results were obtained with gases, and that too, strange to say, before the magnetic character of gases was fully investigated. It was Bancalari's discovery of the extraordinary behaviour of flame between the poles of an electromagnet that led Faraday to resume his magnetic experiments on gases. Flames of all descriptions are strongly repelled from the axial line of a heterogeneous magnetic field,—so much so that it is impossible to induce the flame of a candle to go between the pointed poles of a powerful electromagnet when they are placed at a short distance apart. The flame is blown aside, or even downwards, as if by a strong current of air issuing from between the poles. If a flat pointed flame is placed with its centre a little below the axial line, when the magnet is excited it drops down and spreads out below and around the axial line, assuming a fish-tail shape. It appears that the effect is not due to the solid matter in the flame but simply to the hot gases in it; for the upper and cooler part of the stream of smoke from a freshly extinguished taper is scarcely affected, while the lower and hotter part is most powerfully acted upon, being blown aside and often split into two independent streams. A careful investigation led Faraday to the conclusion that oxygen, carbonic acid, and coal gas are rendered more diamagnetic, or, what is the same thing so far as the resultant differential action is concerned, less magnetic by heat,⁶ and that this effect was much greater than could be accounted for by the mere rarefaction of the gas. He likewise obtained an increase of the susceptibility of oxygen by cooling it with ether and solid carbonic acid. Nitrogen appeared to be altogether indifferent. He found in a later series⁷ of experiments that the magnecrystalline property of bismuth was destroyed at a temperature a little below its melting point, and that the same thing happened to crystalline antimony a little below red heat. In the thirtieth series of his experimental researches he states that between 35° and 142° C. the susceptibility of a specimen of spathic iron ore perpendicular to its magnetic axis decreased by .333 per cent. per degree centigrade of rise of temperature; this agrees very closely with the formula which was found by Wiedemann to

represent very approximately the temperature effect for salt solutions, viz., $k = k_0(1 - .00325 t)$. For the decrease in the magnecrystalline couple, or, which is the same thing, in the difference between the susceptibilities along and perpendicular to the magnetic axis, he found for the spathic iron ore .482 per cent. between 0° and 138° C., and the percentage of decrease was four times as great between -14° and 0° as between 129° and 143°. The corresponding decrease in the case of crystalline bismuth between 36° and 137° C. was .53 per cent. The experiments of Plücker and Matteucci led them to conclude that the susceptibility of diamagnetics diminishes with increase of temperature; in the case of bismuth the decrease between ordinary temperatures and its melting point is said to be about one-sixth or more.

Canton seems to have been one of the first to study the effect of moderate variations of temperature on the permanent magnetism of iron and steel. The results of his and Hallström's experiments went to show that permanent magnetization decreases when the temperature rises, and increases again when the temperature falls. In reality, however, as was shown by Kupffer, Riess and Moser, G. Wiedemann, and others, the phenomenon is complicated; for, if we repeatedly heat a magnet and allow it to cool to its initial temperature, the magnetization lost at each heating is only partially recovered on cooling, and thus a progressive loss goes on, until at last a constant state is reached, in which the magnetization lost on heating is completely recovered on cooling. In this respect, as well as in the effect on the magnetic susceptibility already discussed, there is an analogy between the effect of temperature and the effect of strain; i.e., there is a first or permanent effect and a proper or temporary temperature effect. The permanent effect is that any alteration of temperature, be it increase or decrease, diminishes the permanent magnetization just as a shock or a jar would do, and probably for a similar reason. The proper or temporary effect consists in a decrease of magnetization with increase of temperature, which is completely recovered on decrease of temperature and *vice versa*.⁸ If this be borne in mind, together with what has already been said above, it will not be difficult for the reader to see that the order and amount of the temperature variations, the hardness and form of the bar, and its magnetic history will all influence the temperature coefficient.

To take one example, Wiedemann found that a bar magnetized at 0° C. and then partially demagnetized by an opposite force, lost magnetism when heated; if the demagnetization was not carried too far, it did not when cooled again to 0° wholly recover what it had lost. If the demagnetization was carried a certain length, it recovered all that it had lost; if farther still, more than it had lost. It was in fact found possible to demagnetize a bar, so as to render it apparently unmagnetic, and then to restore part of its original magnetism by merely heating and cooling it again. Similar phenomena were observed with a bar magnetized and demagnetized at 100°, and then alternately cooled and heated. Unverdorben,⁹ who arrived somewhat later at similar results, represents the matter by saying that the bar in this case has two magnetizations superposed, each having its own temperature coefficient.

The following are a few additional references to sources of information concerning the present subject: Mauritius, *Pogg. Ann.*, 1863, and *Phil. Mag.*, 1864; Jamin and Gauguin, *Comptes Rendus*, *passim*; Favé, *ib.*, 1876; Poloni, *Wied. Beibl.*, 1878.

⁸ To give the reader an idea of the magnitude of this effect, we may mention that Whipple in determining the temperature corrections for magnetometer magnets at Kew, according to the formula

$$K_t = K_0 \{1 - q(t - t_0) - q'(t - t_0)^2\},$$

found for the coefficient q values varying from .000762 to .000044, with a mean of .000161; and for q' from .00000398 to .00000001, with a mean of .00000048 (*Proc. Roy. Soc. Lond.*, 1877).

⁹ Quoted in Lamont, *Handb. d. Mag.*, § 82. An account of Lamont's own researches will be found in the same place.

¹ *Wied. Ann.*, xi., 1880.

² *Wien. Ber.*, 1880, 1881, 1882.

³ See the results of Faraday, *Exp. Res.*, ser. xxx. 3424, &c.

⁴ *Phil. Mag.*, 1869, 1870.

⁵ *Exp. Res.*, 2359, 2397, 1845.

⁶ *Exp. Res.*, vol. iii. p. 486.

⁷ *Exp. Res.*, 2570 sq., 1848.

Development of Heat during Magnetization.—Reasoning on purely thermodynamic principles from the results of Faraday, as to the influence of temperature on the magnetic properties of bodies, Thomson¹ has concluded—(1) that a piece of soft iron at a moderate or low red heat, when drawn gently away from a magnet, experiences a cooling effect, and, when allowed to approach, a heating effect, and that nickel at ordinary temperatures and cobalt at high temperatures (between the melting point of copper and some lower temperature) experience the same kind of effect; (2) that cobalt at ordinary temperatures and up to the temperature of maximum permeability experiences a cooling effect when allowed to approach a magnet, and heating when drawn away; (3) that a crystal in a magnetic field experiences cooling when the axis of greatest paramagnetic or of least diamagnetic susceptibility is turned from along to across the lines of force, and *vice versa*.

Besides these considerations, the fact that those who adopt the molecular magnet theory are obliged to assume something of the nature of a frictional resistance to the turning of the magnetic molecules, and generally, without reference to any particular theory, many of the phenomena of coercive force,² lead us to suppose that some specific development of heat may accompany magnetization and demagnetization. The experimental verification of this suspicion is, however, a matter of great difficulty, owing to the enormous generation of heat arising secondarily from induced currents in the mass of the metal. The development caused by magnetization and demagnetization was taken advantage of by Joule in one of his determinations of the mechanical equivalent of heat, but he makes no attempt to separate the effect of the two causes, indeed it did not concern his purpose to do so.³ Notwithstanding that several experimenters have attacked the problem, it cannot be said that it is yet completely solved. It will therefore be best simply to call the reader's attention to some of the papers that have been published on the subject, and leave him to form his own judgment.

See Von Breda, *Pogg. Ann.*, 1846; Grove, *Phil. Mag.*, 1849; Edlund, *Pogg. Ann.*, 1864; Villari, *N. Cim.*, 1870; Cazin, *Comptes Rendus*, 1874; Herwig, *Wied. Ann.*, iv., 1878; Trowbridge, *Wied. Beibl.*, 1879.

Miscellaneous Relations of Magnetism to other Physical Properties.—According to Maggi⁴ the thermal conductivity of magnetized iron is less along the lines of force than across them. Naccari and Bellati⁵ were unable to verify this result; Tomlinson,⁶ however, found that the conductivity of iron and steel bars was diminished by longitudinal and increased by transversal magnetization.

Abraham, Edlund, Mousson, and Wartmann all made experiments in search of a magnetic alteration of the electric conductivity of iron. Thomson seems, however, to have been the first to arrive at any definite result.⁷ He found the conductivity to be diminished along the lines of magnetization and increased across them. Beez⁸ verified the former result, but doubts the latter, which he is inclined to explain as a secondary effect caused by the compression of the iron arising from the external magnetic action on the plates used in Thomson's experiments.

Thomson also found⁹ that the thermoelectric quality of iron was affected by magnetization; the thermoelectric current flowed from unmagnetized to longitudinally magnetized, and from transversely magnetized to unmagnetized or longitudinally magnetized iron through the

hot junction. In the case of nickel, the current flowed from longitudinally magnetized to unmagnetized through the hot junction, i.e., nickel behaved oppositely to iron. Thomson's results have been in part confirmed by a recent investigation of Strouhal and Barus.¹⁰

A relation between magnetism and light was first established by Faraday's discovery of the magnetic rotation of the plane of polarization of a ray passing along the lines of force. This subject belongs more properly to physical optics, but there is one magnetic phenomenon apparently closely connected with it which falls to be mentioned here. This is Hall's discovery¹¹ that, if an electric current flow in a thin metallic strip in a direction AB, the effect of placing the strip in a magnetic field with its plane perpendicular to the lines of force is to cause a transverse electromotive force perpendicular to AB, which changes in sign when the direction either of the current or of the magnetic field is changed. This transverse electromotive force is proportional to the product of the current intensity and the strength of the magnetic field; *ceteris paribus*, its direction in the case of iron is opposite to that in other metals, and its magnitude is also greatest with iron. This discovery establishes the existence of the rotatory coefficient of resistance mentioned by Maxwell¹² in his discussion of æolotropic conductivity; and Rowland has shown that the phenomenon is probably due to the same cause as the magnetic rotation of the plane of polarization.¹³

If, as modern physicists suppose, magnetism be a dynamical phenomenon, time must enter as a conditioning element. The question has been raised how long any magnetizing force takes to develop the maximum magnetization that it is capable of producing. There are many facts that go to prove that this time is very small, or, at all events, that any force develops a very large fraction of the total magnetization due to it in a very short period of time. Perhaps the most wonderful evidence on this head is the fact that the telephone, which depends essentially on varying magnetic action, can reproduce the sounds of human speech even to the consonants.¹⁴ Experiments bearing directly on the subject have been made by Villari.¹⁵ A flat circular disk of flint glass was placed between the poles of a Ruhmkorff's apparatus for measuring the magnetic rotation of the plane of polarization. The axis of the disk was perpendicular to the axial line, so that rotation brought the different radii successively into the line of sight. When the disk was at rest the magnetic action in one experiment caused a rotation of 19 divisions; spinning the disk at the rate of 110, 121, 143, and 180 turns per second reduced the magnetic rotation of the plane of polarization by 2, 5, 10, and 17 divisions respectively; the reduction was less the greater the magnetic force. From this Villari concluded that in flint glass not less than 0.001244 second is required to produce such a diamagnetic intensity as can be observed by the rotation of the plane of polarization, and that 0.00241 second at least is required to develop the greatest diamagnetization of which this substance is capable; he also states that the diamagnetism lasts for less than 0.00018 second after the inducing force is withdrawn. A series of interesting experiments on the oscillation of the plane of polarization caused by the oscillatory discharge from a Leyden jar recently made by Bichat and Blondlot¹⁶ led them to a different conclusion, viz., that if any lagging of the induced magnetization behind the magnetizing force

¹ *Phil. Mag.*, 1878; or Nichol's *Cyclopædia*, 2d ed., 1860.

² See the paper of Warburg quoted above, p. 260.

³ Joule, *Phil. Mag.*, 1843.

⁴ *Archives de Genève*, 1850; or *Wied. Galv.*, ii. § 510.

⁵ *Wied. Beibl.*, 1877.

⁶ *Proc. Roy. Soc. Lond.*, 1878.

⁷ *Phil. Trans.*, 1856.

⁸ *Pogg. Ann.*, 1866.

⁹ *Loc. cit.*

¹⁰ *Wied. Ann.*, xiv., 1881. ¹¹ *Phil. Mag.* [5], ix. and x., 1880.

¹² *El. and Mag.*, vol. i. § 303. See also Stokes, *Camb. and Dub. Math. Jour.*, vi., 1851; and Thomson, *Trans. R.S.E.*, vol. xxi. p. 165, 1854.

¹³ *Am. Jour. of Math.*, 1880; *Phil. Mag.* [5], x., 1880, and xi., 1881.

¹⁴ See also an article by the writer, *Phil. Mag.* [5], ii. 1876.

¹⁵ *Pogg. Ann.*, 1879. ¹⁶ *Comptes Rendus*, 1882.

Direct
influence
of light
upon
magnet-
ization.

exists it is less than 0.000033 second. No explanation has been given of the discrepancy of these results.

In the early part of this century there was an animated controversy as to whether light exerted a direct influence upon magnetization, in which Morichini, Mrs Somerville, Christie, Riess and Moser, and many others took part. Nothing definite, however, was established. A similar fate befell the attempts to trace the influence of magnetic force upon crystallization, and to detect a relation between magnetism and gravity,¹ although both quests at one time or another engaged the skill of Faraday.

FORMS, CONSTRUCTION, AND PRESERVATION OF MAGNETS.

This subject occupied a large portion of most of the earlier treatises on magnetism. Much of the information given, however, either has now been recognized to be of questionable value or has been superseded by recent progress, and retains a merely antiquarian interest; a few brief remarks, mainly historical, will therefore be sufficient.

Load-
stones.

The oldest form of magnet was a piece of magnetic iron ore or loadstone. The power of these natural magnets varied exceedingly from one specimen to another. An elaborate discussion of the various kinds of loadstone will be found in Gilbert's *De Magnete*.² In order to increase the carrying power, the loadstone was usually fitted with armatures of soft iron upon its polar regions; figure 45, taken from Gilbert, represents one of the oldest arrangements.

Figure 46 is taken from a loadstone in the collection of physical apparatus belonging to the university of Edinburgh, the carrying power of which is 205 lb. A loadstone in the Teylerian Museum at Haarlem has a carrying power of 230 lb; and one at Lisbon, pre-

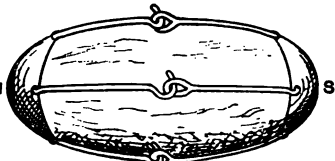


Fig. 45.

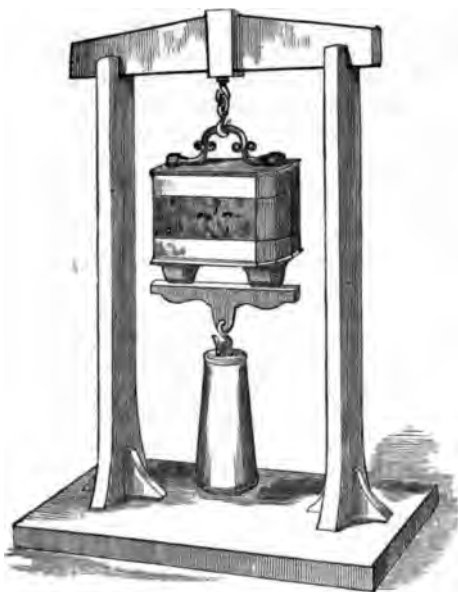


Fig. 46.

sented by the emperor of China to King John V. of Portugal, is said to support as much as 300 lb. Small loadstones are often very powerful in proportion to their weight; e.g., Newton is said to have worn in a ring one that weighed only 3 grains, and yet was able to carry about 746 grains; and one in the physical collection at Edinburgh, formerly belonging to Sir John Leslie, weighing itself 3½ grains, had at one time a carrying power of 1560 grains.

Steel
magnets.

The introduction of steel magnets, and the perfection to which they were gradually brought, caused the loadstone to fall into disuse. It is said that Galileo possessed the art of making steel magnets about the beginning of the 17th century. It was early discovered that the earth's force could be utilized in magnetizing steel.

¹ For the literature, see Wied., *Galv.*, §§ 688, 689.

² See also Gehler's *Physikalisches Wörterbuch*, art. "Magnetismus."

Gilbert was aware that a feeble magnetization could be produced in this way; and Michell, in his treatise on artificial magnets, minutely describes how weak magnets may be made by means of the earth's force, then combined into bundles or "magazines" and used in turn to produce stronger magnets, these used to produce still stronger, and so on.

The earliest process of all was no doubt the method of rubbing or touching by another magnet. This method of making magnets was studied with much attention by the natural philosophers of the 18th century, among whom we may mention Savery, Knight, Duhamel, Le Maire, Canton, Michell, Äpinus, Coulomb, and Euler. The method of single touch consists simply in stroking the bar to be magnetized alternately toward its two halves with the south and north poles of a loadstone or bar magnet, the stroke beginning always at the middle and ending at the end. According to Lamont, the best plan is to lay the magnet flat, overlapping one half of the bar to be magnetized, and then draw it off; when the magnet is held perpendicular to the bar during the process, the result is apt to give an irregular magnetization: e.g., we may even get a magnet with its two ends north poles and with a south pole in the middle, or one with four poles, a north and south pole at the two ends and a south and north pole in the middle.³

The first improvement on single touch was double touch with two separate magnets. This consists in using two magnets simultaneously on the two halves of the bar undergoing magnetization. The north pole of one and the south pole of the other are placed either close together, or at a small distance apart near the middle of the bar, and then each is drawn towards the end of the half on which it lies; according to Lamont, here, as in single touch, the magnets should be laid flat on the bar.⁴ Michell introduced the further improvement of using two bar magnets (or bundles of such) fastened together and kept parallel at a small distance apart by means of small pieces of wood, the north pole of one being continuous with the south pole of the other. This pair is placed vertical with one end on the middle of the bar, drawn towards one end and slipped off, then replaced on the middle and drawn to the other end, and so on alternately until the moment of the bar ceases to increase any farther. Instead of the pair of bar magnets a horse-shoe magnet might of course be used.

Le Maire⁵ introduced the essential improvement of placing the bar to be magnetized upon a larger bar, and then magnetizing the two together. The advantage of this is best seen in the form of the same device adopted by Canton⁶ and Duhamel,⁷ who magnetized steel bars in pairs, connecting them up parallel to each other by means of two pieces of soft iron, and then magnetizing them in opposite directions. It is easy to see that the magnetization of the one reacts on the magnetization of the other and strengthens it. Michell⁷ obtained a similar advantage by magnetizing a number of bars placed end to end in a line; he found, as was to be expected, that the end bars were weaker, but this defect he remedied by repeating the process with the bars arranged in a different order. Coulomb's method was to place the ends of the bar on the north and south poles of two bar magnets arranged in line at the proper distance apart. This process of connecting up the bars to be magnetized in a closed magnetic circuit is sometimes called circular touch; it can be applied to horse-shoe magnets by placing a pair of them with their ends together, and then passing round and round upon them a horse-shoe magnet or a pair of bar magnets arranged as already described.⁸

Immediately after Ørsted's discovery of the magnetic action of the galvanic current, Arago,⁹ Boisgiraud,¹⁰ and Davy almost simultaneously applied this property to the magnetization of iron and steel.¹¹ Powerful electromagnets, with cores of soft iron, were first constructed a few years later by Sturgeon and Brewster. Pohl, Moll, and Pfaff in Germany, and Henry and Ten Eyck in America, may be mentioned as the most successful of the early constructors. One of the electromagnets of Henry and Ten Eyck reached a carrying

³ Poles situated abnormally in this way are called "consecutive points."

⁴ This method appears to have been invented by Knight (about 1740), and used in producing the powerful magnets for which he was famous. The secret of his process was never divulged by himself, but was published by Wilson after his death. See art. MAGNETISM, 8th edition of *Encyclopædia Britannica*.

⁵ *Mém. d. l'Acad. d. Paris*, 1745 and 1750.

⁶ *Phil. Trans.*, 1751.

⁷ *Treatise of Artificial Magnets*, 1750.

⁸ For fuller information on the present subject, see Gehler's *Physikalisches Wörterbuch*, art. "Magnetismus," xv.

⁹ *Ann. d. Chim. et d. Phys.*, 1820.

¹⁰ *Phil. Trans.*, 1820-21.

¹¹ The anomalous magnetization of needles by the discharge from Leyden jars had been observed earlier, but not properly understood. See art. ELECTRICITY, vol. viii. p. 82.

power of 2061 lb; but magnets specially constructed for carrying power have surpassed this limit. As a specimen of scientific toys of this description may be mentioned the electromagnet of Roberts (fig. 47), which consists of a square block of iron deeply slotted with four parallel grooves into which three layers of copper wire cable are wound in zigzag fashion so that the current converts the flanges alternately into north and south poles; the armature is a square block planed to fit the face of the magnet. The carrying power of a machine of this kind was 2949 lb, i.e., more than 1½ tons!

The forms of electromagnet used in the arts, e.g., in electric bells, fire alarms, telegraphs, telephones, electric light regulators, dynamo machines, &c., are simply innumerable. It will be sufficient to allude to those constructed for the purpose of producing an intense magnetic field, uniform or non-uniform, over a larger or smaller area; these find their practical application in the construction of dynamo-electric machines, but they are mainly interesting to purely scientific men on account of their use in the investigation of the properties of weakly magnetic bodies. Figure 40 shows the usual arrangement adopted for large laboratory magnets. In considering the greatest available strength of such magnets, it is necessary to bear in mind the fact that magnetic saturation of iron is practically reached with magnetic forces much under the greatest that we can command. The strength of field in a narrow crevasse perpendicular to the lines of magnetization in saturated iron is less than 18,000 C.G.S. units;¹ and this is practically the utmost at present attainable, for any addition to the strength of the field, arising from direct action of the magnetizing helix, would not under ordinary circumstances affect the hundreds in this number. Further increase of magnetizing current after we have reached within a small percentage of the limit of saturation is a waste of power.

Elias² of Haarlem seems to have been the first who applied the electric current directly with success in the manufacture of powerful permanent magnets. He used a short flat magnetizing coil which was pushed backwards and forwards along the bar, the ends of which were caused to abut against two pieces of iron, which becoming inductively magnetized reacted on the bar, and also served to keep the magnetization at the ends more uniform. The famous Logeman magnets were constructed by this process.

By far the most convenient way of magnetizing steel is to use an electromagnet.³ The bar to be magnetized may be laid flat on the pole of the magnet before it is excited, and after excitation drawn slowly off. By repeating this process several times, with the north pole of the electromagnet for one half and the south pole for the other half, saturation can be very quickly obtained. Perhaps a better plan is to lay the bar with its ends on the two poles, and then excite the electromagnet. For reasons already sufficiently explained, it is advisable to hammer the bar with a mallet while the magnetizing force is in action, and to turn the current off and on several times in succession.⁴

On account of the difficulty of tempering steel to any great depth from the surface, and for specific magnetic reasons as well, it has been customary in constructing powerful permanent magnets to build them up of thin laminæ of steel, each of which is separately magnetized. Figure 48 represents an arrangement of this nature

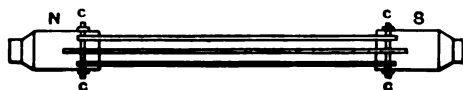


Fig. 48.

adopted by Coulomb, and figure 49 a horse-shoe magnet constructed in the same way. It will be observed that the ends of the laminæ are not exactly continuous, the middle ones projecting more than the others; this arrangement was adopted with the view of getting rid to some extent of the weakening effect which the induction of one lamina has upon the other. That such an effect exists and is very great was conclusively shown by Coulomb; how far the modification

¹ See above, p. 256.

² *Pogg. Ann.*, 1844 and 1846.

³ Frick, *Pogg. Ann.*, 1849, was one of the earliest who practised this method.

⁴ It has been several times proposed to magnetize steel bars by heating them red hot, allowing them to cool to the proper temperature under the magnetizing force, and then tempering while the force is still acting. Gilbert, Knight, Robison, Haman, Gauguin, Aimé, and Holz (*Wied. Ann.*, vii., 1879) have all experimented with this method, but it does not appear to possess any advantage over the ordinary modern process, and need not be discussed here.

in question cures it is another matter; much no doubt depends on the purpose for which the magnet is required; but it is scarcely worth while to discuss the subject here. We may call attention to a farther point in the construction of Coulomb's magnet, viz., that the ends of the laminæ are embedded in two soft iron terminals N and S; there can be no doubt that, for some purposes at least, this is an advantageous arrangement. Among the famous modern makers of permanent magnets Häcker of Nuremberg, Logeman and Wetteren of Haarlem,⁵ Willward, and Jamini deserve to be specially mentioned.⁶

In the preservation of permanent magnets it is essential to avoid extreme changes of temperature and shocks. When the magnet is laid aside it should be made part of a closed magnetic circuit; in the case of a horse-shoe magnet this is attained by simply laying a piece of soft iron, called the keeper, across the poles; bar magnets should be kept in parallel pairs, north pole to south pole and south pole to north pole, with two pieces of soft iron between the poles. When this is done the induced magnetism reacts on the magnets and diminishes the demagnetizing force; the action of shocks then ceases to destroy the permanent magnetism, and may even increase it.

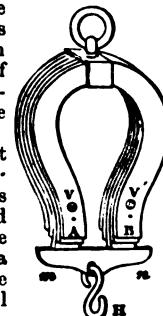


Fig. 49.

Preservation of magnets.

ULTIMATE THEORIES OF MAGNETIC PHENOMENA.

If we pass over the stream theory, which, although Stream theory. partially developed by Euler, has never taken root in modern physical science, the first great theory that we find proposed with a view to the explanation of magnetism is the two-fluid theory of Coulomb and Poisson. This is not an ultimate theory in the modern sense, inasmuch as it is not dynamical; but it was, doubtless, looked upon as ultimate in the days when the imponderable fluids had a recognized role in the physical sciences. In the two-fluid Two-fluid theory. theory the imaginary positive and negative attractive agents (called magnetism in the empirical theory developed above) are regarded as imponderable fluids; but the essential point in the definite form of the theory due to Poisson is that he regards a body susceptible to magnetic induction as made up of an infinite number of particles of infinite permeability immersed in an impermeable medium. After pointing out that, if the particles were of elongated form, and arranged so that the axes of elongation had one preponderating direction, or if they were arranged so that the linear density in different directions varied, the result would be anisotropy, he assumes that they are spheres uniformly distributed in the impermeable medium so that the volume of the magnetic particles in unit volume of the substance is the fraction k . The problem of magnetic induction under the influence of a uniform force is then the same as the problem of electric induction for an infinite number of perfectly conducting spheres uniformly distributed in a non-conducting medium. He finds for the permeability $\mu = (1 + 2k)/(1 - k)$.

Maxwell has pointed out one fundamental objection to this theory, viz., that the value of k calculated from the formula just given by means of observed values of μ in the case of iron is greater than it would be even if the magnetic spheres were packed in the closest possible manner. Another objection is that the theory affords no explanation of the variability of k with different forces. We might of course modify the hypothesis, as was done by Pliicker, by supposing that a resistance depending on the magnitude of the force opposes the separation of the fluids in the magnetic molecules, and that in certain cases a frictional resistance tends to prevent their reunion. We might in this way explain magnetic saturation and permanent magnetism; but the theory thus burdened has no more scientific value than the purely empiric theory, and, moreover, affords no clue to the phenomena of diamagnetism.

⁵ Advised by Elias and Van der Willigen; see *Nature*, vol. xix. p. 552, 1879.

⁶ Further details as to the advantages and disadvantages of various forms of magnets will be found in Wiedemann's *Galvanismus*, and Lamont's *Handbuch des Magnetismus*. See also a recent paper by W. Holz, *Wied. Ann.*, 1880, on hollow cylindrical magnets, and another by Gray on the moments attainable with hard steel bars, *Phil. Mag.*, 1878.

Molecular magnet theory.

In a very important class of modern theories, the fundamental assumption is that the molecules, or at all events a certain proportion of the molecules, of magnetic substances are small permanent magnets. In a body which is to outward appearance unmagnetized, the axes of these molecular magnets are turned indifferently in all directions; in a body which is magnetized in a certain direction a larger proportion than usual of the molecular magnets have their axes more or less in that direction. Magnetic induction is supposed to consist, not in any alteration of the molecular magnets themselves, but in the orientation of their axes under the action of the inducing force. The reader may figure to himself the nature of the action by imagining a line of small magnetic needles with their axes all horizontal, but all pointing in different directions; the whole system thus arranged will have no determinate magnetic moment, and will represent an unmagnetized body. Next, suppose a magnetizing force to act parallel to the line joining the centres of the needles, they will then arrange themselves in that line, and the magnetic moment of the system will be the sum of the moment of the different parts; we have thus an image of a body magnetized by induction.

Weber's form.

The notion of molecular magnets seems to have been suggested by Kirwan; but it was not until a definite form was given to it by Weber that it acquired any importance. The mathematical problem presented is one of great complexity. In the position of equilibrium any molecule is acted on by the magnetizing force, by a magnetic force due to the combined action of the other molecules, and possibly by a force arising from the displacement as well. Weber assumes that the couple tending to restore the molecule to its original position is that due to a constant magnetic force D , parallel to the original direction of its axis. If m be the magnetic moment of a molecule, and there be n molecules in a unit of volume, then the magnetic intensity \mathfrak{H} due to the magnetizing force \mathfrak{H} is given by $\mathfrak{H} = 2mn\mathfrak{H}/3D$, if $\mathfrak{H} < D$; and by $\mathfrak{H} = mn(1 - D^2/3\mathfrak{H}^2)$, if $\mathfrak{H} > D$. In other words, the curve (\mathfrak{H} , \mathfrak{H}) is straight till it reaches the point ($\frac{3}{2}mn$, D), it then becomes concave towards the axis of \mathfrak{H} , and rises towards an asymptote parallel to the axis of \mathfrak{H} ; the maximum value of \mathfrak{H} is mn . The theory does, therefore, give a general explanation of the phenomena of magnetic induction. The reader will be able by comparison with the experimental data given above to see how far it falls short of a complete explanation.

Maxwell's form.

If the magnetic substance be devoid of coercive force, we must suppose that the molecules return to their original positions when the magnetizing force is removed. In substances capable of being permanently magnetized, we must imagine something of the nature of a frictional resistance to the motion of the magnetic molecules; so that, when they are deflected through more than a certain angle, they retain a permanent set after removal of the magnetizing force. Maxwell has worked out the particular hypothesis that each molecule which is deflected through an angle less than β_0 returns when the magnetizing force ceases to act, but that a molecule deflected through an angle $\beta > \beta_0$ retains the deflexion $\beta - \beta_0$. Denoting $D \sin \beta_0$ by L , he finds as the result of the above supposition that the curve of temporary magnetization is a straight line from $\mathfrak{H} = 0$ to $\mathfrak{H} = L$; after that it is concave to the axis of \mathfrak{H} , and rises to an asymptote, the maximum value of \mathfrak{H} being mn as before. The curve of residual magnetization begins when $\mathfrak{H} = L$; it is concave to the axis of \mathfrak{H} , and rises to an asymptote corresponding to the maximum $\mathfrak{H} = \frac{1}{2}mn\{1 + \sqrt{1 - L^2/D^2}\}$. It results from the hypothesis that, when a bar is permanently magnetized by a positive force \mathfrak{H}_1 , its magnetism cannot be increased by a positive force $< \mathfrak{H}_1$, but may be diminished by a negative force $< \mathfrak{H}_1$; and, when the bar is exactly demagnetized by a negative force \mathfrak{H}_2 , it cannot be magnetized in the opposite direction without the application of a force $> \mathfrak{H}_2$; but a positive force $< \mathfrak{H}_2$ is sufficient to begin to remagnetize the bar in the original direction.

Ampère's hypothesis.

Behind the molecular magnet theory there arises the question, What is the nature of the magnetic molecule? One answer to this question is given by the hypothesis of Ampère, that around each such molecule a current circulates in planes perpendicular to the axis of the molecule. That such an arrangement will be equivalent to an infinitely small magnet in the axis of the molecule, so far as external action is concerned, we know from the laws of electrodynamics. It remains only to inquire what the nature and properties of these molecular currents must be, to trace the full logical

consequences of the assumption, and to compare them with experience. This was first done by Weber, and afterwards more completely by Clerk Maxwell.

It is obvious in the first place that the circuits in which the molecular currents flow must be perfectly conducting; for otherwise the electrokinetic energy of the molecular currents would be continually transformed into heat, and a constant supply of energy from without would be necessary to support the magnetism of a permanent magnet, which is contrary to experience. Let A be the effective area of a molecular circuit, L its coefficient of self-induction, θ the inclination of its axis to the inducing force \mathfrak{H} , γ the primitive current, and γ' the current after the inducing force is in action. Then $\gamma' = \gamma_0 - \mathfrak{H}A \cos \theta / L$; and the component of the moment parallel to \mathfrak{H} will be $A(\gamma_0 - \mathfrak{H}A \cos \theta / L) \cos \theta$. There are three different cases to consider.

1. Let either γ_0 be so great, or $\mathfrak{H}A/L$ be so small, that the effect due to the electromagnetic induction may be neglected in comparison with the effect due to the deflexion of the molecule; putting $m = A\gamma_0$, we have thus merely the theory of molecular magnets already explained.

2. Let the force resisting the turning of the molecules be infinitely great, we then find for the magnetic susceptibility the value $\kappa = -\frac{1}{2}nA^2/L$. This is the theory originally proposed by Weber to explain diamagnetism.

3. If the effects due to deflexion of the molecules and to electromagnetic induction in the molecular circuits be both considered, we have a theory intermediate to (1) and (2), inclining to the one or the other according to the assumptions made as to the relative values of γ_0 , A , and L .

The reader will find a full discussion of the different cases in Maxwell's *Electricity and Magnetism*, vol. ii. chap. xxii.

The most important attempt that has yet been made to realize a mechanism affording a dynamical explanation of magnetic phenomena is the theory of molecular vortices, published by Clerk Maxwell in the *Philosophical Magazine* for 1861 and 1862 (4th ser., vols. 21 and 23). The general results, stripped of all particular assumptions, will be found embodied in his great treatise on *Electricity and Magnetism*; but the following summary, taken from the original paper, may be of some interest.

1. Magnetolectric phenomena are due to the existence of matter under certain conditions of motion or of pressure in every part of the magnetic field. The substance producing these effects may be a certain part of ordinary matter, or it may be an æther associated with matter.

2. The condition of any part of the field through which lines of magnetic force pass is one of unequal pressure in different directions, the pressure being least along the lines of force, so that they may be considered as lines of tension.

3. This inequality of pressure is due to vortices coaxial with the lines of force. The density of the revolving matter is proportional to the magnetic permeability of the medium. The direction of rotation is related to the direction of the line of force; and the velocity at the circumference of the vortex is proportional to the resultant magnetic force.

4. The vortices are separated from each other by a single layer of round particles; so that a system of cells is formed, the partitions being layers of these particles, and the substance of each cell being capable of rotating as a vortex.

5. The particles forming the layer are in rolling contact with both the vortices which they separate, but do not rub against each other. They are perfectly free to roll between the vortices and so to change their place, provided they keep within one complete molecule of the substance; but in passing from one molecule to another they experience resistance and generate irregular motions which constitute heat. These particles play the part of electricity. Their motion of translation constitutes an electric current; their rotation serves to transmit the motion of the vortices from one part of the field to another; the tangential pressures thus called into play constitute electromotive force; and the elastic yielding of the connecting particles constitutes electric displacement.

Maxwell deduces without difficulty all the principal electrical and magnetic phenomena from this theory; and he points out that its general conclusions have a value which does not depend upon the somewhat intricate kinematical arrangements supposed to exist in the magnetic medium. The theory certainly affords us a most instructive dynamical picture of the phenomena of electricity and magnetism; and it remains, so far as we know, the only successful attempt of its kind. (G. CH.)

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